Electron acoustic solitary waves in a magnetized plasma with nonthermal electrons and an electron beam

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A theoretical investigation is carried out to study the obliquely propagating electron acoustic solitary waves having nonthermal hot electrons, cold and beam electrons, and ions in a magnetized plasma. We have employed reductive perturbation theory to derive the Korteweg-de-Vries-Zakharov-Kuznetsov (KdV-ZK) equation describing the nonlinear evolution of these waves. The two-dimensional plane wave solution of KdV-ZK equation is analyzed to study the effects of nonthermal and beam electrons on the characteristics of the solitons. Theoretical results predict negative potential solitary structures. We emphasize that the inclusion of finite temperature effects reduces the soliton amplitudes and the width of the solitons increases by an increase in the obliquity of the wave propagation. The numerical analysis is presented for the parameters corresponding to the observations of "burst a" event by Viking satellite on the auroral field lines. *Published by AIP Publishing.*
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I. INTRODUCTION

The advent of spacecraft observations and availability of high time resolution instrument onboard has open up new era in the field of space plasmas. These observations made it possible to identify different populations of electrons and ions in the space plasma environment. Wave phenomena observed in space plasmas are no longer looked upon from the point of view of pure electron-ion plasmas. It gave a new dimension to study the wave phenomena in multi-component plasmas. One of the most common and naturally occurring waves in multi-component plasmas is electron-acoustic wave (EAW). Though it can exist in pure electron-ion plasmas but then it would require very restrictive condition of ion temperature to be much larger than the electron temperature, i.e., $T_i \gg T_e$. However, it can easily be excited in two-electron (hot and cool) and ion plasmas. Here, cool electrons play an important role in providing the inertia, and the restoring force comes from the hot electrons. A multi-component electron, i.e., two or three component electrons with distinct temperatures and ion plasmas exists in different regions of the Earth's magnetosphere. This leads to the possibility of the excitation of electron-acoustic waves in these regions. The electron-acoustic waves (EAWs) play an important role in the generation of broadband electrostatic noise (BEN) observed in various regions of the Earth's magnetosphere. Linear EAWs have been widely studied in multi-component space plasmas.[1–9] The high-time resolution measurements onboard Geotail spacecraft in the Earth's magnetosphere have shown that BENs consist of electrostatic solitary waves (ESWs).[10] These ESWs can have high electric field amplitudes sometimes in excess of 100 mV/m.[11–16]

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Dubouloz *et al.*[12,17] studied the BEN observed in the dayside auroral zone and suggested that nonlinear effects play an important role in the generation of BEN. They showed that electron-acoustic solitons can explain the high-frequency part (above the electron plasma frequency) of the BEN. Mace *et al.*[11] studied the electron-acoustic solitons in two-electron (hot and cold) and ion unmagnetized plasmas using Sagdeev potential as well as reductive perturbation methods. They showed that reductive perturbation method through which KdV solitons are studied is not suitable for intermediate strength solitons. Berthomier *et al.*[18] examined electron acoustic solitons in a four-component unmagnetized plasma consisting of cool, hot, beam electrons, and ion. They showed that electron-acoustic solitons with positive polarity potentials are possible due to the presence of an electron beam. In a similar model, Singh *et al.*[7] studied the generation of electron-acoustic solitons and applied their results to explain the Viking spacecraft observations in the dayside auroral zone. Mamun *et al.*[19] studied electron-acoustic solitary waves in a plasma consisting of cold electron, trapped/vortex like hot electrons, and ions. Singh and Lakhina[20] studied the arbitrary amplitude electron-acoustic solitons in a three-component unmagnetized plasma with cool and hot electrons and ions. The hot electrons were considered to be having Cairn's type distribution. They found that soliton solutions are restricted by nonthermal parameter, α for a chosen set of plasma parameters. This study was further extended by Singh *et al.*[16] to include an electron beam and existence domains for positive as well as negative potential electrostatic solitons/double layers were explored for auroral region parameters. Kakad *et al.*[21,22] studied electron-acoustic solitons in the magnetotail and plasma sheet boundary layer (PSBL) region. Latter have shown the existence of both rarefactive and compressive solitary potential structures. Ion- and electron-acoustic solitary waves in multi-fluid plasmas have been studied by Lakhina *et al.*[23,24] They have shown that depending on the Mach number, three types of solitons, i.e., ion-acoustic, slow ion acoustic, and electrons acoustic solitons, can be generated. It was also shown that electron-acoustic solitons can have either positive or negative potential depending on the fractional number density of the cold electrons relative to total ion or total electron number density. Lakhina *et al*.[25,26] developed theoretical models to study electron-acoustic solitons/double layers in a multi-component unmagnetized plasma which can explain the generation of ESWs in the magnetosheath and PSBL.

