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Arbitrary amplitude fast electron-acoustic solitons in three-electron component space plasmas

L. N. Mbuli,[1,2] S. K. Maharaj,[1,2] R. Bharuthram,[2] S. V. Singh,[3,2] and G. S. Lakhina[3,2]
[1]*South African National Space Agency (SANSA) Space Science, P.O. Box 32, Hermanus 7200, Republic of South Africa*
[2]*Department of Physics, University of the Western Cape (UWC), Robert Sobukwe Road, Bellville 7535, Republic of South Africa*
[3]*Indian Institute of Geomagnetism, New Panvel (West), Navi Mumbai 410218, India*

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We examine the characteristics of fast electron-acoustic solitons in a four-component unmagnetised plasma model consisting of cool, warm, and hot electrons, and cool ions. We retain the inertia and pressure for all the plasma species by assuming adiabatic fluid behaviour for all the species. By using the Sagdeev pseudo-potential technique, the allowable Mach number ranges for fast electron-acoustic solitary waves are explored and discussed. It is found that the cool and warm electron number densities determine the polarity switch of the fast electron-acoustic solitons which are limited by either the occurrence of fast electron-acoustic double layers or warm and hot electron number density becoming unreal. For the first time in the study of solitons, we report on the coexistence of fast electron-acoustic solitons, in addition to the regular fast electron-acoustic solitons and double layers in our multi-species plasma model. Our results are applied to the generation of broadband electrostatic noise in the dayside auroral region. *Published by AIP Publishing.*
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I. INTRODUCTION

Since the observation of electrostatic solitary wave (ESW) signatures in various regions of the magnetosphere, several experimental and theoretical studies have been conducted to explain the generation mechanisms of the non-linear wave structures. The solitary wave structures have been suggested to be the source of the high frequency component of broadband electrostatic noise (BEN) varying up to the total electron plasma frequency and sometimes above the electron gyro-frequency.[1–6] BEN manifests itself as bipolar pulses in the electric field which are called electrostatic solitary waves (ESWs).

A popular theoretical interpretation of the ESWs is given in terms of Bernstein-Green-Kruskal (BGK) solitons, in which a positive potential pulse is interpreted as a moving population of trapped electrons.[7] A different theoretical interpretation is that the ESWs can be considered to be solitons which are obtained in the Sagdeev pseudo-potential formalism[8] which yields non-linear solutions for arbitrary amplitude solitons. The fluid equations are combined with Poisson's equation to yield the energy integral for a particle of unit mass in a potential well. The potential energy or Sagdeev potential or pseudo-potential determines whether solitons (or bell shaped wave structures in potential having bipolar electric field signatures) are possible. For certain parameter combinations, double layers (or kinks in potential) may also be admitted which appear as monopolar structures in the electric field. In another formalism, the non-linear wave structure function, which is the negative of the Sagdeev potential, is analysed to determine the existence of solitons and double layers in the fluid-dynamic paradigm.[9] Whilst the Sagdeev pseudo-potential technique and fluid-dynamic paradigm yield solutions for non-linear wave structures of arbitrary amplitude, analytical solutions for small amplitude solitons are governed by the Korteweg-de Vries (KdV) equation which can be derived using a reductive perturbation technique.

There are a number of reports on theoretical studies of small and large amplitude solitary waves which are based on a wide variety of plasma models in connection with BEN.[4,10,11] It was shown by DuBois *et al.*[12] that the compression of the magnetised laboratory plasma by the electric field generates broadband electrostatic noise (BEN).

Non-linear wave structures associated with high frequency waves involving electron dynamics have been studied for many plasma models. High speed electron-acoustic solitons and double layers have been theoretically studied in various plasma models. Mace *et al.*[13] studied arbitrary amplitude electron-acoustic solitons in a plasma with hot (Boltzmann) and cool (adiabatic) electrons as well as cool ions (inertial). Their theoretical study showed that only negative potential solitons are supported. The findings in the study by Cattaert *et al.*[14] revealed that the positive potential electron-acoustic solitons can be obtained when the inertia of the hot electron component is retained in the model. This was further verified by the results of Maharaj *et al.*,[15] which show that the positive potential electron-acoustic solitons are supported when the hot electrons are treated as an inertial (adiabatic) species as opposed to inertialess and Boltzmann distributed.

Lakhina *et al.*[16,17] conducted a study of solitons associated with the fast and slow electron-acoustic modes (driven by counter-streaming cool and warm electron beams) in a three-electron component plasma model (i.e., a plasma model with cool (anti-field-aligned beam), warm (field-aligned beam) and hot electron components as well as background cool ions) where all species are treated as adiabatic fluids. It was found that both positive and negative potential fast electron-acoustic solitons and double layers of both polarities are supported in the model.[16,17] On the other hand, the existence of only negative potential slow electron-acoustic solitons was reported for their model.[16–18]

Recently, Verheest and Hellberg[19] conducted a theoretical study of electron-acoustic solitons by revisiting the model with cool and hot electrons and ions. This very recent study was motivated by the findings in a paper by Rice *et al.*[20] that the existence regions for ion-acoustic solitons are significantly reduced and the amplitudes of the supported wave structures are smaller when finite electron mass effects are included in the two-temperature electron model. An analytical form of the Sagdeev pseudo-potential was explicitly derived, and numerically analysed for the electron-acoustic solitons with phase speed lying between $v_{tce} \leq v_{\phi} \leq v_{the}$ where v_{tce} and v_{the} are the thermal speeds of the cool and hot electrons. For the case where the hot electrons were treated as isothermal and inertia was also retained, it was found that a switch from negative polarity to positive polarity solitons is possible in the model. The upper Mach number limits in the order of increasing cool electron concentration was the cool electron, hot electron sonic points, negative double layers, and then positive double layers, which was very similar to the findings in Refs. 14 and 15. On the other hand, if the hot electrons were considered to be Boltzmann-distributed (neglecting inertial effects), then only negative potential electron-acoustic solitons were found to be supported, limited by the cool electron sonic point.

