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Intermediate coupling model description of ^{55}Fe and ^{57}Fe

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Abstract. The properties of the negative parity states of ^{55}Fe and ^{57}Fe are investigated in the framework of the intermediate coupling model. In the model, a neutron or a quasineutron is coupled to anharmonic vibrations of the core. Anharmonicities of the vibrations are estimated through the observed properties of the core. Energy levels, spectroscopic factors and electromagnetic properties have been calculated. The results of the present calculations are also compared with available experimental results and other theoretical results. The model reasonably accounts for many of the properties of the low-lying states.

Keywords. Nuclear spectroscopy; transition rates; intermediate coupling model; ^{55}Fe; ^{57}Fe.

1. Introduction

In recent years, f-p shell nuclei have been extensively studied, both experimentally as well as theoretically. An exact shell model calculation for ^{55}Fe and ^{57}Fe will need diagonalization of a prohibitively large energy-matrix and even then the physical picture is rather difficult to comprehend. In all shell model calculations reported so far, ^{48}Ca is assumed as core (Ohnuma 1966, Vervier 1966, Hamamoto and Arima 1962, McGrory 1966). Also, the proton space is truncated. The residual two-body interaction is either parametrized or derived from the spectra of neighbouring nuclei. Though they give a reasonable fit so far as the energy spectrum is concerned, the same is not true for other spectroscopic properties.

Although it remains possible that a careful admixture of different configurations through an enlarged basis space may yield better results, it is equally legitimate to search for an alternative, simpler approach to this problem. Such a search becomes particularly imperative in view of the consistent enhancement by an order of magnitude of some of the electromagnetic transition rates in this region of the f-p shell.

A configuration mixing in the shell model can be simulated by introducing deformed basis and a Coriolis interaction. Comfort *et al* (1971) have studied ^{57}Fe using this model, but the results are not encouraging.

Vibrations are another mode of nuclear core-excitations. Intermediate coupling model (ICM) has been successful in explaining the observed spectroscopic properties of nuclei in s-d shell (Thankappan and Pandya 1960, Bailey and Choudhury 1970, Castel *et al* 1971), in f-p shell (Carola and Ohnuma 1971, Castel *et al* 1972, Paar 1972, Dayras and Čujec 1972) and in s-d-g shell (Almar *et al* 1973, Naquib *et al* 1973). (The list is only representative). The f-p shell nuclei, for which ICM has achieved credibility, lie, in the periodic table, on both sides of nuclei ^{55}Fe and ^{57}Fe. Carola and Ohnuma (1971) and Csürös *et al* (1971) have studied ^{55}Fe and Paar (1972) has studied ^{57}Fe in ICM prescription. Carola and Ohnuma (1971) and Csürös *et al* (1971) consider harmonic vibrations; Paar (1972) uses Alaga model which tries to take into account the anharmonicities of the core. For consistency and simplicity we couple a neutron to ^{54}Fe and a quasineutron to ^{56}Fe to study ^{55}Fe and ^{57}Fe respectively. This difference has its origin in the fact that ^{54}Fe has 28 neutrons and since 28 is a magic number the $f_{7/2}$ shell is considered closed. However, in ^{56}Fe the neutron number is 30 and the last two neutrons may occupy any of the $p_{3/2}$, $f_{5/2}$ or $p_{1/2}$ orbitals. In that case one should use quasiparticle states rather than particle states. Our calculation for ^{55}Fe differs from those of Carola and Ohnuma (1971) and of Csürös *et al* (1971) in inclusion of the anharmonicity of the core and that for ^{57}Fe differs from calculation of Paar (1972) in the choice of configuration. Though Paar (1972) treats ^{54}Fe as a vibrating core there is *a priori* no reason to consider ^{54}Fe a better vibrator than ^{56}Fe.

In section 2 we briefly give the essential features of formalism of ICM. In section 3 the results of present calculations are discussed. Section 4 contains the conclusions.