Several authors have examined electron-acoustic solitary waves in magnetized plasmas. Mohan and Buti[27] studied electron-acoustic solitons in a current carrying a magnetized collisionless plasma having ion temperature much greater than the electron temperature. Later on, Buti *et al*.[28] examined the nonlinear propagation of electron-acoustic waves in magnetized plasmas by considering exact electron and ion nonlinearities. Further, nonlinear propagation of electron-acoustic waves in a strongly magnetized, multi-component plasma with two ion species was investigated.[29] It was shown besides solitons, supersonic holes (density depressions) are produced by sufficiently large- amplitude perturbations. Dubouloz *et al*.[13] studied the turbulence generated in the dayside auroral zone by electron-acoustic solitons in magnetized plasmas. Berthomier *et al*.[30] showed that in a current carrying auroral plasmas the small-amplitude positive potential structures can be described by 3D electron acoustic beam solitons. Further, it was suggested that large amplitude solitary structures observed by FAST and Polar satellites evolve from the small amplitude electron-acoustic solitons. Shukla *et al*.[31] investigated multi-dimensional electron-acoustic solitons in a magnetized plasma composed of stationary ions, magnetized cold, beam fluid electrons, and hot electrons having a vortex-like distribution. Tagare *et al*.[32] studied small amplitude rarefactive electron acoustic solitons in a magnetized plasma consisting of two types of electrons (cold electron beam and background electrons) and two temperature ion plasma. Ghosh *et al*.[33] studied the existence domain of positive potential electron acoustic solitary waves in a four-component plasma composed of warm magnetized electrons, warm electron beam, and energetic multi-ion species with ions hotter than the electrons.

In recent years, a great deal of interest has been generated in the study of wave phenomena in plasmas with nonthermal distribution of particles. These nonthermal distributions which deviate from Maxwellian distributions can be found where collisions are infrequent. The nonthermal distributions such as kappa,[34] Tsallis[35] and Cairns type distributions[36] have pronounced energetic particle tails. Nonthermal distributions can also exist in the regions of strong electric fields or intense horizontal shears.[37] Cairns *et al*.[36] proposed a nonthermal distribution with excess energetic electrons to model the Freja satellite observations and showed that electrostatic ion-acoustic solitary waves with both negative and positive polarity can be generated. In the past, most of the studies involving nonthermal distribution of electrons have focussed on unmagnetized plasmas. In this paper, we study the evolution of electron-acoustic solitary waves in a four component magnetized plasma consisting of cool electrons, Cairns type hot electrons, an electron beam, and ions. We employ small amplitude approach to study these nonlinear waves in magnetized plasmas. The paper is organized as follows: In Section II, we present the theoretical model, in Section III, numerical results are discussed, and conclusions are given in Section IV.

II. THEORETICAL MODEL

We consider a four-component, magnetized collisionless plasma consisting of hot electrons, fluid cool electrons, a field aligned electron beam, and ions. In our theoretical model, cool and beam electrons as well as ions are considered to be adiabatic. An ambient magnetic field B_0 is taken along z-direction, i.e., $\mathbf{B}_0 = B_0\hat{z}$ where $\hat{z}$ is the unit vector along the z axis. The hot, nonthermal electrons follow the Cairn's type distribution given as[36]

$$f_{0h}(v) = \frac{N_{0h}}{\sqrt{2\pi v_{th}^2}} \frac{\left(1+\frac{\alpha v^4}{v_{th}^4}\right)}{(1+3\alpha)} \exp\left(-\frac{v^2}{2v_{th}^2}\right), \tag{1}$$

where N_{0h}, $v_{th} = \sqrt{T_h/m_e}$, m_e, and T_h are the equilibrium density, thermal speed, mass, and temperature of the hot electrons, respectively, and α is a non-thermal parameter which determines the population of energetic hot electrons. In the presence of non-zero potential, the distribution of electrons can be found by replacing v^2/v_{th}^2 by $v^2/v_{th}^2 - 2e\Phi/T_h$, where Φ is the electrostatic potential and e is the electronic charge. Further, the resulting distribution function can be integrated to obtain the normalized hot electron number density which can be written as[16,20]

$$n_h = n_{0h}(1-\beta\phi+\beta\phi^2)\exp(\phi). \tag{2}$$

The other normalized governing equations for electron acoustic waves in a four-component magnetized plasma can be written as