To the best of our knowledge, to date the coexistence of negative and positive polarity (electron-acoustic type) high frequency non-linear waves has not been reported in a three-electron plasma. Not only do we report here on the phenomenon of coexistence for the case where the non-KdV like solitons occur at the acoustic speed but also when solitons at the acoustic speed are not possible. The former situation describes coexistence of non-KdV-like solitons with the opposite polarity KdV-like solitons, whereas both polarities of solitons are KdV-like in the latter. The terminology "KdV-like" describes solitons where the amplitudes tend to zero for decreasing wave structure speed approaching the phase speed of the linear wave. On the other hand, "non-KdV like" describes solitons which exist at the acoustic speed.

In this paper, we investigate the properties of the fast (higher phase speed) electron-acoustic solitons and compare with the characteristic of the slow electron-acoustic solitons discussed by Mbuli *et al.*[21] In presenting and discussing the results, we report on the coexistence of negative polarity with positive polarity solitons. This study on fast electron-acoustic solitons and our earlier published results on slow electron-acoustic solitons follows from kinetic studies of low and high frequency instabilities in a single warm electron beam model[22,23] and also a two beam model with counter-streaming electrons of different temperatures.[24] The studies in Refs. 22 and 24 attempt to explain the generation mechanisms of electrostatic turbulences such as broadband electrostatic noise, electrostatic hiss, and auroral kilometric radiation (AKR) in the auroral and polar cusp regions.[1,23] In order to be consistent with our earlier study on the slow electron-acoustic solitons, the focus in this study will be on sets of plasma parameters which are closely aligned with the measurements of number density and temperature of plasmas which occur in the auroral zone.[21] We have not included beams in this study. The effect of the beams will be deferred for future studies.

The paper is arranged in the following manner: in Section II, we present the model and give details of the theory. In Section III, we discuss our numerical results. Finally, in Section IV we present a summary of our detailed numerical findings.

II. THEORETICAL MODEL

We study arbitrary amplitude fast electron-acoustic solitons supported in a four-component unmagnetised, homogeneous and collisionless plasma model with cool, warm, and hot electron components, as well as cool ions. In our study, we retain the pressure and inertia of all species considered. The dynamics of the system is governed by the fluid equations, viz., the continuity, momentum, and pressure equations given by

$$\begin{aligned} &\frac{\partial n_j}{\partial t} + \frac{\partial (v_j n_j)}{\partial x} = 0, \\ &\frac{\partial v_j}{\partial t} + v_j \frac{\partial v_j}{\partial x} = -\frac{Z_j}{\mu_j}\frac{\partial \Phi}{\partial x} - \frac{1}{\mu_j n_j}\frac{\partial P_j}{\partial x}, \\ &\frac{\partial P_j}{\partial t} + v_j \frac{\partial P_j}{\partial x} + 3P_j \frac{\partial v_j}{\partial x} = 0, \end{aligned} \tag{1}$$

and the system of equation is enclosed by the Poisson's equation written as

$$\frac{\partial^2 \Phi}{\partial x^2} = -\Sigma Z_j n_j, \tag{2}$$

where the plasma parameters such as $Z_j(=-1(+1))$, n_j, v_j, P_j, $\mu_j = m_j/m_e$ denote electrons (protons), number density, speed, pressure, and mass ratio of species j(j = ce, we, he, i, representing the cool, warm, and hot electrons, and the cool ions). Details of the theory of the solitons for a four-component plasma were given by Mbuli *et al.*[21] in a study of slow electron-acoustic solitons, but it is repeated here for completeness. It is important to note that in the general formalism we treat all the species to have an equilibrium drift v_{dbj}. However, in the numerical analysis for this particular study we consider a total stationary plasma with $v_{dbj} = 0$. All the numerical results are presented in normalised form, with the proton-electron mass ratio taken as $m_i/m_e = 1836$, i.e., for a hydrogen plasma. The number densities are normalised by the total equilibrium plasma density $n_o = n_c + n_h + n_w$ (or alternatively $n_{co} + n_{ho} + n_{wo} = 1$) and temperatures by the hot electron temperature, T_{he}. Time is normalised by the inverse of the total electron plasma frequency $\omega_{pe}^{-1} = (4\pi n_o e^2/m_e)^{-1/2}$, speeds by the hot electron thermal speed, $C_h = (T_{he}/m_e)^{1/2}$, and spatial lengths by the Debye length $\lambda_d = (T_{he}/4\pi n_o e^2)^{1/2}$.

