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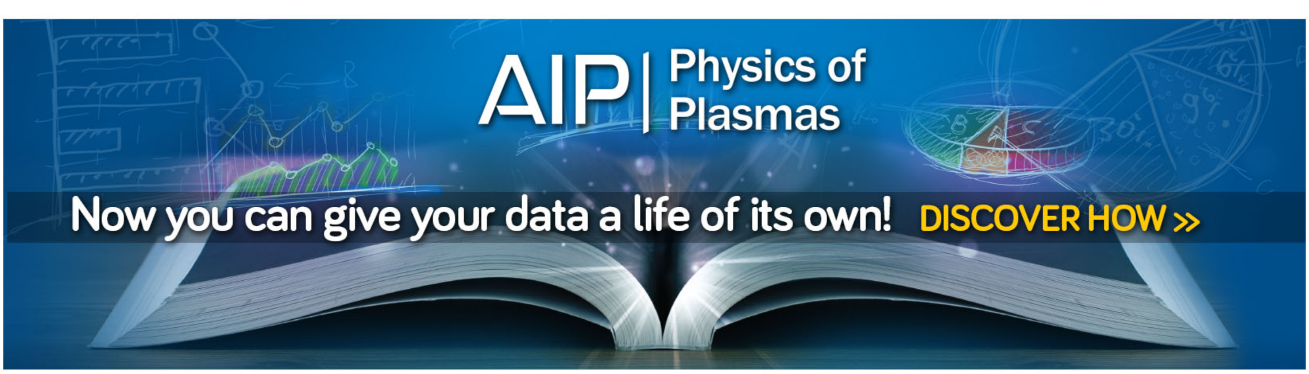

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Nonlinear low frequency electrostatic structures in a magnetized two-component auroral plasma

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Finite amplitude nonlinear ion-acoustic solitons, double layers, and supersolitons in a magnetized two-component plasma composed of adiabatic warm ions fluid and energetic nonthermal electrons are studied by employing the Sagdeev pseudopotential technique and assuming the charge neutrality condition at equilibrium. The model generates supersoliton structures at supersonic Mach numbers regime in addition to solitons and double layers, whereas in the unmagnetized two-component plasma case only, soliton and double layer solutions can be obtained. Further investigation revealed that wave obliqueness plays a critical role for the evolution of supersoliton structures in magnetized two-component plasmas. In addition, the effect of ion temperature and nonthermal energetic electron tends to decrease the speed of oscillation of the nonlinear electrostatic structures. The present theoretical results are compared with Viking satellite observations. © *2016 AIP Publishing LLC*.
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I. INTRODUCTION

The study of energetic particles has experienced significant progress, due to its important role in studying nonlinear electrostatic fluctuations phenomena in space and astrophysics plasmas. The Vela satellite mission reported observations of excess energetic particles near the Earth's bow shock region[1] and around the Earth's foreshock.[2] Freja and Viking satellite observations[3–5] also confirmed the presence of excess energetic particles in the auroral plasmas. Plasmas with excess energetic or superthermal electrons deviated significantly from standard Maxwellian behavior, due to the presence of accelerated particles attributed to the solar radiation.[6]

Cairns *et al.*[7] presented a non-Maxwellian energy distribution model for the density of hot electrons to describe the nonlinear electrostatic structures observed by Freja and Viking satellites. Their analysis predicted the possibility of obtaining both positive and negative potential soliton structures in an unmagnetized auroral plasma. Using small amplitude theory, Carins *et al.*[8] revisited the Freja and Viking spacecraft measurements in the auroral region to investigate the existence of nonlinear low frequency electrostatic structures in a magnetized plasma. The plasma model is composed of an adiabatic warm ions fluid and nonthermal energy distributed hot electron species. Later, Mamun[9] showed the coexistence of compressive and rarefactive ion-acoustic solitary waves in an unmagnetized plasma consisting of fluid dynamics warm ions and nonthermal electron species. Gill *et al.*[10] applied the reductive perturbation technique to investigate nonlinear small amplitude ion-acoustic solitons and double layers in an unmagnetized plasma of positive and negative ions with nonthermal electron species. Choi *et al.*[11] investigated the existence condition of compressive and rarefactive soliton and double layer structures for nonlinear low frequency waves in nonthermal plasmas with heavy ion species. Pakzad[12] studied the existence of rarefactive and compressive ion-acoustic solitons in unmagnetized plasmas composed of warm ions fluid, energetic nonthermal hot electrons, and Boltzmann distributed positrons.

Recently, Dubinov and Kolotkov[13] introduced the concept of nonlinear electrostatic supersolitons in unmagnetized four component plasmas. Verheest *et al.*[14] showed the possibility of obtaining nonlinear supersoliton structures in unmagnetized three-component nonthermal plasmas. In two-temperature electrons (Boltzmann and Kappa distributions), Verheest *et al.*[15] investigated the characteristic and the existence domain of ion-acoustic supersoliton structures in unmagnetized plasmas. Later, Verheest *et al.*[16] used different types of energetic electron velocity distributions to study nonlinear electrostatic structures in unmagnetized two-component auroral plasmas. They concluded that no supersoliton solutions could be found in two-component unmagnetized plasmas. Rufai *et al.*[19] reported the existence of supersoliton solutions in a magnetized three-component plasma consisting of cold ion fluid with Boltzmann cool electrons and nonthermal energetic hot electron species. In three-component unmagnetized plasma model composed of Boltmann distribution electrons, energetic nonthermal hot electrons, and fluid cold ions, Singh and Lakhina[20] described the properties of the supersoliton electric field structures which have twist on their wings. These structures have more distorted electric field potential than the convectional solitons/double layers and their Mach number values are found to exist beyond the double layer solutions. Using the auroral satellite data, Rufai[21] predicted the possibility of obtaining

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supersoliton solutions in a magnetized plasma model composed of two-ion species and nonthermal hot electrons.

