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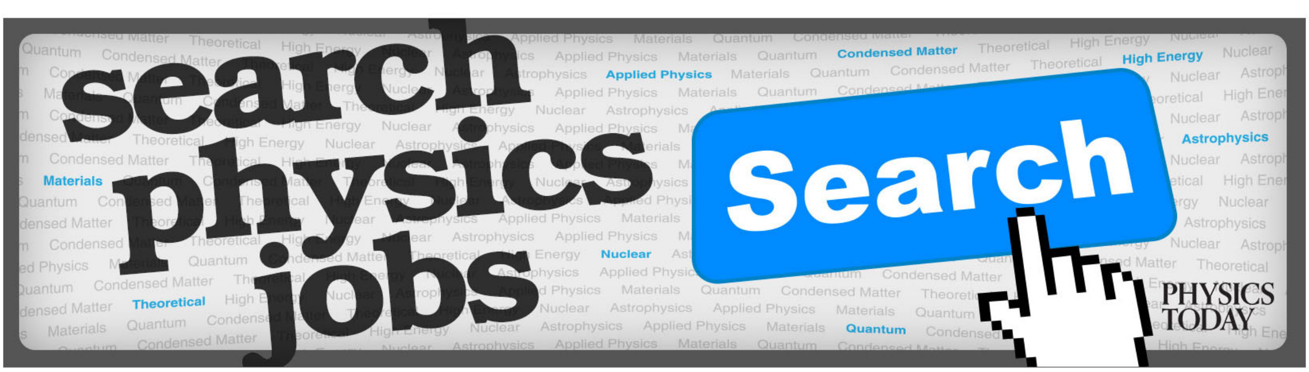

Response to "Comment on 'Existence domains of slow and fast ion-acoustic solitons in two-ion space plasmas'" [Phys. Plasmas 23, 064701 (2016)]

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In a note, Olivier *et al.*[1] have commented on the paper entitled, "Existence domains of slow and fast ion-acoustic solitons in two-ion space plasmas," Phys. Plasmas **22**, 032313 (2015), by Maharaj *et al.*[2] claiming that expressions for the Sagdeev potential given here and in two other papers by Maharaj *et al.*[3,4] are incorrect. Usually, it is the duty mainly of the first author, not of the coauthors, to defend any comments on his work. This is a peculiar and unique situation where the first and the second authors are party to a Note commenting on their own work. We are indeed in a very awkward position to write this Response. We believe that the Sagdeev potential given in Refs. 2–4 is correct. Through this Response, we shall try to clarify the misunderstanding which prompted Olivier *et al.*[1] to write the note.

Olivier *et al.*[1] mentioned at least two times in the note that despite the expression for Sagdeev potential (Eq. (11) in Ref. 2) being incorrect, all results are correct. How is it possible to get correct results from an incorrect expression? The only answer is that these authors have not understood the implications of how the Sagdeev potential has been derived.

We must point out that expressions for the densities (Eqs. (6)–(9)) and Sagdeev potential, $V(\Phi, M)$, given by Eq. (11) in Ref. 2, represent symbolic algebraic equations, and they have been derived without any assumptions on the relative temperature (or thermal velocity) of a plasma species with respect to other species or with the Mach number. In fact, the Mach number is arbitrary and is not known to start with; it is found much later from the solution of Equation (14) in Ref. 2 which arises from the soliton condition $d^2V(\Phi, M)/d\Phi^2 = 0$ at $\Phi = 0$. The choice of the plasma parameters, such as densities and temperatures of plasma species, decides the critical Mach numbers, and thus the wave mode(s).

Since the equations are in symbolic form, the operation of square rooting a squared term returns the original term as an output in a symbolic form, i.e., $\sqrt{(A-B)^2} = (A-B)$ always irrespective of the relative magnitude of A and B as both are treated just as symbols. This follows from the fact that ordinarily $\sqrt{(A-B)^2} = \pm|A-B|$ (two choices). Then, if one takes the positive (+) sign for $A > B$ and the negative (−) sign for $A < B$, this immediately leads to a single symbolic operation $\sqrt{(A-B)^2} = (A-B)$ irrespective of whether A is greater or smaller than B. For example, in a symbolic operation, we have $(A-B) - \sqrt{(A-B)^2} \equiv 0$. When this operation is done in Eqs. (6)–(9), one recovers the proper density limit for each species when $\Phi = 0$. Actually, we have faced a problem when we were computing $V(\Phi, M)$ given by Eq. (11) of Ref. 2 numerically. The computer softwares are not capable of doing a simple symbolic square rooting operation. They return only a positive quantity, e.g., $\sqrt{(A-B)^2} = (A-B)$ for $A > B$, and $(B - A)$ for $A < B$. Therefore, to do this basic symbolic square rooting operation during numerical computing, we needed to put a minus sign before $(B - A)$ for the latter case. This point was taken care of while doing the numerical computations of Equation (11) by pulling out factor equivalent of $A_{j\pm} = (M \pm C_j)$ from the square root terms appearing in expressions for densities and Sagdeev potential. If one does this operation on Eq. (11) of Ref. 2, one immediately recovers Equation (8) of Ref. 1. Had the Sagdeev potential $V(\Phi,M)$ of Ref. 2 been wrong, neither $V(\Phi,M)$ nor its first derivative with respect to Φ would vanish at $\Phi = 0$, thereby violating the first condition for the soliton solution to exist. Therefore, one cannot proceed further (nonexistence of the paper by Maharaj *et al.*[2]).