The dynamic fluid equations (1) and (2) are transformed to a frame moving with the non-linear wave structure by considering $\zeta = x - Mt$ where $M(=V/C_h)$ is the Mach number and V is the speed of the solitary wave. In this theoretical study, we consider the following boundary conditions: $n_j \to n_{jo}$, $v_j \to v_{dbjo}$, $n_i \to 1$, $\Phi \to 0$, and $P_j \to P_{jo} \sim n_{jo}T_j$ as $\zeta \to \pm\infty$.

The general expression for the number density of the plasma constituents considered in our four component plasma is then given by

$$n_j = \frac{n_{jo}}{\sqrt{6\delta_j}}\left(\left((M - v_{dbjo})^2 + 3\delta_j - \frac{2Z_j\Phi}{\mu_j}\right) \pm \sqrt{\left((M - v_{dbjo})^2 + 3\delta_j - \frac{2Z_j\Phi}{\mu_j}\right)^2 - 12(M - v_{dbjo})^2}\right)^{1/2}, \quad (3)$$

where $\delta_j = P_{jo}/n_{jo}\mu_j$. Then following Mbuli *et al.*,[21] we obtain the energy integral-like equation given by

$$\frac{1}{2}\left(\frac{d\Phi}{d\zeta}\right)^2 + V(\Phi) = 0, \quad (4)$$

where the expression for the pseudo-potential, $V(\Phi)$, is given by

$$V(\Phi) = \sum_j \frac{n_{jo}\mu_j}{6\sqrt{3\delta_j}}\left[\left[\left((M - v_{dbjo}) + \sqrt{3\delta_j}\right)^3 - \left(\sqrt{\left((M - v_{dbjo}) + \sqrt{3\delta_j}\right)^2 - \frac{2Z_j\Phi}{\mu_j}}\right)^3\right] \pm \left[\left((M - v_{dbjo}) - \sqrt{3\delta_j}\right)^3 - \left(\sqrt{\left((M - v_{dbjo}) - \sqrt{3\delta_j}\right)^2 - \frac{2Z_j\Phi}{\mu_j}}\right)^3\right]\right]. \quad (5)$$

The second derivative of Equation (5) which is evaluated at $\Phi = 0$ reads

$$\frac{d^2V(\Phi)}{d\Phi^2} = \sum_j \frac{Z_j^2 n_{jo}}{\mu_j\left((M - v_{dbjo})^2 - 3\delta_j\right)}, \quad (6)$$

and the third derivative at $\Phi = 0$ is written as

$$\frac{d^3V(\Phi)}{d\Phi^3} = \sum_j \frac{3Z_j^3 n_{jo}/\mu_j^2\left((M - v_{dbjo})^2 + \delta_j\right)}{\left((M - v_{dbjo})^2 - 3\delta_j\right)^3}. \quad (7)$$

In the limit of small amplitude non-linear waves, with appropriate expansions, Equation (5) reduces to

$$V(\Phi) \approx C_2\Phi^2 + C_3\Phi^3, \quad (8)$$

with the solution of (4) given by

$$\Phi = -\left(\frac{C_2}{C_3}\right)\operatorname{sech}^2\left(\sqrt{\left(-\frac{C_2}{4}\right)\zeta^2}\right), \quad (9)$$

where $C_2 = \frac{1}{2}(\frac{d^2V(\Phi)}{d\Phi^2})|_{\Phi=0}$ and $C_3 = \frac{1}{6}(\frac{d^3V(\Phi)}{d\Phi^3})|_{\Phi=0}$. It should be noted that the expression (6) gives roots corresponding to the phase speeds of the three different linear wave modes, viz., ion-acoustic, fast, and slow electron-acoustic waves. From Equations (8) and (9), the polarity of the small amplitude super-acoustic solitons in general is determined by the sign of the third derivative of the Sagdeev potential at $\Phi = 0$.

The lower limits (critical Mach number) M_{crit} are evaluated numerically by setting $C_2 = 0$ and solving for M-values. On the other hand, we numerically compute the upper Mach number limit, M_{max}, beyond which the fast electron-acoustic solitary waves solution does not exist. The M-values between the lower and upper M limits is defined as the existence domains, i.e., the region where valid soliton solutions can be found. Soliton solutions exist when the following conditions are satisfied, namely, $V(\Phi) = 0$, $\frac{dV(\Phi)}{d\Phi} = 0$, and $\frac{d^2V(\Phi)}{d\Phi^2} < 0$ at $\Phi = 0$; $V(\Phi) = 0$ at $\Phi = \Phi_o$; and $V(\Phi) < 0$ for Φ_o ranging between 0 and Φ_o. If $\Phi_o < \Phi < 0$, negative potential solitons exist and when $0 < \Phi < \Phi_o$ positive potential solitons exist, where Φ_o is the amplitude of the solitons. For the formation of the double layers, one more condition must be satisfied, i.e., $\frac{dV(\Phi)}{d\Phi}|_{\Phi=\Phi_o} = 0$.