In this paper, a magnetized two-component plasma model consisting of warm adiabatic ions fluid and nonthermal distributed electron species is considered. The present model is an extension of the earlier work of Mamun[9] to investigate the effect of the obliqueness of the waves on finite amplitude nonlinear low frequency electrostatic structures in the auroral plasmas. For the first time in two-component magnetized plasmas, we are announcing here the existence of obliquely propagating ion-acoustic supersoliton structures in addition to solitons and double layers in an auroral plasma, contrary to the unmagnetized plasma case studied by Verheest *et al.*[16] This paper is organized in the following sequence. In Section II, the theoretical model is presented and the localized solution of nonlinear structures is derived using the Sagdeev pseudo-potential approach. The nonlinear characteristics are discussed in detail in Section III and numerical results in Section IV. Finally, possible applications to the satellite observations in the auroral regions are presented.

II. THEORETICAL MODEL

We consider finite amplitude low frequency waves, propagating in the (x, z)-plane, in a collisionless, homogeneous and magnetized two-component auroral plasma composed of energetic nonthermal electrons (N_e, T_e) and adiabatic warm ions fluid $(N_i, T_i \neq 0)$ in the presence of an external static magnetic field $(\mathbf{B_o} = B_o\hat{z})$ along the z-direction. The phase velocity of the ion-acoustic wave is assumed much less than the thermal velocity of hot electrons, i.e., $\frac{\omega}{k} \leq v_e$, where $v_e = (T_e/m_e)^{1/2}$ is the thermal velocity of electrons, T_e is the electron temperature, and m_e is the electron mass. The velocity distribution for the hot electron species is governed by the Cairns *et al.*[7] distribution function

$$f_e(v) = \frac{N_{e0}}{(3\alpha+1)\sqrt{2\pi v_e^2}}\left(1+\frac{\alpha v^4}{v_e^4}\right)\exp\left(-\frac{v^2}{2v_e^2}\right), \quad (1)$$

where N_{e0} is the hot electron density, $v_e = \sqrt{T_e/m_e}$ is the thermal speed of the hot electrons, and α is a parameter, which determines the population of energetic nonthermal electrons. By integrating Equation (1) over the velocity space v after replacing v^2/v_e^2 by $v^2/v_e^2 - 2e\phi/T_e$ gives an expression for the electron density of the nonthermal electron as

$$N_e = N_{e0}\left[1-\beta\left(\frac{e\phi}{T_e}\right)+\beta\left(\frac{e\phi}{T_e}\right)^2\right]\exp\left(\frac{e\phi}{T_e}\right), \quad (2)$$

where $\beta = \frac{4\alpha}{1+3\alpha}$ and ϕ is the electrostatic potential. The Maxwell-Boltzmann equilibrium can be obtained in the limit $\alpha \to 0$.

For the adiabatically heated ions (temperature $T_i \neq 0$), the dynamics are governed by the fluid continuity, momentum, and pressure equations

$$\frac{\partial N_i}{\partial t} + \nabla.(N_i\mathbf{V_i}) = 0, \quad (3)$$

$$\left(\frac{\partial}{\partial t}+\mathbf{V_i}.\nabla\right)\mathbf{V_i} = -\frac{e\nabla\phi}{m_i} + e\frac{\mathbf{V_i}\times\mathbf{B_0}}{m_i c} - \frac{1}{N_i m_i}\nabla.P_i, \quad (4)$$

$$\frac{\partial P_i}{\partial t} + \mathbf{V_i}.\nabla P_i + \gamma P_i.\nabla\mathbf{V_i} = 0. \quad (5)$$

Here, N_i, $\mathbf{V_i}$, and m_i are the number density, fluid velocity, and mass of the ions, e is the magnitude of the electron charge, c is the speed of the light in vacuum, and the ion pressure p_i is given by the balance pressure equation (5). Then, for the adiabatic ions, the ion pressure can be written as

$$p_i = p_{i0}\left(\frac{N_i}{N_{i0}}\right)^\gamma, \quad (6)$$

where $\gamma = \frac{(N+2)}{N}$ is the specific heat ratio (for N degrees of freedom). For a magnetized adiabatic ions plasma, $N=3$. Therefore, $\gamma = \frac{5}{3}$ and the ion pressure at equilibrium is $p_{i0} = N_{i0}T_i$.

At equilibrium, the quasi-neutrality condition is given by $N_{i0} = N_{e0} = N_0$. Then, the densities are normalized by N_0, velocity is normalized by the ion-acoustic speed $c_s = (T_e/m_i)^{1/2}$, $\psi = e\phi/T_e$ is the normalized electrostatic potential, distance by effective ion Larmor radius, $\rho_i = c_s/\Omega$, time t normalized by inverse of ion gyro-frequency Ω^{-1} $(\Omega = eB_0/m_i c)$, and the temperature ratio $\sigma = T_i/T_e$.