2. Formalism

2.1 *Harmonic version*

In the classical intermediate-coupling model, the total Hamiltonian H_t is assumed to consist of three terms.

$$H_t = H_c + H_{sp} + H_{int} \tag{1}$$

Here H_c is the Hamiltonian of a harmonically vibrating core, H_{sp} is the single-particle shell model Hamiltonian and H_{int} accounts for the phonon-particle interaction energy. The chosen basis wavefunctions are those which make $H_c + H_{sp}$ diagonal and are written as $|\alpha j; NR : IM\rangle$; here α is the set of quantum numbers which completely describe the state of the odd nucleon of angular momentum j; N and R are the number of phonons and the spin of the core state respectively and I is the total angular momentum of the coupled system ($\boldsymbol{I} = \boldsymbol{j} + \boldsymbol{R}$). The Z-component of I is represented by M. Thus, ignoring zero-point energy,

$$(H_c + H_{sp})\,|\alpha j; NR : IM\rangle = (N\hbar\omega + \epsilon_j)\,|\alpha j; NR : IM\rangle \tag{2}$$

where $\hbar\omega$ is the energy of the quadrupole-phonon and ϵ_j is that of the odd nucleon. (Only quadrupole oscillations are considered here). The particle-phonon interaction, which is taken to be linear in the collective co-ordinates of the core has the form:

$$H_{\text{int}} = \mp k(r)\left(\frac{\hbar\omega}{2C}\right)^{\frac{1}{2}} \sum_{\mu} Q_{2\mu} Y_{2\mu}(\theta, \phi) \tag{3}$$

which in the chosen basis, has the matrix elements

$$\begin{aligned} &\langle \alpha' j'; N'R' : IM \mid H_{\text{int}} \mid \alpha j; NR : IM \rangle \\ &= \mp (-)^{I-j'} \left(\frac{\hbar\omega}{2C}\right)^{\frac{1}{2}} \langle \alpha' j' \mid k(r) \mid \alpha j \rangle\, W(RR'jj'; 2I) \\ &\quad \times \langle l'sj' \parallel Y_2 \parallel lsj \rangle \langle N'R' \parallel Q_2 \parallel NR \rangle \end{aligned} \tag{4}$$

where C is the nuclear stiffness parameter and $W(RR'jj'; 2I)$ is a Racah coefficient. The expression for the reduced matrix elements of the single-particle operator $Y_{2\mu}(\theta, \phi)$ has been given by de-Shalit and Talmi (1963); the choice of sign in (3) and (4) which originates from this matrix element is negative for a particle and positive for a hole. The selection rules for the interaction matrix element (4) are: $\triangle N = \mp 1$, $\triangle R \leq 2$, $\triangle j \leqslant 2$ and $\triangle l = 0, 2$.

The matrix elements $\langle \alpha' j' \mid k(r) \mid \alpha j \rangle$ are a measure of the coupling strength. Their magnitude was treated as a parameter k. This is usually expressed in terms of a dimensionless parameter ξ defined by

$$\langle \alpha' j' \mid k(r) \mid \alpha j \rangle = k = \left(\frac{2\pi\hbar\omega C}{5}\right)^{\frac{1}{2}} \xi \tag{5}$$

The diagonalization of H_t gives the energies and wave-functions of the coupled system. The latter are expressed as:

$$\mid E; IM \rangle = \sum_{\substack{\alpha j \\ NR}} a_E(\alpha j; NR : I) \mid \alpha j; NR : IM \rangle \tag{6}$$

In $\mid E, IM \rangle$, E denotes the energy of the state with spin I and the Z-component M. This form of the wave-function is then used to calculate the E2 and the M1 matrix elements.

Detailed expressions for the E2 and the M1 transition rates and static moments can be found elsewhere in the literature (*e.g.*, Heyde and Brussard 1967); they are given in the following abbreviated form:

$$\begin{aligned} &B(\text{E2}, I \to I') \\ &= (2I'+1)\left\{\frac{3}{5} R_0^2 \left(e_n + \frac{Ze}{A^2}\right) A_1 + \frac{3}{4\pi} ZeR_0^2 \left(\frac{\hbar\omega}{2C}\right)^{\frac{1}{2}} B_1\right\}^2 \end{aligned} \tag{7}$$