III. NUMERICAL RESULTS AND DISCUSSION

We conduct a numerical study of arbitrary potential amplitude fast electron-acoustic solitons with phase speed, v_ϕ, ranging between the thermal speeds of the warm and hot electron components such that $v_{twe} \leq v_\phi (= (\sqrt{n_{co}/n_{ho}})v_{the}) < v_{the}$.[16,17] In order to gain insight into the evolution of the

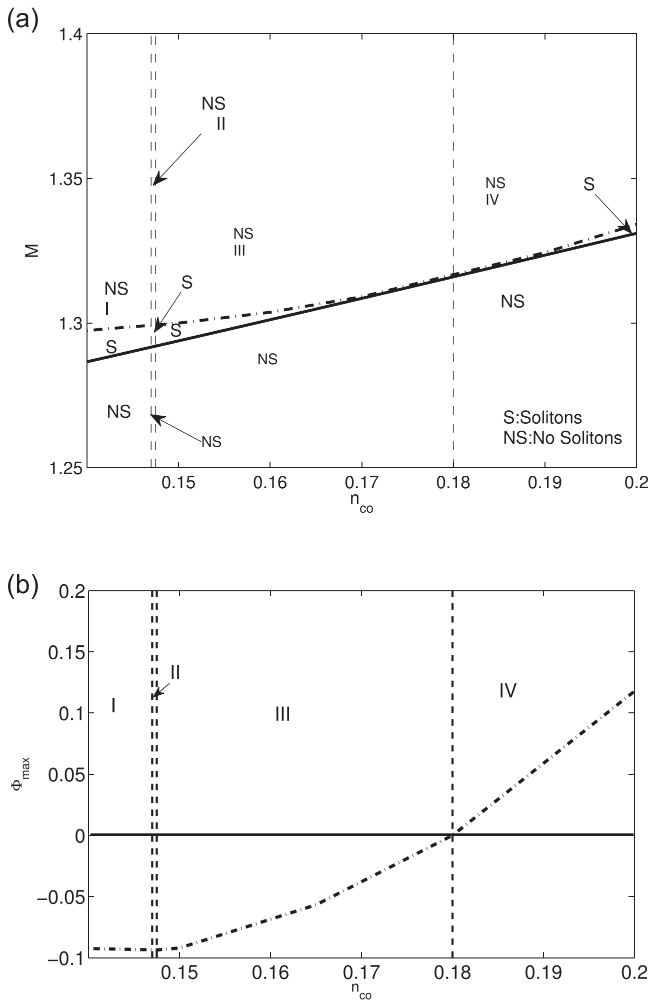

FIG. 1. Fast electron-acoustic solitons existence domains as a function of equilibrium cool electron number density, n_{co}. From the top panel of the figure the solid line (—) represents the critical Mach number limit and the dashed-dotted line (– · – · –) represents upper Mach number limit determined in regions I and II by the warm and hot electron number densities becoming complex valued. In regions III and IV, the dashed-dotted line (– · – · –) represents M or M_{max} values for the negative and positive potential double layers, respectively. In the lower panel, the behaviour of the corresponding maximum potential amplitudes, Φ_{max}, is illustrated by the dashed-dotted line (– · – · –) for warm and hot electron number density limits and both positive and negative potential fast electron-acoustic double layers. The absolute value of the maximum potential amplitude of the fast electron-acoustic solitons from the figure maximises as the cool electron density, n_{co}, increases. The other fixed parameters are: $T_{we} = 0.25$, $T_{ce} = T_i = 0.001$, $n_{wo} = 0.3$, $m_i/m_e = 1836$.

non-linear wave structures in this paper, all species are treated stationary. The effect of particle drifts on solitons is deferred for later investigations. Our main focus is to discuss the numerical result for the fast electron-acoustic solitons for the same parameter set (applicable to the dayside auroral zone[22]) which was used to study the slow electron-acoustic solitons.[21] The input plasma parameters used in this section are as follows: $T_{we}=0.25$, $T_i=T_{ce}=0.001$, $n_{co}/n_o=0.3, n_{wo}/n_o=0.3, m_i/m_e=1836$ for an electron-proton plasma[22] but the warm electrons are treated as stationary in this study. The Sagdeev pseudo-potential formalism is adopted in this paper to investigate the non-linear wave structures of arbitrary amplitudes. All results have been generated using the expression for the Sagdeev potential given by Equation (5).

The existence regions for fast electron-acoustic solitons are shown as a function of the cool electron concentration in Figure 1. The lower M limits for solitons on solid line (—) correspond to the phase speed of the linear fast electron-acoustic waves which were obtained by solving (6).

Figure 1 has been demarcated into four regions (I, II, III, and IV) according to the reason for the upper Mach number limits represented by the dashed-dotted line (– · – · –) in all regions of the figure. The physical reason for the upper M limits represented by the dashed-dotted line (– · – · –) in region I is that the expression for the warm electron number density given by

$$n_{we} = \frac{n_{wo}}{2\sqrt{3T_{we}}}\left[\sqrt{\left(M+\sqrt{3T_{we}}\right)^2+2\Phi} - \sqrt{\left(M-\sqrt{3T_{we}}\right)^2+2\Phi}\right], \tag{10}$$

becomes unreal when $\Phi < \Phi_{min,warm}(= -\frac{1}{2}\left(M-\sqrt{3T_{we}}\right)^2)$ ruling out the possibility of the fast electron-acoustic solitons solution for Mach number values, M, greater than M_{max}(or $\Phi > \Phi_{max}$). The limiting values of the potential $\Phi_{min,warm}$ corresponding to the solitons which occur in region I are shown in the lower panel of the figure. On the other hand, in region II of Figure 1 it is the expression for the hot electron number density given by