The normalized form of Equations (2)–(5) are

$$n_e = n_{e0}(1-\beta\psi+\beta\psi^2)e^\psi, \quad (7)$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i v_x)}{\partial x} + \frac{\partial(n_i v_z)}{\partial z} = 0, \quad (8)$$

$$\frac{\partial v_x}{\partial t} + \left(v_x\frac{\partial}{\partial x}+v_z\frac{\partial}{\partial z}\right)v_x = -\frac{\partial\psi}{\partial x} + v_y - \frac{\sigma}{n_i}\frac{\partial}{\partial x}.(n_i)^{5/3}, \quad (9)$$

$$\frac{\partial v_y}{\partial t} + \left(v_x\frac{\partial}{\partial x}+v_z\frac{\partial}{\partial z}\right)v_y = -\frac{\partial\psi}{\partial y} - v_x, \quad (10)$$

$$\frac{\partial v_z}{\partial t} + \left(v_x\frac{\partial}{\partial x}+v_z\frac{\partial}{\partial z}\right)v_z = -\frac{\partial\psi}{\partial z} - \frac{\sigma}{n_i}\frac{\partial}{\partial z}.(n_i)^{5/3}. \quad (11)$$

For a low frequency study, the quasi-neutrality condition is defined as

$$n_i = n_e = (1-\beta\psi+\beta\psi^2)e^\psi, \quad (12)$$

which justified the assumption that the wavelength is much longer than the Debye length λ_D.

The linear dispersion relation for the above set of equations can be obtained from harmonic oscillations varying as $e^{i(k_x+k_z-\omega t)}$; $k_x = k\sin\theta$ and $k_z = k\cos\theta$, i.e., the wave vector $\mathbf{k}$ makes an angle θ with the magnetic field $\mathbf{B_0}$, and $\frac{\partial}{\partial t} \to -i\omega$, $\frac{\partial}{\partial x} \to ik_x$, $\frac{\partial}{\partial z} \to ik_z$. Using the quasi-neutrality condition in Equation (12), we obtain

$$\frac{\omega^2}{k^2} = \frac{T_e}{m_i}\left(\frac{3+5\sigma(1-\beta)}{3(1-\beta)}\right)\left[\frac{1-\frac{\Omega^2}{\omega^2}\cos^2\theta}{\left(1-\frac{\Omega^2}{\omega^2}\right)}\right]. \quad (13)$$

Equation (13) is the dispersion relation for small amplitude ion-cyclotron and ion-acoustic waves in a magnetized plasma consisting of adiabatic warm ions and nonthermal hot electrons. For low frequency waves in a magnetized plasma ($\omega \ll \Omega\cos\theta$), Equation (13) becomes

$$\frac{\omega}{k} \approx c_s \cos\theta \left(\frac{3+5\sigma(1-\beta)}{3(1-\beta)}\right)^{1/2}. \tag{14}$$

For $\sigma = 0$, the dispersion relation equation (14) gives the cold ions temperature limit

$$\frac{\omega}{k} \approx c_s \cos\theta \left(\frac{1}{1-\beta}\right)^{1/2}. \tag{15}$$

Also, for the Boltzmann electron limit, that is, $\alpha = \beta = 0$, the linear dispersion relation equation (14) reduces to

$$\frac{\omega}{k} \approx c_s \cos\theta \left(1+\frac{5\sigma}{3}\right)^{1/2}. \tag{16}$$

Further, the dispersion relation reduces to that obtained by Choi *et al.*[24] in the absence of adiabatic hot ion temperature ($\sigma = 0$) and at the Maxwellian equilibrium from the electron species. The above dispersion relation in Equations (14)–(16) describes the linear structures of the low frequency electrostatic waves in a magnetized two-component plasma.

For the nonlinear localized structures, the ion fluid equations (7)–(12) are transformed in terms of the coordinate $\xi = (\mu x + \delta z - Mt)/M$, for convenience, the Mach number[26,27] is simply defined as $M = V/c_s$, where $V = \omega/k$ is the wave phase speed, $\mu = \sin\theta$, $\delta = \cos\theta$; θ is the angle between the direction of the wave propagation and the magnetic field $\mathbf{B_0}$. Applying the appropriate boundary conditions for solitary wave structures (namely, $n_i \to 1$, $\psi \to 0$, and $d\psi/d\xi \to 0$ at $\xi \to \pm\infty$), we can reduce Equations (7)–(12) to

$$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2 + V(\psi, M) = 0, \tag{17}$$

where $V(\psi, M)$ is the Sagdeev potential given by

$$\begin{aligned} V(\psi,M) = &-\left[1-\left(\frac{M^2}{n_i^3}-\frac{5\sigma}{3n_i^{1/3}}\right)((e^{\psi}+\beta(\psi^2+\psi-1)e^{\psi})\right]^{-2} \times \left(-\frac{M^4}{2n_i^2}(1-n_i)^2 - M^2(1-\delta^2)\psi + M^2H(\psi)\right. \\ &+ M^2\sigma\left(n_i^{5/3}-\frac{5}{2}n_i^{2/3}+\frac{3}{2}\right) + \delta^2\sigma H(\psi) + M^2\delta^2\sigma\left(\frac{1}{n_i}+\frac{3}{2}n_i^{2/3}-\frac{5}{2}\right) + \delta^2\sigma^2\left(n_i^{5/3}-\frac{1}{2}n_i^{10/3}-\frac{1}{2}\right) \\ &\left.-\frac{\delta^2}{2}H^2(\psi) - \frac{M^2\delta^2}{n_i}H(\psi) - \delta^2\sigma n_i^{5/3}H(\psi)\right), \end{aligned} \tag{18}$$

where

$$H(\psi) = (1+3\beta)(e^{\psi}-1) + \beta\psi(\psi-3)e^{\psi}. \tag{19}$$

Equation (17) is known as "energy integral" of an oscillating particle of a unit mass with a velocity $d\psi/d\xi$ at position ψ in a potential well $V(\psi, M)$.