$$\frac{\partial n_j}{\partial t} + \vec{\nabla}.(n_j\vec{v}_j) = 0, \tag{3}$$

$$\mu_j n_j\left(\frac{\partial \vec{v}_j}{\partial t} + \vec{v}_j.\vec{\nabla}\vec{v}_j\right) = -\vec{\nabla}p_j + Z_j n_j\vec{\nabla}\phi - Z_j\mu_j n_j\Omega_j(\vec{v}_j\times\hat{z}), \tag{4}$$

$$\frac{\partial p_j}{\partial t} + \vec{v}_j.\vec{\nabla}p_j + 3p_j\vec{\nabla}.v_j = 0, \tag{5}$$

$$\nabla^2\phi = n_h + n_c + n_b - n_i. \tag{6}$$

Here, n_j, v_j, and ϕ are normalized number density, velocity, and electrostatic potential of the jth species, respectively. The subscript $j = c, b, i$ stands for cool and beam electrons and ions, respectively, $\mu_j = m_j/m_e; m_e$ is the mass of the electrons, $Z_j = \pm 1$ for electrons and ions, respectively, $\Omega_j = \omega_{cj}/\omega_{pe}; \omega_{cj} = eB_0/m_jc$ is the cyclotron frequency of the jth species and $\omega_{pe} = (4\pi N_0 e^2/m_e)^{1/2}$ is the electron plasma frequency.

In Equations (2)–(6), we have used the following normalizations: densities are normalized by total electron equilibrium density $N_0 = N_{0c} + N_{0b} + N_{0h} = N_{0i}$; N_{0c}, N_{0b}, N_{0h} and N_{0i} are the equilibrium cool, beam, hot electron, and ion densities, respectively, time by inverse of total electron plasma frequency ω_{pe}, velocities by hot electron thermal speed, v_{th}, lengths by effective Debye length, $(T_h/4\pi N_0 e^2)^{1/2}$ electrostatic potential by T_h/e and thermal pressure by $N_0 T_h$. We have taken adiabatic index, $\gamma = 3$.

In order to find a solution to the set of Equations (2)–(6) in the small amplitude limit, following stretching coordinates are used which have been described by Mace and Hellberg[39] and Devanandhan *et al.*[40]

$$\xi = \epsilon^{1/2}x, \quad \eta = \epsilon^{1/2}y, \quad \zeta = \epsilon^{1/2}(z - Vt), \quad \tau = \epsilon^{3/2}t, \tag{7}$$

where ϵ is a smallness parameter. All physical variables are expanded as under

$$\begin{aligned} n_j &= n_{j0} + \epsilon n_{1j} + \epsilon^2 n_{2j} + \dots\dots\dots, \\ p_j &= p_{j0} + \epsilon p_{1j} + \epsilon^2 p_{2j} + \dots\dots\dots, \\ \phi &= \epsilon\phi_1 + \epsilon^2\phi_2 + \dots\dots\dots\dots\dots\dots\dots, \\ v_{jx} &= \epsilon^{3/2}v_{1jx} + \epsilon^2 v_{2jx} + \dots\dots\dots\dots, \\ v_{jy} &= \epsilon^{3/2}v_{1jy} + \epsilon^2 v_{2jy} + \dots\dots\dots\dots, \\ v_{jz} &= v_{0j} + \epsilon v_{1jz} + \epsilon^2 v_{2jz} + \dots\dots\dots, \end{aligned} \tag{8}$$

where v_{jx}, v_{jy}, and v_{jz}, respectively, are the x, y, and z components of the velocity v and v_{0j} is the beam speed of the jth species along the magnetic field.

Substituting stretchings given by Equation (7) and variable expansions given by Equation (8) in Equations (2)–(6) and collecting the different order terms in ϵ, we get a number of relationships. From zeroth order term in ϵ, i.e., ϵ^0 terms, we obtain the quasi-neutrality condition, $n_{0c} + n_{0h} + n_{0b} = 1$. Further, from the subsequent higher order terms in ϵ linear and nonlinear set of equations are obtained and can be written as

Linear equations:

$$n_{1h} = n_{0h}(1-\beta)\phi_1, \tag{9}$$

$$-(V - v_{0j})\frac{\partial n_{1j}}{\partial\zeta} + n_{0j}\frac{\partial v_{1jz}}{\partial\zeta} = 0, \tag{10}$$

$$Z_j n_{0j}\frac{\partial\phi_1}{\partial\xi} - \frac{\partial p_{1j}}{\partial\xi} - Z_j\mu_j n_{0j}\Omega_j v_{1jy} = 0, \tag{11}$$

$$Z_j n_{0j}\frac{\partial\phi_1}{\partial\eta} - \frac{\partial p_{1j}}{\partial\eta} + Z_j\mu_j n_{0j}\Omega_j v_{1jx} = 0, \tag{12}$$