$$n_{he} = \frac{n_{ho}}{2\sqrt{3T_{he}}}\left[\sqrt{\left(M+\sqrt{3T_{he}}\right)^2+2\Phi} + \sqrt{\left(M-\sqrt{3T_{he}}\right)^2+2\Phi}\right], \tag{11}$$

which imposes the upper limits on the numerical solution for fast electron-acoustic solitons. For solitons which are supported in region II, the hot electron number density imposes the limit $\Phi_{min,hot}(= -\frac{1}{2}\left(M-\sqrt{3T_{he}}\right)^2)$ on the amplitudes of the supported wave structures. If the amplitudes of the soliton structures become so large such that $\Phi < \Phi_{min,hot}$, then (11) becomes unreal and solitons are no longer possible. Limiting values of the potential $\Phi_{min,hot}$ are shown in region II in the lower panel of the figure.

Moving to higher cool electron concentrations, it is the occurrence of negative potential double layers which limits the negative potential solitons in region III. The limiting Φ values (Φ_{max}) corresponding to the negative potential double layers are shown in the lower panel of the figure as a function of the cool electron number density, n_{co}. It is noted that a switch in polarity to positive potential fast electron-acoustic solitons occurs for very large values of the cool electron density, n_{co}.

Positive potential fast electron-acoustic solitons which are limited by positive potential double layers are supported in region IV. The positive double layer potentials are shown in the lower panel (region IV) of the figure. From the lower panel in Figure 1, it is seen that the crossover from negative to the positive potential wave structures occurs at $n_{co} = 0.18$.

The lower panel in Figure 1 is very useful because it provides information on the polarity of the supported solitons and clearly shows where a crossover to the opposite polarity occurs. As observed in the lower panel of Figure 1 that the crossover point for the fast electron-acoustic solitons occurs at $n_{co}/n_o = 0.18$ which is much lower than the value $n_{co} = 0.3$ found for the slow electron-acoustic solitons[21] for the same fixed plasma parameter set used in Figure 1 due to the difference in the phase speed of the wave structures. However, our numerical analysis shows that by fixing the value of warm electron number density at $n_{wo}/n_o = 0.186471$ the polarity switches of the fast electron-acoustic solitons occurs at the same value as for the slow electron-acoustic solitons studied by Mbuli *et al.*,[21] i.e., at $n_{co} = 0.3\,n_o$. We find in this study that in the regions which are adjacent to the crossover point in Figure 1 the positive and negative potential fast electron-acoustic solitons are limited by the formation of the fast electron-acoustic double layers which is consistent with the results for slow electron-acoustic solitons in our earlier study conducted by Mbuli *et al.*[21]

To obtain the limitations on the Mach number ranges of the fast electron-acoustic solitary waves in the existence domains plots (Figure 1), we have constructed the typical Sagdeev pseudo-potential plots by fixing the value of the cool electron density, n_{co}, obtained in each region of Figure 1 for the fast electron-acoustic solitons. We have chosen not to show these Sagdeev pseudo-potential plots so that we do not overload the paper. However, we refer the reader to the Sagdeev potentials in Figures 2–5 in Ref. 21 as the realization of the lower and upper Mach numbers limits for fast electron-acoustic solitons is very similar to the results presented for slow electron-acoustic solitons in Ref. 21.

In Figure 2, the admissible Mach number ranges are presented as a function of the warm electron density (n_{wo}). For the fast electron-acoustic solitons, we find that a single crossover from negative to positive polarity solitons occurs at $n_{wo} = 0.28034$. However, in a theoretical study of the slow electron-acoustic solitons, Mbuli *et al.*[21] found that the crossover between two polarities occurs at two different values of n_{wo}. There is a crossover in slow electron-acoustic soliton polarity from negative to positive polarity at $n_{wo} = 0.004$ but there is a return to negative polarity slow electron-acoustic solitons at $n_{wo} = 0.08$. For the fast electron-acoustic solitons, our findings indicates in region I of Figure 2 that only the warm electron number density for $M > M_{max}$ become unreal. However, in region II(III) of Figure 2 negative (positive) potential fast electron-acoustic double layers limit the solution of the fast electron-acoustic solitons. Our numerical results in Figure 2 (for $n_{co} = 0.2$) indicate that the hot electron number density (11) does not impose a limit on the fast electron-acoustic soliton solutions as was the case for the solitons which occur in region II in Figure 1.

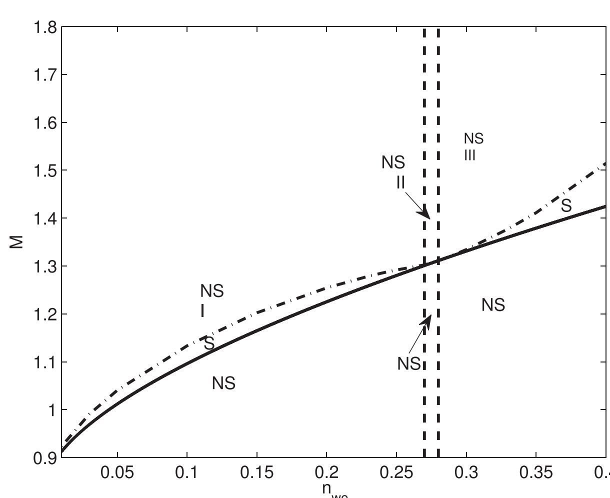

FIG. 2. Fast electron-acoustic solitons existence domains as a function of the equilibrium warm electron number density, n_{wo}. The lower and upper Mach numbers are represented by solid line (—) and dashed-dotted line (– · – · –) in the figure. The dashed-dotted line (– · – · –) in region I represents M-values at which the warm electron number density becomes complex valued, and in regions II and III represent the M-values corresponding to the double layer solution. The other fixed parameters are similar to those used in Figure 1 with $n_{co} = 0.2$.