III. NONLINEAR CHARACTERISTICS

The soliton solutions of Equation (17) exist when the following conditions are satisfied, namely, $V(\psi, M)$ satisfies the following conditions: $V(\psi, M) = 0$, $dV(\psi, M)/d(\psi) = 0$, $d^2V(\psi, M)/d(\psi)^2 < 0$ at $\psi = 0$; $V(\psi, M) = 0$ at $\psi = \psi_m$; and $V(\psi, M) < 0$ for ψ lying between 0 and ψ_m. If $\psi_m < \psi < 0$, rarefactive solitary (RS) waves exist and when $0 < \psi < \psi_m$, compressive solitary (CS) waves exist, where ψ_m is the maximum amplitude of the solitons. For the formation of the double layers (dl), one more additional condition must be satisfied, i.e., $\frac{dV(\psi,M)}{d\psi}\big|_{\psi=\psi_m} = 0$.[17,18,22,23] A supersoliton exists when there is an accessible root of Sagdeev potential beyond the double layer (dl) solution, i.e., $V(\psi, M) = 0$ for $\psi > \psi_{dl}$.

The condition $d^2V(\psi, M)/d\psi^2 < 0$ at $\psi = 0$ can be written as

$$\left.\frac{d^2V(\psi,M)}{d\psi^2}\right|_{\psi=0} = \frac{M^2-M_0^2}{M^2-M_1^2} < 0, \tag{20}$$

where

$$M_o^2 = \frac{\delta^2(3+5\sigma(1-\beta))}{3(1-\beta)} \tag{21}$$

is the critical Mach number and the upper limit is

$$M_1^2 = \frac{(3+5\sigma(1-\beta))}{3(1-\beta)}, \tag{22}$$

for $\delta, \beta > 0$.

Since $\delta^2 = \cos^2\theta < 1$, this would imply that $M_0 < M_1$. Further, if $M > M_1 \Rightarrow M > M_0$, which means that $M^2 - M_o^2 > 0$ and $M^2 - M_1^2 > 0$, consequently Equation (20) is not satisfied. Similarly, if $M < M_0$, then $M < M_1$ from which $M^2 - M_0^2 < 0$ and $M^2 - M_1^2 < 0$, once again (20) is not satisfied.

The low frequency wave structures in a magnetized plasma may exist within the interval

$$M_0 < M < M_1. \tag{23}$$

From Equation (23), we obtain a condition

TABLE I. Properties of ion-acoustic solitons, such as Mach number range M, soliton velocity V (km/s), electric field E (mV/m), soliton width W (m), and pulse duration $\tau = W/V$ (ms), for various values of the nonthermal contribution (α) with propagation angle $\theta = 15°$ and temperature ratio $\sigma = T_i/T_e = 0.0246$ corresponds to the parameters observed by the Viking satellite in the auroral zone.[25]

α	$M_0 - M_1$	V (km s^{-1})	E (mV m^{-1})	W (m)	$\tau = W/V$ (ms)
0.0	0.988 – 1.020 (CS)	25.59 – 26.42	0.14 – 4.05	377.00 – 157.04	14.73 – 5.94
0.01	1.008 – 1.039 (CS)	26.11 – 26.91	0.24 – 4.32	325.00 – 156.62	12.45 – 5.82
0.05	1.085 – 1.118 (CS)	28.10 – 28.96	0.43 – 6.15	304.20 – 156.00	10.83 – 5.39
0.1	1.182 – 1.219 (CS)	30.61 – 31.57	0.81 – 11.59	301.08 – 148.72	9.84 – 4.71
0.15	1.282 – 1.321 (CS)	33.20 – 34.21	4.43 – 26.37	261.04 – 134.16	7.86 – 3.92
0.2	1.383 – 1.428 (RS)	35.82 – 36.99	0.49 – 14.06	398.84 – 180.70	11.13 – 4.86

$$\delta\sqrt{\left(\frac{1}{1-\beta}+\frac{5\sigma}{3}\right)} < M < \sqrt{\left(\frac{1}{1-\beta}+\frac{5\sigma}{3}\right)}, \qquad (24)$$

which gives the Mach number M values for the fixed values of δ, σ, β. For $\sigma = 0$, condition (24) reduces to

$$\delta\sqrt{\left(\frac{1}{1-\beta}\right)} < M < \sqrt{\left(\frac{1}{1-\beta}\right)}. \qquad (25)$$

Further, for $\beta = \alpha = 0$, the condition reduces to

$$\delta\sqrt{\left(1+\frac{5\sigma}{3}\right)} < M < \sqrt{\left(1+\frac{5\sigma}{3}\right)}. \qquad (26)$$

It is obvious that the consideration of the cold ion limit ($\sigma = 0$) and the Boltzmann distribution of hot electron species (i.e., $\alpha = \beta = 0$) correspond to the earlier work of Choi *et al.*[24]

IV. NUMERICAL RESULTS

Nonlinear low frequency electrostatic structures are examined numerically for magnetized plasmas in which the dominant species are energetic hot electrons. The typical auroral parameters considered for the numerical computation are Mach number M, ion temperature $\sigma = T_i/T_e$, nonthermal contribution α, and wave obliqueness $\delta = \cos\theta$, where θ is the angle of propagation.