$$-\mu_j n_{0j}(V - v_{0j})\frac{\partial v_{1jz}}{\partial\zeta} + \frac{\partial p_{1j}}{\partial\zeta} - Z_j n_{0j}\frac{\partial\phi_1}{\partial\zeta} = 0, \tag{13}$$

$$-(V - v_{0j})\frac{\partial p_{1j}}{\partial\zeta} + 3p_{0j}\frac{\partial v_{1jz}}{\partial\zeta} = 0, \tag{14}$$

$$n_{1c} + n_{1h} + n_{1b} = n_{1i} \tag{15}$$

and nonlinear set of equations

$$n_{2h} = n_{0h}\left[\frac{\phi_1^2}{2} + (1-\beta)\phi_2\right], \tag{16}$$

$$\begin{aligned} &\frac{\partial n_{1j}}{\partial\tau} + n_{0j}\left(\frac{\partial v_{2jx}}{\partial\xi} + \frac{\partial v_{2jy}}{\partial\eta} + \frac{\partial v_{2jz}}{\partial\zeta}\right) \\ &\quad - (V - v_{0j})\frac{\partial n_{2j}}{\partial\zeta} + \frac{\partial}{\partial\zeta}(n_{1j}v_{1jz}) = 0, \end{aligned} \tag{17}$$

$$(V - v_{0j})\frac{\partial v_{1jx}}{\partial\zeta} - Z_j\Omega_j v_{2jy} = 0, \tag{18}$$

$$(V - v_{0j})\frac{\partial v_{1jy}}{\partial\zeta} + Z_j\Omega_j v_{2jx} = 0, \tag{19}$$

$$\begin{aligned} &\mu_j n_{0j}\frac{\partial v_{1jz}}{\partial\tau} + \frac{\partial p_{2j}}{\partial\zeta} - \mu_j(V - v_{0j})\left(n_{0j}\frac{\partial v_{2jz}}{\partial\zeta} + n_{1j}\frac{\partial v_{1jz}}{\partial\zeta}\right) \\ &\quad + \mu_j n_{0j}v_{1jz}\frac{\partial v_{1jz}}{\partial\zeta} - Z_j\left(n_{1j}\frac{\partial\phi_1}{\partial\zeta} + n_{0j}\frac{\partial\phi_2}{\partial\zeta}\right) = 0, \end{aligned} \tag{20}$$

$$\begin{aligned} &\frac{\partial p_{1j}}{\partial\tau} + v_{1jz}\frac{\partial p_{1j}}{\partial\zeta} - (V - v_{0j})\frac{\partial p_{2j}}{\partial\zeta} \\ &\quad + 3p_{0j}\left(\frac{\partial v_{2jx}}{\partial\xi} + \frac{\partial v_{2jy}}{\partial\eta} + \frac{\partial v_{2jz}}{\partial\zeta}\right) + 3p_{1j}\frac{\partial v_{1jz}}{\partial\zeta} = 0, \end{aligned} \tag{21}$$

$$\frac{\partial^2\phi_1}{\partial\xi^2} + \frac{\partial^2\phi_1}{\partial\eta^2} + \frac{\partial^2\phi_1}{\partial\zeta^2} = n_{2c} + n_{2h} + n_{2b} - n_{2i}. \tag{22}$$

Using a set of Equations (9)–(15), a linear dispersion relation for electron-acoustic waves for a model having cool and hot electrons and a drifting electron beam and ions is given by

$$n_{0h}(1-\beta) - \frac{n_{0c}}{V^2 - 3\sigma_c} - \frac{n_{0b}}{(V - v_{0b})^2 - 3\sigma_b} - \frac{1}{\mu_i V^2 - 3\sigma_i} = 0, \tag{23}$$

where $\mu_i = m_i/m_e$ is the ion to electron mass ratio, $\sigma_c = T_c/T_h$, $\sigma_i = T_i/T_h$, and $\sigma_b = T_b/T_h$. It must be pointed out here that dispersion relation (23) is the same as given by Singh *et al.*[16] for an unmagnetized four-component plasma consisting of cool electrons, hot Cairns type nonthermal electrons, beam electrons, and ions. Further, it is worth mentioning that the present model is valid for a weakly magnetized plasma as magnetic field term appears through higher order terms in "C" (see Equation (25)). In the absence of beam electrons and for Maxwellian ($\alpha = 0$) hot electrons, the above Equation (23) reduces to that of Mace and Hellberg.[39] The Korteweg-de-Vries-Zakharov-Kuznetsov (KdV-ZK) equation for nonlinear electron-acoustic waves in a magnetized plasma is obtained by using a linear set of Equations (9)–(15) in Equations (16)–(22) and eliminating the second order variables, which after some algebraic manipulations can be written as

$$\frac{\partial\phi_1}{\partial\tau} + a\phi_1\frac{\partial\phi_1}{\partial\zeta} + b\frac{\partial^3\phi_1}{\partial\zeta^3} + c\frac{\partial}{\partial\zeta}\left(\frac{\partial^2\phi_1}{\partial\xi^2} + \frac{\partial^2\phi_1}{\partial\eta^2}\right) = 0. \tag{24}$$