$$\begin{aligned} Q(E; I) &= \left[\frac{I(2I-1)(2I+1)}{(I+1)(2I+3)}\right]^{\frac{1}{2}} \left(\frac{16\pi}{5}\right)^{\frac{1}{2}} \\ &\quad \times \left\{\frac{3}{5} R_0^2 \left(e_n + \frac{Ze}{A^2}\right) A_1 + \frac{3}{4\pi} ZeR_0^2 \left(\frac{\hbar\omega}{2C}\right)^{\frac{1}{2}} B_1\right\} \end{aligned} \tag{8}$$

$$B(\text{M1}; I \to I') = \frac{3}{4\pi} \mu_N^2 (2I'+1)(g_R D + g_l E + g_s F)^2 \tag{9}$$

$$\mu(E; I) = \left[\frac{I(2I+1)}{(I+1)}\right]^{\frac{1}{2}} \mu_N (g_R D + g_l E + g_s F) \tag{10}$$

where μ_N is the nuclear magneton. The symbols A_1, B_1, D, E and F stand for expressions which involve double summations over the components of initial and final states.

2.2 *Anharmonic version*

So far we have discussed a simple picture where the vibrations of the core are harmonic and these are coupled to particle states. However, the properties of low-lying states of both ^{54}Fe and ^{56}Fe show large deviations from pure harmonic picture and also the partial occupancies of single particle states observed in reality make the concept of particle states rather naive. Actually we should talk of quasiparticle states coupled to anharmonic vibrations. This concept has been elaborately discussed by Castel *et al* (1971) and we give here only its salient features.

(i) The core Hamiltonian H_c is modified such that the energy of a two-phonon core state of spin-R is given by $(2+\eta_R)\,\hbar\omega$, where the anharmonicity parameters η_R are derived from the experimental core spectrum.

(ii) The assumption that the quadrupole operator $Q_{2\mu}$ is proportional to the phonon operator $[b_\mu + (-)^\mu b_{-\mu}]$, which underlines the harmonic approximation, is dropped. Thus instead of using the harmonic values for the off-diagonal and the diagonal matrix elements of $Q_{2\mu}$, these elements are taken from experimental B(E2) and quadrupole moment values of the core respectively. [Equations (7) and (8) in Castel *et al* 1971].

(iii) The single-particle reduced matrix-elements which appear in the formulae for the interaction Hamiltonian (eq. 4) and for the E2 matrix elements (expressions for A_1 in eqs (7) and (8)) should be multiplied by the factor $(u_j u_{j'},\ - v_j v_{j'},)$ where u_j and v_j represent the quasihole and quasiparticle amplitudes respectively in the state j. The sign in eq. (3) should now be positive as the dependence on whether a hole or a particle is being considered is now taken care of by the quasiparticle factor. A multiplication factor $(u_j u_{j'} + v_j v_{j'})$ should be used for the single-particle matrix elements in the formulae for M1 matrix elements [expression E and F in eqs (9) and (10)].

For ^{55}Fe we found a good fit with coupling of particle states to the ^{54}Fe-core states but for ^{57}Fe we coupled quasineutron states to the ^{56}Fe core vibrations.

3. Results and discussion

3.1 *Nucleus ^{55}Fe*

Here ^{54}Fe is taken as a vibrating core. Core vibrations only up to two-phonons are considered. In the core, $f_{7/2}$ is considered completely filled by neutrons. In this sense, the model consists a neutron-particle (and not quasi-particle)-coupled to the core, ^{54}Fe. The neutron is allowed to be in single particle states $p_{3/2}$, $f_{5/2}$ and $p_{1/2}$.

Energy spectrum: To get a good fit to the experimental spectrum, dimensionless parameter ξ, and single particle energies of states $f_{5/2}$ and $p_{1/2}$ relative to $p_{3/2}$ were taken as free parameters. The value of ξ is so chosen that the level order and separation between the various levels in the calculated spectrum bears a close resemblance to the observed one. Then the value of $\hbar\omega$—which is a scale factor—is selected such that the two spectra match best. Earlier study by Castel *et al.* (1972) has shown that the energy spectra are very sensitive to the quadrupole moment of the 2_1^+ state. Therefore, this was also used as a free parameter. The stiffness parameter C is deduced from the electromagnetic data. (This will be discussed later). The set of parameters finally obtained are shown in table 1. The value of $\hbar\omega$ is 1·0 MeV which is lower than the excitation energy (1·4 MeV) of 2_1^+ state in ^{54}Fe. The value of ξ is 3·0 which is within the reasonable range. These values along with the value of C give for the coupling strength k (eq. 5), the value 40·1 MeV.