First, we apply our theoretical results to the Viking satellite observations in the auroral region of the Earth's magnetosphere. The observations[25] reported an electric field amplitude of less than 100 mV/m, spatial width of about 100 m, pulse duration of about 20 ms, and soliton velocities in the range of about 5–50 km/s. The following parameters are taken from the Viking satellite observations, namely, total equilibrium electron density $N_{e0} = 2\,\text{cm}^{-3}$, ion temperature, $T_i = 0.64$ eV, and electron temperature, $T_e = 26$ eV which gives the temperature ratio, $\sigma = T_i/T_e = 0.0246$. For different values of α (nonthermality), Table I shows the summary of our theoretical results.

Table I describes the behavior of the nonlinear ion-acoustic structures for the temperature ratio $\sigma = 0.0246$ (corresponds to the parameters observed by the Viking satellite in the auroral zone[25]) and the angle of propagation $\theta = 15°$. It is clear from the Table I that rarefactive solitons (RS) and compressive solitons (CS) are obtained. As α increases, both minimum and maximum electric field amplitude of the positive potential soliton increase also the soliton velocity, but the soliton width as well as pulse duration decreases with α. The new and surprising results, which emerged from the current study, are the case of nonthermal electron contribution $\alpha = 0.16$ (see Figures 3–6). For the first time, we are announcing the existence of nonlinear ion-acoustic supersoliton structures in magnetized two-component plasmas. For $\alpha = 0.17$, 0.18, and 0.19, both rarefactive and compressive solitons are found to coexist. As shown in Table I, only negative potential soliton structures are found to exist at $\alpha = 0.2$.

For different values of nonthermal contribution, α with fixed value of propagation angle, $\theta = 15°$, Figure 1 shows the Mach number range (critical Mach number M_0 and upper limit M_1) supported for the existence of finite amplitude ion-acoustic solitons and supersolitons in magnetized two-component nonthermal plasmas, as a function of normalized ion temperature σ. The lower M_0 and upper M_1 limiting curves for each value of α were found by the evaluation of the algebraic expressions in Equations (21) and (22), respectively (where $\beta = \frac{4\alpha}{(1-3\alpha)}$). The nonlinear structures can exist both in the subsonic and supersonic Mach number regions.

Figure 2 shows the variation of the Sagdeev potential $V(\psi, M)$ with the normalized electrostatic potential ψ for different values of the Mach number M and for $\sigma = 0.0$ and 0.01. Other fixed parameters are angle of propagation

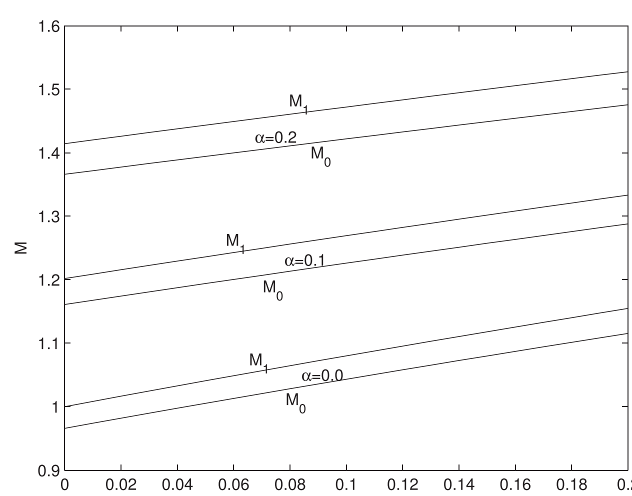

FIG. 1. Critical M_0 and upper limit M_1 of the Mach number for nonlinear electrostatic structures shown as a function of ion temperature ratio σ, for $\theta = 15°$.

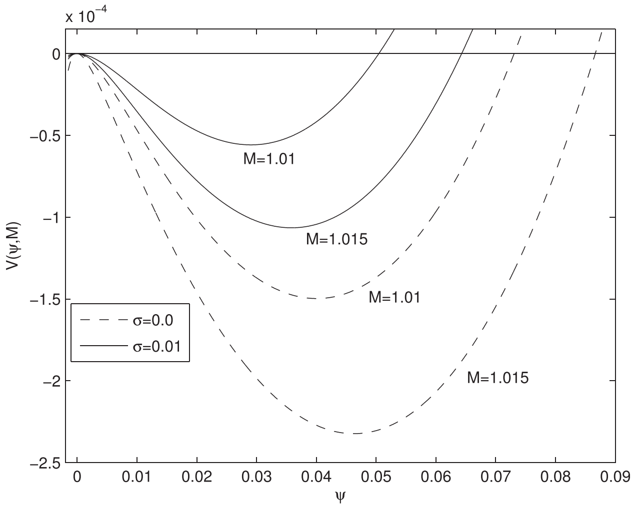

FIG. 2. Sagdeev potential, $V(\psi, M)$ vs. normalized electrostatic potential ψ, for $\alpha = 0.01$, $\theta = 15°$, and varying different values of M.

$\theta = 15°$ and nonthermal electron contribution $\alpha = 0.01$. The solid (−) and dashed (- - -) curves are corresponding to the ion temperature ratio, $\sigma = T_i/T_e = 0.0$ and 0.01, respectively. The positive potential ion-acoustic soliton amplitude, ψ, increases with increasing Mach number M for both categories. It is clearly shown that the soliton amplitude decreases with an increase in ion temperature σ.