It seems that Olivier *et al.*[1] have looked at $V(\Phi,M)$ given by Eq. (11) of Ref. 2 from computation point of view. Second, they have borrowed terminology of supersonic and subsonic species from the gas-dynamics description of plasma[5–7] which is indeed a complementary description for studying solitons and has provided new insight in understanding some basic soliton properties. We have also studied soliton properties using this formalism.[8,9] The use of this terminology in Sagdeev potential approach may cause some confusion. Generally, the terms supersonic (or subsonic) refer to the motion of a body (say a jet liner) or a disturbance (like a shock wave) passing through a medium when its speed is greater (lesser) than the acoustic speed or some other characteristic speed of the medium. In Ref. 1, the terms supersonic (subsonic) are used for the plasma species depending upon whether their thermal speeds are small (large) compared to the velocity of the solitary wave. Therefore, one needs to know the velocity of the soliton or the Mach number before hand to know which species is going to behave as supersonic or subsonic. Further, in a multispecies plasma, there can be more than one wave mode, and a particular species which is supersonic with respect to a certain wave mode could be subsonic for another wave mode. Therefore, in our opinion, it is not necessary to use the terminology of the gas dynamics approach in the Sagdeev potential

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technique for studying solitons. It serves little purpose but may cause big confusion. This could have created the misconception in their mind about the correctness of Equation (11) in Ref. 2.

We agree with Oliver *et al.*[1] that equilibrium value of the densities of all species should be recovered in the limit of Φ tending to zero. Actually, taking a proper sign in the density (and Sagdeev potential) equation has been a matter of concern in several studies.[10–13] The idea of writing the Sagdeev potential with a unique sign for plasma species grew when we came across the work of Ghosh *et al.*[14] who presented densities in a different way than given in our earlier papers.[15,16] Equation (6) in Ref. 1 is written in the form first given by Ghosh *et al.*[14] If one considers the lower sign (i.e., minus sign) in this equation, and do the symbolic algebraic operation on the terms under the square root, all densities regain their equilibrium value when $\Phi = 0$. Therefore, the statement in Ref. 2 that "The choice of the lower sign (minus) in the density expressions given by Eqs. (6)–(9) is consistent with the boundary conditions" is indeed correct. One can recover Equation (7) of Ref. 1, the so called "correct expression" by just pulling out the factor $A_{j\pm}$ out of the square root terms in their Eq. (6) in just one symbolic operation without bothering about the subsonic or supersonic nature of the species. In fact, the Appendix in the Ref. 17 paper gives the procedure to do the symbolic square rooting operation, all discussions about the supersonic and subsonic are really not necessary. Similarly, one recovers Equation (8) of Ref. 1 (or Eq. (24) of Ref. 17) by pulling out $A_{j\pm}$ like terms out of the square root terms in $V(\Phi,M)$ given by Equation (11) of Ref. 2.

This brings us to the question, "Is the format of the Sagdeev potential given in Equation (8) of Ref. 1 (or Equation (24) of Ref. 17) better than other expressions in the literature?" The answer is yes and no. It may score over others as far as the ease of doing numerical computations are concerned in the sense that the computer program will be a few step shorter. At the same time, their form of Sagdeev potential has a drawback that it has introduced unnecessary singularities in the Sagdeev potential which are not real. The expression for the Sagdeev potential in Ref. 2 does not suffer from this deficiency and is superior to Equation (8) of Ref. 1 (or Equation (24) of Ref. 17). But both these forms of Sagdeev potential cannot handle a cold ($T = 0$) plasma species, and are inferior to the expression of Sagdeev potential as given in a series of paper by Lakhina *et al.*[10–12,15,16] which can handle cold as well as warm plasma species, but then here, one needs to keep track of proper signs for the density expressions for different plasma species.

To conclude, we hope our Response clears all doubts about the correctness of the Sagdeev potential given in a series of papers by Maharaj *et al.*[2–4]

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