From the study, we also calculate the quadrupole moment of 2_1^+ state to be (− 0·014) barn and this should be close to the experimental quadrupole moment of the 2_1^+ state in ^{54}Fe. The experimental value of the same is not known.

The resulting spectrum is shown in figure 1. One sees that the agreement with the experimental spectrum is quite good. Also, the present calculations yield better density of states than others. Modifications to classical ICM due to anharmonicities have proved successful in getting $1/2^-$ state at $\simeq$ 2 MeV. This is a major achievement compared to the work of other authors. However, the model does not account for the $7/2^-$ and $9/2^-$ states observed at 1·41 MeV and 2·21 MeV respectively. They are strongly excited in pickup reactions (Pilt *et al* 1970) and are strongly connected electromagnetically (Robertson *et al* 1972). Therefore, they seem to arise from the creation of a hole in the $f_{7/2}$ sub-shell. Such hole states fall outside the scope of present prescription. We obtain in our spectrum some high spin states also. Though Carola and Ohnuma (1971) consider core excitations up to three phonons, they do not report in their spectrum such high spin states. Further experimental data is awaited to test, in our spectrum, the position of $13/2^-$ state, the only possible $13/2^-$ state in the present prescription.

Table 2 gives the spectroscopic factors for the reaction ^{54}Fe (d, p) ^{55}Fe. Our model has obtained remarkable improvement over the harmonic ICM calculations of Carola and Ohnuma (1971). The state at 1·93 MeV is assigned spin $1/2^-$ (Kocher and Haeberli 1972). However, the $(2J+1)\,S$ listed for this level in table 2 for the calculation of Carola and Ohnuma (1971) is based on a spin assignment of 3/2. Their calculation does not predict any level with spin 1/2 around 2·0 MeV excitation energy. We note that, in the present prescription we have considered $f_{7/2}$ orbit full and this results in zero spectroscopic factor for the $7/2^-$ level.

Electromagnetic properties: From the wavefunctions obtained, the electromagnetic properties are studied. No effective charge is attributed to the neutron, orbital gyromagnetic ratio g_l is zero and the core gyromagnetic ratio g_R has its hydrodynamical value as given by Z/A. Over-all good fit to the experimental results is obtained by using stiffness parameter C and spin gyromagnetic ratio g_s as free parameters. The stiffness parameter C had the value 142 MeV. Wong (1968) reports this to be (113 ± 10) MeV. Our value of C yields the deformation parameter β_2 given by $\sqrt{5\hbar\omega/2C}$ equal to (0·13) for the ^{54}Fe core. The experimental value as compiled by Verheul (1970) is (0·17). Magnetic properties are fitted by using g_s as a free parameter. The value finally obtained is about 0·4 times g_s for a free neutron.

In the absence of experimental results, we report only ground state moments for ^{55}Fe. We have obtained (− 0·09) barn for electric quadrupole moment and (− 0·86) n.m. for magnetic dipole moment of the ground state. The values obtained by Carola and Ohnuma (1971) for the same are (− 0·125) barn and

Table 1. Values of the parameters used in computations

Core	$Q_{2_1^+}$(e.b)	$\hbar\omega$ (MeV)	η_0	η_2	η_4	$\epsilon_1 = p_{1/2} - p_{3/2}$ (MeV)
^{54}Fe	−0·014*	1·0*	0·18	−0·1	0·2	1·56*
^{56}Fe	−0·18*	0·85	1·46	1·13	0·45	2·63

Core	$\epsilon_2 = f_{5/2} - p_{3/2}$ (MeV)	$v_{1/2}$	$v_{3/2}$	$v_{5/2}$	$v_{7/2}$	ξ	C (MeV)
^{54}Fe	1·42*	0·0	0·0	0·0	1·0	3·0*	142*
^{56}Fe	0·93	0·19	0·57	0·32	1·0	2·5*	80*

* These were used as free parameters.