Figure 3 shows the Sagdeev potential $V(\psi, M)$ vs. the real potential ψ for different values of M for fixed parameters, angle of propagation $\theta = 15°$, ion temperature ratio $\sigma = 0.0246$, and the nonthermal electron contribution $\alpha = 0.16$. The plotted curves show the coexistence of compressive and rarefactive ion-acoustic structures and the upper limit of Mach numbers, for positive potential soliton, is $M = 1.342$. At Mach number value of $M = 1.31055373$, the curves show that a positive potential soliton structure coexist with a negative potential double layer (followed by a third root). It must be noted that $M = 1.311$ is a supersoliton curve. In the case of unmagnetized

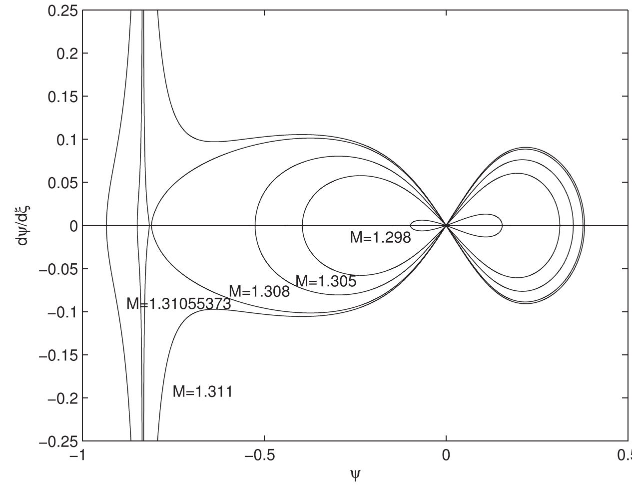

FIG. 4. The phase portrait of ion-acoustic waves for the same parameters of Figure 3.

plasma,[9] only compressive and rarefactive soliton solutions are found to coexist. The ion-acoustic soliton amplitude increases with the increase in the Mach number M. The phase portrait curves of pseudoparticle of the finite amplitude nonlinear ion-acoustic waves are plotted in Figure 4 for the same fixed parameters of Figure 3. The closed curves in the phase space are for solitons. The curves for $M = 1.31055373$ where separatrices appear for negative values of ψ represent the negative double layer. The curves for $M = 1.311$ having no closed contour represent the supersolitons. Thus from looking at the phase space diagram, one can immediately say, positive potential double layer and supersolitons cannot exist in our model.

Interestingly, the curves plotted in Figure 5 show that as the ion temperature ratio σ increases, the ion-acoustic wave amplitude decreases. The chosen parameters for the Sagdeev potential curves are Mach number $M = 1.311$, angle of propagation $\theta = 15°$, and nonthermal electron contribution $\alpha = 0.16$. In addition, for different values of σ, positive and negative potential structures coexist with the upper bound

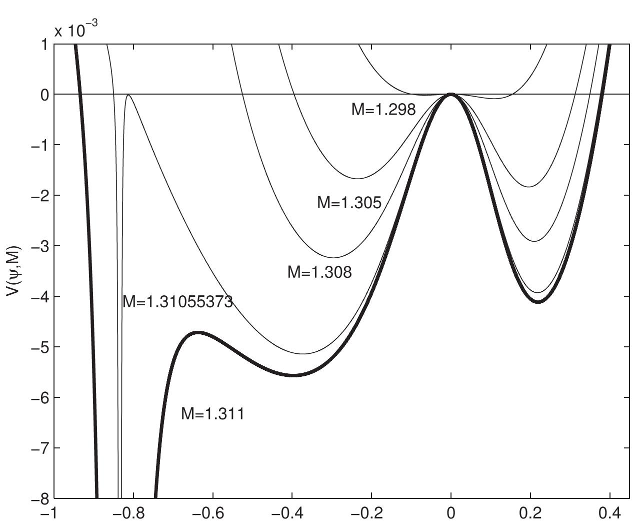

FIG. 3. Sagdeev potential $V(\psi, M)$ vs. normalized electrostatic potential ψ, for $\sigma = 0.0246$, $\alpha = 0.16$, and $\theta = 15°$ varying different values of M.

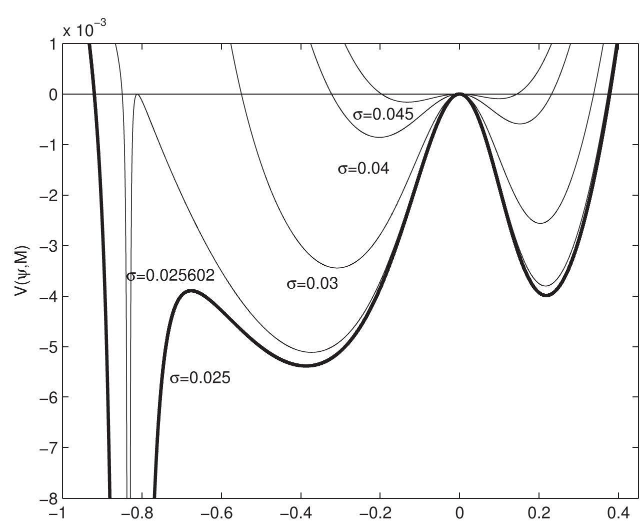

FIG. 5. Sagdeev potential $V(\psi, M)$ vs. normalized electrostatic potential ψ, for $M = 1.311$, $\alpha = 0.16$, and $\theta = 15°$ varying with different values of σ.