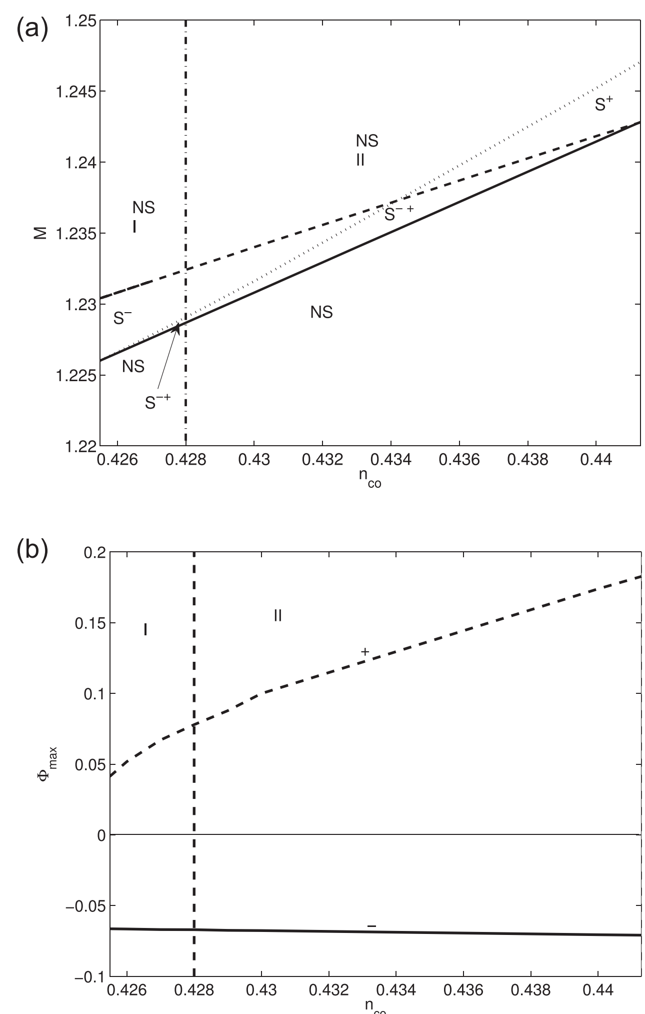

FIG. 3. Existence domains of the electron-acoustic solitons as a function of the cool electron density, n_{co} (upper panel). The upper Mach number limits in the upper panel are represented by dashed line (– – –) and dotted line (· · ·) for negative and positive potential solitons limited by the warm electron number density becoming complex valued and the occurrences of double layers in regions I and II. The solid line (—) represents the lower limit or critical Mach numbers. In the lower panel, the dashed line (– – –) and solid line (—) represent the positive and negative potential solitary waves maximum amplitudes. The fixed plasma parameters are similar to those used in Figure 2 with $n_{wo} = 0.05$.

We also follow the idea of Verheest and Hellberg[19] to study the ordering of the upper M limits on the solitary waves existence domains. It is found here that the ordering of the upper Mach number limits in the existence domains of the fast electron-acoustic solitons as depicted in Figure 2 is the same as that for slow electron-acoustic solitons studied by Mbuli *et al.*[21]

For the very first time, we report that for high frequency non-linear disturbances that we have found the coexistence of negative polarity fast electron-acoustic solitons with positive polarity structures in a three-electron plasma. These have been reported before for ion-acoustic[25,26] and dust-acoustic solitons.[27] Coexistence of high frequency electron-acoustic solitons has been reported by Kakad *et al.*[28] in a two electron temperature plasma when cool and hot ion dynamics were neglected. The region in Figure 3 marked "S^-" ("S^+") indicates where only negative (only positive) polarity solitons are supported. Both polarities of soliton are supported in the regions marked "S^{-+}." The phenomenon whereby both polarities of solitons are supported for the same set of parameters is referred to as coexistence and occurs because there is some overlap between the regions where negative polarity and positive polarity solitons are admitted.

The limits on the soliton potential (—) imposed by the warm electron density, viz., $\Phi_{min/warm}$ and double layer amplitudes (---) are shown in the lower panel of Figure 3. Referring to the upper panel, the critical values of the Mach number lie on a solid line (—). Negative polarity solitons are supported between the solid line (—) and dashed line (---). The M limits on the dashed line (---) are imposed by the warm electron density becoming unreal (Figure 3). Positive polarity solitons are also supported and these occur for increasing M above (—) until double layers are encountered on (…).