The coefficients a, b, and c in Equation (24) are given by

$$a = B/A, \quad b = 1/A \quad \text{and } c = C/A,$$

where

$$A = \sum_j \frac{2Z_j^2 \mu_j n_{0j}(V - v_{0j})}{\left[\mu_j (V - v_{0j})^2 - 3T_j\right]^2},$$

$$B = -\left[n_{0h} + \sum_j \frac{3Z_j^3 n_{0j}\left[\mu_j (V - v_{0j})^2 + T_j\right]}{\left[\mu_j (V - v_{0j})^2 - 3T_j\right]^3}\right],$$

$$C = 1 + \sum_j \frac{\mu_j n_{0j}(V - v_{0j})^4}{\Omega_j^2 \left[\mu_j (V - v_{0j})^2 - 3T_j\right]^2}. \tag{25}$$

It is worth mentioning that the coefficients A, B, and C look similar to Mace and Hellberg[39] for their three-component plasma model. However, our model is a four-component which is essentially more general and takes care of beams for all species except for hot electrons which follow Cairn's type distribution, whereas their model considers Maxwellian distribution for hot electrons. Also, variable V appearing in the coefficients A, B, and C must satisfy the dispersion relation given in Equation (23). Further, in the absence of beam electrons and non-thermality, we recover all analytical results of Mace and Hellberg.[39]

For simplicity, we consider the waves to be propagating in the x–z plane, so that there are no variations along y-axis (i.e., $\frac{\partial}{\partial y} = 0$). The plane solitary wave solution is obtained by using the transformation $Z = \zeta \cos\psi + \xi \sin\psi - U\tau$ in the KdV-ZK Equation (24) and can be written as

$$\phi_1 = \frac{3U}{a\cos\psi} \sec h^2 \left[\frac{1}{2}\left(\frac{U}{\cos\psi\left(b\cos^2\psi + c\sin^2\psi\right)}\right)^{1/2} Z\right]. \tag{26}$$

The stationary solution of the above equation is obtained by assuming $\phi = \epsilon\phi_1$

$$\phi = \frac{3\delta V}{a} \sec h^2 \left[\frac{1}{2}\left(\frac{\delta V}{b\cos^2\psi + c\sin^2\psi}\right)^{1/2} \times (x\sin\psi + z\cos\psi - M\cos\psi t)\right], \tag{27}$$

where $\delta V = \epsilon U/\cos\psi$. Let us define the amplitude ϕ_0 and width Λ of the soliton in Eq. (27) as

$$\phi_0 = \frac{3\delta V}{a}, \tag{28}$$

$$\Lambda = 2\left[\frac{b\cos^2\psi + c\sin^2\psi}{\delta V}\right]^{1/2}. \tag{29}$$

The solution ϕ and its first derivative vanish at $Z \to \pm\infty$. Here, U and ψ correspond to the soliton speed and the wave propagation direction, respectively.

From Equation (29), we find that the soliton width, Λ increases with the increase in angle of propagation, ψ and it is bounded by $(4b/\delta V)^{1/2} \le L \le (4c/\delta V)^{1/2}$. The soliton width is found to be maximum at perpendicular propagation (i.e., $\psi = 90°$).[38–40] In Section III, numerical results are discussed.