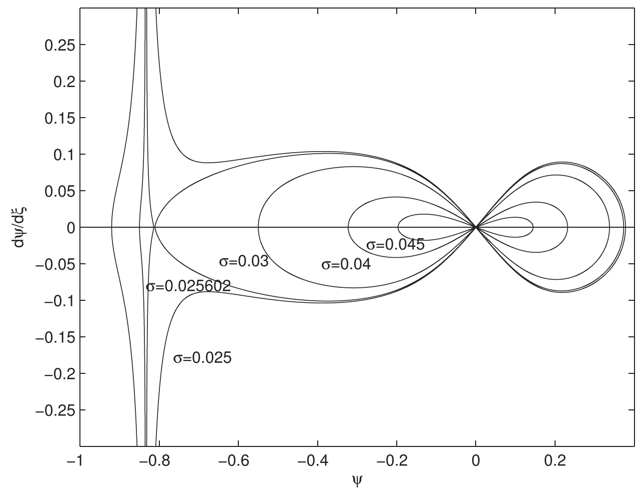

FIG. 6. The phase portrait of ion-acoustic waves for the same parameters of Figure 5.

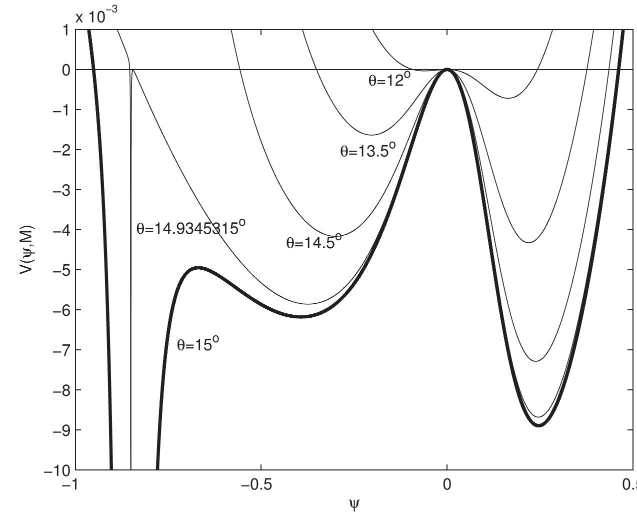

FIG. 8. Sagdeev potential, $V(\psi, M)$ vs. normalized electrostatic potential ψ, for $M = 1.30$, $\alpha = 0.16$, and $\sigma = 0$ varying different values of θ.

limit of $\sigma = 0.045$. It is important to specify that at $\sigma = 0.025602$, a double layer solution is found followed by a supersoliton (the third root) curve at $\sigma = 0.025$. For the same fixed parameters, the phase space portrait in Figure 6 clearly shows that compressive and rarefactive solitons and negative potential double layer and supersolitons can only be found to exist in our plasma model. The closed curves $\sigma = 0.045$, 0.04, and 0.03 are solitons, while $\sigma = 0.025602$ is a double layer and $\sigma = 0.025$ is a supersoliton.

Figure 7 shows the Sagdeev potential $V(\psi, M)$ with real electrostatic potential ψ for different values of the propagation angle ($\theta = 0°$ to $90°$) (where $\cos\theta = \delta$ for the obliqueness of the waves) for $\alpha = 0.01$, and other fixed parameters are $\sigma = 0.0246$ and $M = 1.02$. The curves show that as the angle of propagation θ increases, the positive potential soliton amplitude increases. It is observed that for a parallel propagation (i.e., $\theta = 0°$), the soliton solution does not exist as the solution existence condition given by Eq. (23) cannot be satisfied. On the other hand, at $\theta = 90°$, the solitons have the largest amplitude.

At the cold ion domain (that is ion temperature ratio, $\sigma = 0$), the curves plotted in Figure 8 show the characteristics of the Sagdeev potential $V(\psi, M)$ with normalized potential ψ for different values of θ for other fixed parameters, namely, Mach number $M = 1.30$ and nonthermal energetic electron contribution $\alpha = 0.16$. Compressive and rarefactive low-frequency structures are found to coexist. It is important to mention that at $\theta = 14.9345315°$, both positive potential soliton and negative potential double layer structures appear and supersoliton (the third root) solution are found to exist at $\theta = 15°$. Clearly, the ion-acoustic wave amplitude ψ increases with increasing θ.

Figure 9 shows the variation of electrostatic potential amplitude ψ against ξ for different values of M for $\sigma = 0.02$. Other fixed parameters are $\theta = 15°$ and $\alpha = 0.2$. The rarefactive ion-acoustic soliton amplitude as well as its width increases with an increase in the Mach number M.

Figure 10 shows the variation of electrostatic potential ψ against ξ for different values of nonthermal electron

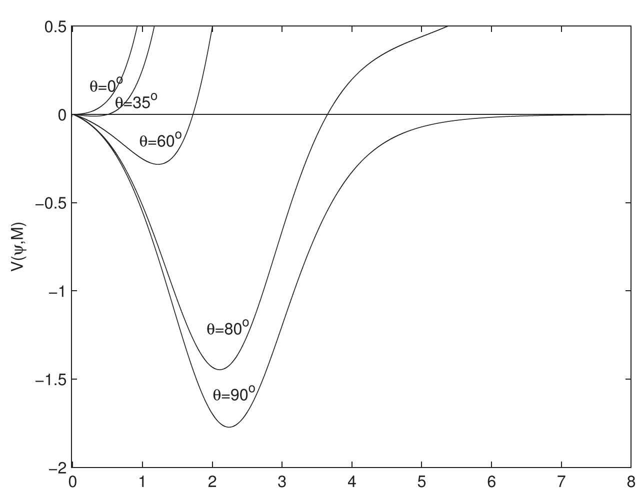

FIG. 7. Sagdeev potential $V(\psi, M)$ vs. normalized electrostatic potential ψ, for $\alpha = 0.01$, $\sigma = 0.0246$, and $M = 1.01$.