In Figure 3, regions I and II have not been demarcated according to the reason for the upper Mach number limit but rather the regions have been separated according to where two KdV-like solitons coexist (region I) and coexistence of a non-KdV like soliton with a KdV like soliton (region II). For coexistence of two KdV-like solitons, we refer the reader to the Sagdeev potentials depicted in the upper and lower panels of Figure 4 ($n_{co} = 0.428$) where M-values vary between 1.228679 and 1.23241. The upper panel is a magnified view (to clearly illustrate the coexistence of the positive and negative potential solitons). A small increase in M above M_{crit} results in wells in the Sagdeev potential for both negative ($\Phi < 0$) and positive ($\Phi > 0$) potential values, signifying the existence of both negative and positive polarity fast electron-acoustic solitons. The negative and positive polarity solitons are both KdV-like, because neither polarity soliton exists at the acoustic speed ($M = M_{crit} = 1.228679$).

It is important to mention that to the best of our knowledge the coexistence of two KdV-like solitons was never reported before for high frequency non-linear disturbances. We are aware of one occurrence of this phenomenon reported for fast ion-acoustic solitons.[25] In region II ($n_{co} > 0.428$) of Figure 3, the behaviour for the coexistence is different from that demonstrated in Figure 4. In the upper panel of Figure 5 ($n_{co} = 0.43$), there exists a negative potential fast electron-acoustic solitons at the acoustic speed ($M_{crit} = 1.230807$). The non-KdV-like character of the solitons is clearly shown in the upper panel (magnified view) of Figure 5. The negative polarity solitons are referred to as being non-KdV-like because they exist at the acoustic speed. A slight increase in M above M_{crit} results in the formation of a positive polarity soliton for $M = 1.23161$. On the other hand, such positive polarity solitons are referred to as KdV-like because these do not exist at the acoustic speed.

The lower panels of Figure 4 (Figure 5) show that negative potential solitons for $M_{crit} = 1.228679 < M < M_{max} = 1.23241 (1.230807 = M_{crit} < M < 1.23401 = M_{max})$ are limited by the warm electron density and positive solitons which occur for $M_{crit} = 1.228679 < M < M_{max} = 1.22899$ $(1.230807 = M_{crit} < M < M_{max} = 1.23149)$ limited by the occurrences of a positive potential fast electron-acoustic double layer. We find that both positive and negative potential fast electron-acoustic solitons are supported in our adiabatic model. On the other hand, although not shown here, in an isothermal model (plasma model with at-least one species (hot electrons) treated as Boltzmann distributed) as reported by Verheest and Hellberg,[19] only the negative potential solitons of fast electron-acoustic type limited by warm electron number density, n_{we}, are found to be supported.

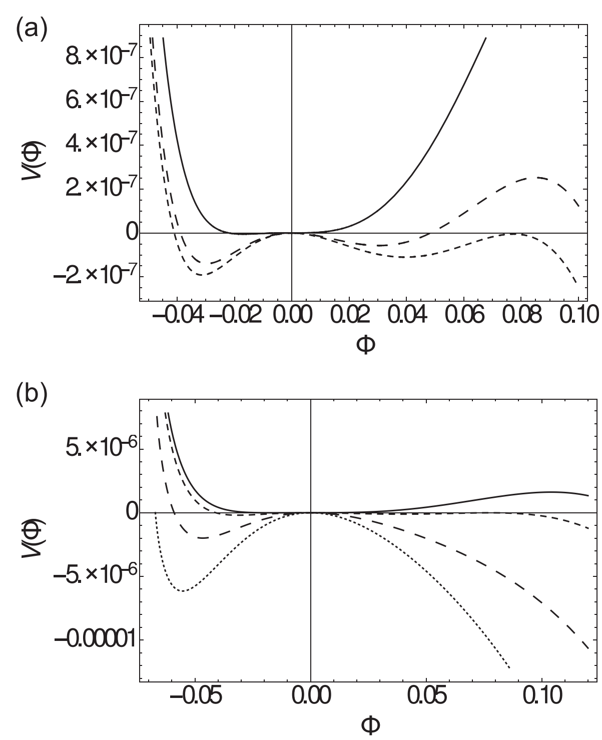

FIG. 4. The Sagdeev potential profile of the fast electron-acoustic solitary waves as a function of the electrostatic potential, Φ. The labelling parameter in the upper panel is the Mach number, M, $M = 1.228679(-)$, $M = 1.229068(- - -)$, and $M = 1.22899(\text{- - -})$. In the lower panel, the labelling parameter is the Mach number, $M = 1.228679(-)$, $M = 1.22899(\text{- - -})$, $M = 1.2305(- - -)$, and $M = 1.23241(\cdots)$. The fixed plasma parameters are those in Figure 3 with $n_{wo} = 0.05$ and $n_{co} = 0.428$. The upper panel is a zoomed-in version of the lower panel. It shows the lower Mach number limit for solitons to occur, as well as the occurrence of a double layer for $M = 1.22899$ as the upper limit for positive potential solitons. In the lower panel for the negative potential solitons, the upper Mach number limit ($M = 1.23241$) corresponds to the warm electron density becoming unreal.