III. NUMERICAL RESULTS AND DISCUSSION

In this section, we present numerical computations of Equations (27) and (29) to evaluate the amplitude and width of the nonlinear electron acoustic structures in a magnetized four component plasma. For the numerical results, auroral region parameters are used from the Viking satellite observation of burst a.[13] These observed parameter are, namely, cold electron density $N_{0c} = 0.5\,\mathrm{cm}^{-3}$, hot electron density $N_{0h} = 2.0\,\mathrm{cm}^{-3}$, beam electron density $N_{0b} = 1.0\,\mathrm{cm}^{-3}$, hot electron temperature $T_h = 250\,\mathrm{eV}$, and ambient magnetic field, $B_0 = 3570\,\mathrm{nT}$. Other normalized parameters used here are: $\delta V = 0.05$, $v_{0b} = 0.1$, $\Omega_{c,b} = 6.78$, $\Omega_i = 3.68 \times 10^{-3}$ and angle of propagation is assumed to be $\psi = 30°$. The effect of nonthermality on electron acoustic solitary structures has been studied in detail for the observed parameters mentioned earlier, and the comparison of soliton profiles of Maxwellian ($\alpha = 0$) with various nonthermal electron population ($\alpha = 0.1$, 0.2) is shown in Figure 1. It is observed from the figure that the solitary wave amplitude decreases with an increase in nonthermality of hot electrons which is similar to the case for unmagnetized plasmas,[20] where the highest amplitude is reported for the Maxwellian ($\alpha = 0$) case. In the Singh and Lakhina's[20] model, arbitrary amplitude analysis has been carried out and computations are done for fixed value of Mach number and varying the nonthermality whereas, in the present case, small amplitude analysis has been carried out where the Mach number, V has to be calculated for each value of nonthermal parameter. Further, the width of the solitons increases with the increase in nonthermality. For the parameters mentioned above, the soliton speed and width are found to be in the range 6339–8684 km s^{-1} and 2890–4775 m, respectively, for $\alpha = 0 - 0.2$. The corresponding electric field amplitudes are found to be in the range 5.8–3.2 mV/m for $\alpha = 0.0$–0.2.

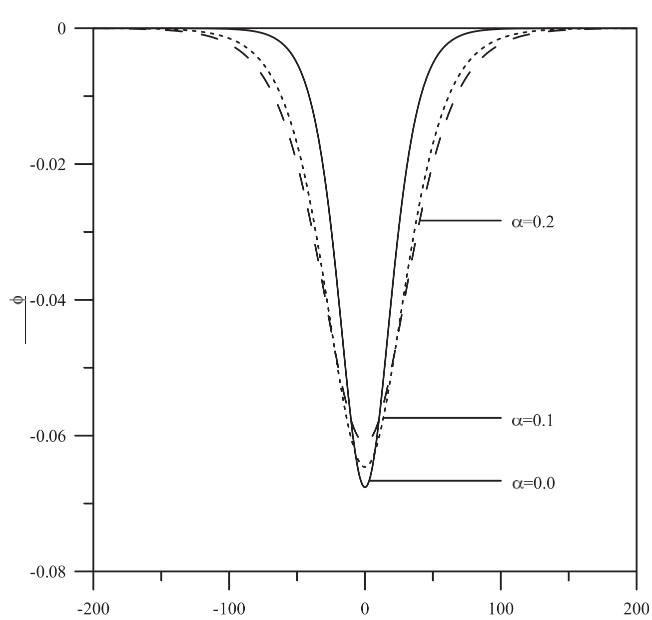

FIG. 1. Variation of a soliton profile against nonthermal parameter $\alpha = 0.0$ ($V = 0.950428$), $\alpha = 0.1$ ($V = 1.22277$), $\alpha = 0.2$ ($V = 1.304062$), where $N_{0b} = 1.0\,\mathrm{cm}^{-3}$, $N_{0h} = 2.0\,\mathrm{cm}^{-3}$, $N_{0c} = 0.5\,\mathrm{cm}^{-3}$, $\sigma_c = \sigma_i = 0.001$, $\sigma_b = 0.01$, $B_0 = 3570\,\mathrm{nT}$, $\psi = 30°$, $\delta V = 0.05$, $v_{0b} = 0.1$, $\Omega_{c,b} = 6.78$, and $\Omega_i = 3.69 \times 10^{-3}$.

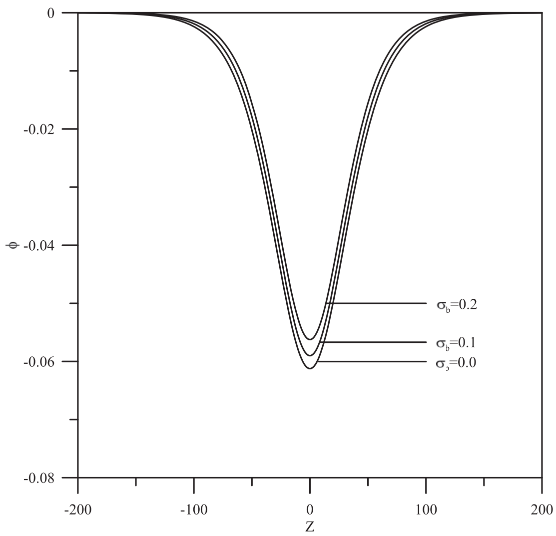

FIG. 2. Variation of a soliton profile against beam electron temperature $\sigma_b = 0.0$ (V = 1.295052), $\sigma_b = 0.1$ (V = 1.386194), $\sigma_b = 0.2$ (V = 1.477983), where $\alpha = 0.2$ and other parameters are same as Fig. 1.