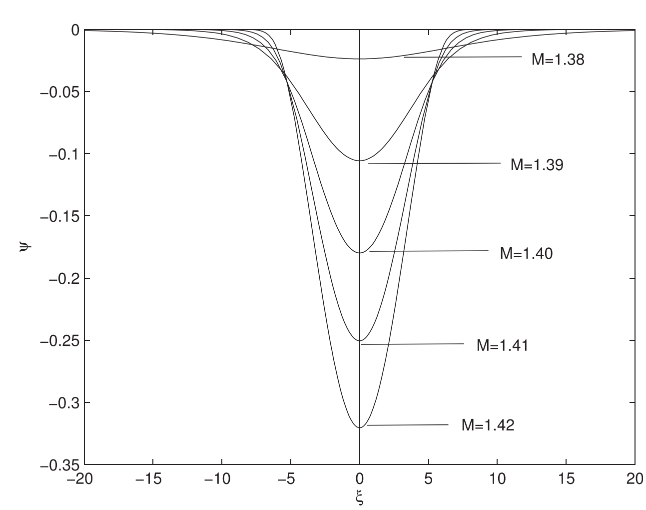

FIG. 9. Normalized electrostatic potential ψ vs. ξ for $\theta = 15°$, $\alpha = 0.2$, and $\sigma = 0.02$.

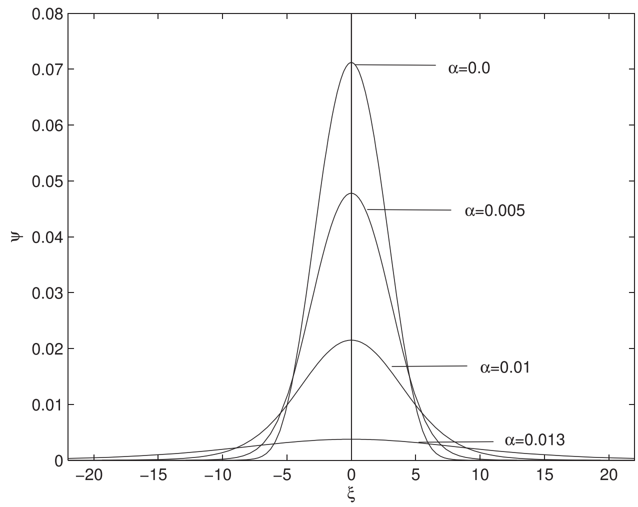

FIG. 10. Normalized electrostatic potential ψ vs. ξ for $\theta = 15°$, $\sigma = 0.01$, and $M = 1.00$.

contribution α. Other fixed parameters are $\theta = 15°$, $\sigma = 0.01$, and $M = 1.00$. It clearly shows that both the soliton amplitude and width decreases with increase in nonthermality. The maximum soliton amplitude corresponds to $\alpha = 0$.

V. CONCLUSION

We have examined the properties of finite amplitude nonlinear ion-acoustic soliton, double layer, and supersoliton structures in a magnetized plasma composed of fluid adiabatic warm ions and nonthermal distribution of hot electron species using the Sagdeev pseudo-potential technique. The coexistence of compressive and rarefactive ion-acoustic soliton and supersoliton structures is found to occur both in the subsonic and supersonic Mach number regime. Whereas, for the case of unmagnetized plasmas, Mamun[9] found the coexistence of compressive and rarefactive soliton solutions only at the supersonic Mach number region. Moreover, this is contrary to "No electrostatic supersolitons in two-component plasmas" reported by Verheest *et al.*[16] for an unmagnetized plasma system. The present plasma model supports the existence of electrostatic supersoliton structures in magnetized two-component plasmas. So, obviously the magnetic field plays a key role in the existence of supersolitons in two component plasmas by increasing the degree of freedom in the system.

It was observed that the wave obliqueness plays an important role for the existence of the supersoliton structures. It must be noted that our analytical expression cannot be reduced to an unmagnetized case by setting the angle of propagation $\theta = 0$. The ion-acoustic wave amplitude increases with an increase in the Mach number and the angle of propagation, whereas ion temperature and nonthermal electrons reduce the amplitude of the electrostatic structures. Additionally, it was found that the Mach number and electric field amplitude increase as the value of energetic electrons α increases, while the soliton width and pulse duration decrease.

Related to the Viking satellite observations,[25] the maximum electric field associated with the ion-acoustic waves for the Mach number $M = 1.31055373$, angle of propagation $\theta = 15°$, nonthermal energetic electron contribution $\alpha = 0.16$, and ion temperature ratio $\sigma = 0.0246$ is about 13.4 mV/m and width, pulse duration, and speed becomes about 444.6 m, 33.9 ms, and 13.1 km/s, respectively. The theoretical model presented here may be relevant to analyzing the spacecraft observations of nonlinear electrostatic structures in the auroral region of the Earth's magnetosphere.

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