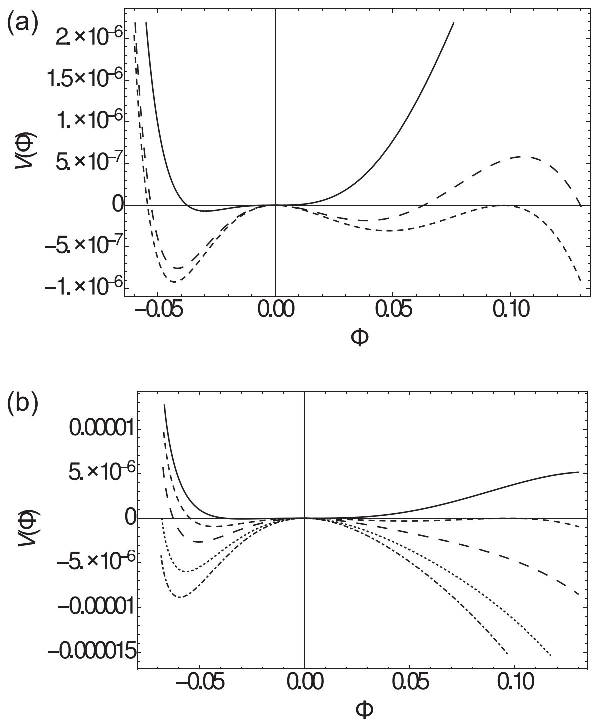

FIG. 5. Sagdeev potential profiles of the fast electron-acoustic solitons as a function of the electrostatic potential. The parameter labelling the curves in the upper panel is the Mach number $M = 1.230807(-)$, $M = 1.23161(---)$, $M = 1.23149(\text{-}\text{-}\text{-})$. In the lower panel, the labelling parameter is the Mach number, $M = 1.230807(-)$, $M = 1.23149(\text{-}\text{-}\text{-})$, $M = 1.23261(---)$, $M = 1.23401(\cdots)$, and $M = 1.23501(-\cdot-\cdot-)$. The fixed plasma parameters are those used in Figure 3 with $n_{wo} = 0.05$ and $n_{co} = 0.43$. The upper panel is a zoomed-in version of the lower panel. It shows the lower Mach number limit for the solitons to occur, as well as the occurrence of a double layer for $M = 1.23149$ as the upper limit for positive potential solitons. In the lower panel for the negative potential solitons, the upper Mach number limit ($M = 1.23401$) corresponds to the warm electron density becoming unreal.

IV. CONCLUSION

Using the Sagdeev pseudo-potential approach, we have studied large amplitude fast electron-acoustic solitary waves in a multi-electron species plasma model composed of adiabatic cool, warm, and hot electrons and cool ions. The study was conducted for parameters which are aligned with number density and temperature measurements consistent with the plasma composition in the dayside auroral zone,[22] but excluding the effects of any drifting species (beams). The existence regions in terms of admissible Mach number ranges for fast electron-acoustic solitons are presented for wide ranges of cool and warm electron densities and compared with the earlier results for slow electron-acoustic solitons.[21] Our findings show that the crossover from negative to positive polarity fast electron-acoustic solitons occurs at smaller values of n_{co} and n_{wo} than for the slow electron-acoustic solitons studied by Mbuli *et al.*[21] This may be due to different phase speed of both fast and slow electron-acoustic modes. For the dayside auroral zone plasma parameters, we found for the first time in the study of the high frequency solitons that the coexistence of two KdV-like fast electron-acoustic solitons is possible. Furthermore, we also found in a certain region of parameter space that both KdV-like (positive potential) and non-KdV-like (negative potential) fast electron-acoustic solitons coexist.

Our results show clearly that a switch in polarity from negative to positive potential solitons associated with the higher phase speed fast electron-acoustic wave is possible when the hot electrons are treated as an inertial (adiabatic) species and not as an isothermal (Boltzmann distributed) inertialess species. This finding is consistent with our earlier published results that positive polarity electron-acoustic[14] (slow electron-acoustic[21]) solitons were found when the inertia of the hot (warm) electrons is retained in a model with two (three) electron temperatures.

It is also found that the ion temperature ratio, T_i, does not have any significant effect on the electrostatic potential amplitudes of the high frequency fast electron-acoustic solitons. The negative potential acoustic speed solitons (viz., non-KdV-like fast electron-acoustic solitons) are found to be supported in an adiabatic model.

Finally, although we presented the numerical results of the fast electron-acoustic solitary waves for a wide range of Mach number M-values, the ranges of M-values for which our results are applicable such that $\sqrt{3T_{we}} \leq M \lesssim \sqrt{3}$. Our findings of the high frequency fast electron-acoustic solitons presented here could provide a better understanding of various electrostatic fluctuations such as broadband electrostatic noise (BEN) and electrostatic hiss observed by satellites in various regions of the terrestrial magnetosphere.

Our earlier nonlinear study on the slow electron-acoustic waves[21] and this study on fast electron-acoustic waves are based on a model in which all species are stationary. We are currently investigating slow and fast electron-acoustic solitons in plasma models with one and two electron beams (magnetic field-aligned) in an attempt to better model the dayside auroral zone and other plasma regions in the Earth's magnetosphere where BEN has been observed. In this paper, we considered an unmagnetised plasma model which is only justified for the electrostatic non-linear waves propagating along the background magnetic field. The numerical investigation conducted here could be extended further by including a background magnetic field and studying the effects of the plasma parameters such as magnetic field strength and propagation angle on the potential amplitudes, $|\Phi_o|$, of the fast electron-acoustic solitons as well as on the existence domains of the fast and slow electron-acoustic solitons and double layers.

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