The numerical results are in agreement with the observed electric field amplitudes of the Viking satellite data.

Next, we study the effect of electron beam temperature on the evolution of these nonlinear electron acoustic waves for the parameters of Figure 1 and $\alpha = 0.2$. The beam to hot electron temperature ratio, $\sigma_b = T_b/T_h$ is varied from 0.0 to 0.2 which is illustrated in Figure 2. It is observed that solitary wave amplitude as well as width decrease with an increase in beam temperature. These results are similar to the one obtained by Singh *et al.*[7] for four-component unmagnetized case where hot electrons are treated as Maxwellian. The solitary wave speed and width are found to be 8624–9837 km s^{-1} and 4775–4398 m, respectively, for $\sigma_b = 0.0$–0.2.

The effect of electron beam speed is studied on the electron acoustic solitary structures for the parameters of Figure 1 and $\alpha = 0.05$ and results are shown in Figure 3. The electron beam speed is varied from $v_{0b} =$ 0.0–1.0. The maximum amplitude occurs for a beam speed, $v_{0b} = 0$ and as the beam speed increases, both soliton amplitude as well as width decrease. Similar results were obtained by Singh *et al.*,[16] where they carried out the arbitrary amplitude theory of electron-acoustic solitons in an unmagnetized four-component plasma having nonthermal hot electrons.

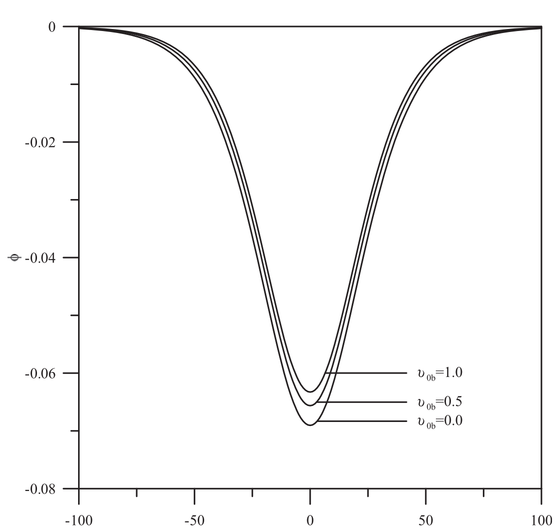

FIG. 3. Variation of soliton profile against beam velocities $v_{0b} = 0.0$ (V = 0.964468), $v_{0b} = 0.5$ (V = 1.367667), $v_{0b} = 1.0$ (V = 1.833919), where $\alpha = 0.05$ and other parameters are the same as Fig. 1.

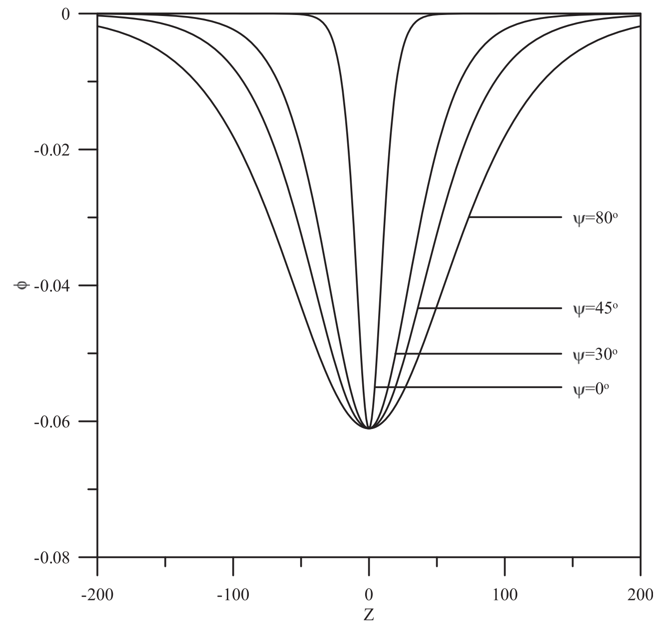

FIG. 4. Variation of soliton profile for different values of propagation angle $\alpha = 0^\circ, 30^\circ, 45^\circ, 80^\circ$, (V = 1.304062), where $\alpha = 0.2$ and other parameters are the same as Fig. 1.

The effect of obliquity (angle of propagation, ψ) on the propagation of solitary structures is shown in Figure 4. It can be seen from the figure that the maximum soliton amplitude remains constant, but width increases with an increase in the value of ψ. This can be attributed to the fact that the Mach

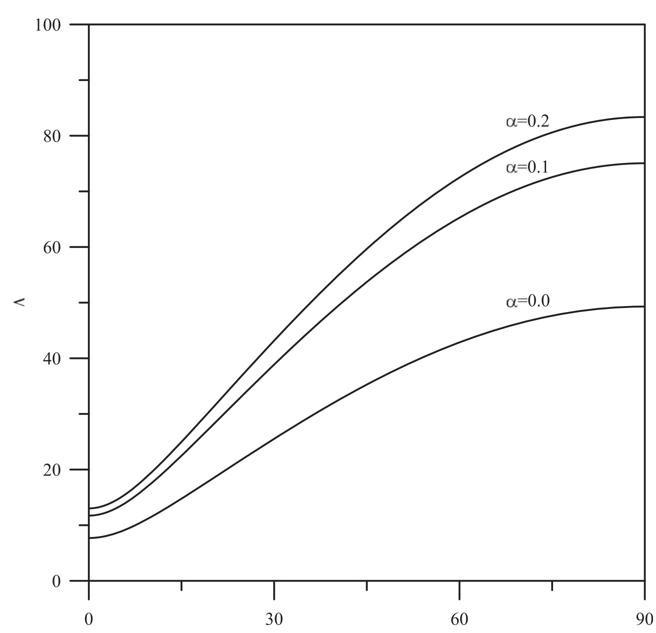

FIG. 5. The soliton width versus propagation angle for various values of nonthermal parameter, α, and other parameters are the same as Fig. 1.

number, V given by Equation (23) does not depend on angle of propagation, therefore, irrespective of any angle of propagation, it remains constant and thus maximum soliton amplitude remains unchanged. For the observed parameters mentioned earlier, the soliton width and electric field amplitudes are found to be in the range 1382–9047 m and 11–1.7 mV/m, respectively, for $\psi = 0^\circ - 80^\circ$.

In Figure 5, we have shown the behaviour of the width of the soliton for various values of the nonthermal parameter, α as shown on the curves. The soliton width increases with an increase in propagation angle as well as with nonthermality which was also evident from Figure 1.

IV. CONCLUSIONS

The present study of nonlinear electron acoustic waves has been motivated by the observations of solitons in dayside auroral zone by Viking satellite[13] and the Cairn's nonthermal distribution function[36] in the auroral zone. We have devoted this study to discuss evolution of electron acoustic solitary waves in a four component, magnetized plasma consisting of cool electrons, hot non-thermal electrons, beam electrons, and adiabatic ions. We have employed a small amplitude theory to derive the KdV-ZK equation and solve it for the plasma parameters corresponding to "burst a" observed by Viking spacecraft to obtain negative potential electron acoustic solitary structures. However, it must be emphasized that sign of coefficient "a" appearing Eq. (24) will determine the polarity of the solitons. Therefore, for some other set of parameters, one may get positive potential solitons. It is found that the soliton velocities and the width increase with an increase in nonthermality, however, their amplitude decreases. The solitary wave amplitude decreases with an increase in electron beam temperature as well as streaming of electrons. On the other hand, the maximum amplitude of the electron acoustic solitons is not affected with the increase in angle of propagation, however, soliton width increases.

Earlier, Singh and Lakhina[20] have studied electron-acoustic solitary waves using the Sagdeev potential method in a three-component unmagnetised plasma consisting of fluid cold electrons, hot electrons having a nonthermal distribution, and fluid ions. They found that the presence of nonthermal electrons modifies the existence regime of electron acoustic solitons and electric field amplitudes were in the range of ∼2–100 mV/m for the auroral region parameters. Later on, the above study was extended to include an electron beam.[16] The focus of the study was on the existence domains of both positive as well as negative potential electrostatic solitons and double layers. It was found that the inertia of the warm electrons is essential for the existence of positive potential solitary structures and not the beam speed. The electric field amplitudes for the parallel propagating negative potential electrostatic solitary waves were found to be in the range of ∼3–30 mV/m. Thus, inclusion of an electron beam reduces the electric field amplitude appreciably. On the other hand, in this paper, the study in Ref. 16 is further extended to a magnetised plasma case, and the reductive perturbation method is used to study the small amplitude electron-acoustic solitary waves. Here, we obtain negative potential solitons for the same parameters as used in the above mentioned Refs. 16 and 20. In the magnetised case studied here, electric field amplitude of the negative potential electrostatic solitary waves is found to be in the range of ∼3–6 mV/m which is significantly lower than the unmagnetized case.[16] It must be emphasized that the electric field amplitude is lowest for the magnetised case as compared to the two unmagnetized cases discussed above.

The present model has a limitation that it can explain solitary potential structures in weakly magnetized plasmas, because the effect of the magnetic field appears in the second order terms only. The model can be improved by considering higher order terms or one has to find different stretching scheme which can bring out magnetic field effects at the linear as well as higher levels.

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