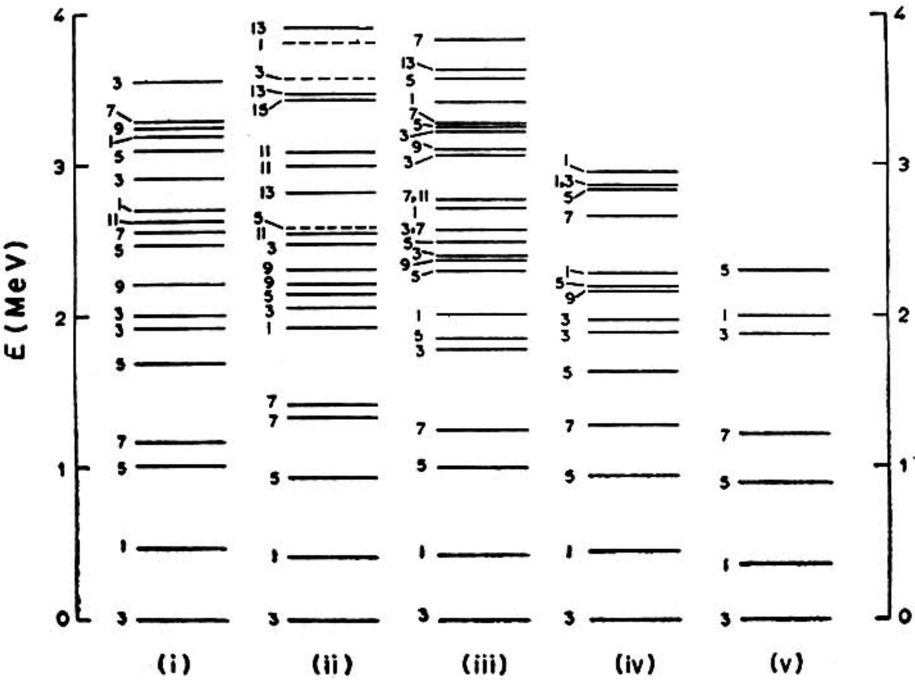

Figure 1. Experimental and theoretical level schemes for ^{55}Fe. All spin values are multiplied by two and refer to negative parity states. (i) ICM calculations by Carola and Ohnuma (1971). (ii) Experimental spectrum (Pilt *et al* 1970 and Sawa 1972). Dotted lines are dominant states observed in stripping reaction by Kocher and Haeberli 1972. (iii) Results of present calculations. (iv) ICM calculations by Csürös *et al* (1970). (v) Results of Thankappan—True (TT) model calculations by Larner (1970).

Table 2. Spectroscopic factors $(2J+1)\ S$ for the $^{54}Fe(d, p)\ ^{55}Fe$ reaction (All states have negative parity)

Experiment[a]			Theory[b]		Theory[c]
Energy (MeV)	J	$(2J+1)\ S$	J	$(2J+1)\ S$	$(2J+1)\ S$
0	3/2	2·92	$3/2_1$	2·92	2·68
0·41	1/2	1·18	$1/2_1$	0·78	0·53
0·93	5/2	4·14	$5/2_1$	2·46	1·09
1·32	7/2	0·40	$7/2_1$	0·0	
1·93	1/2	0·14	$1/2_2$	0·36	0·22[d]
2·06	3/2	0·32	$3/2_2$	0·68	0·82
2·15	5/2	0·96	$5/2_2$	1·11	0·26
2·59	5/2	0·30	$5/2_3$	0·41	1·87
3·04	3/2	0·12	$3/2_3$	0·14	
3·80	1/2	1·00	$1/2_3$	0·56	

(*a*) Kocher and Haeberli (1972); (*b*) present work; (*c*) Carola and Ohnuma (1971); (*d*) Based on a spin assignment of 3/2.

(− 1·62) n.m. respectively. Thus the anharmonic version of ICM gives consistently enhanced values compared to the harmonic version.

Experimentally, only the reduced transition probabilities for transitions from 0·93 MeV $5/2_1^-$ and 1·32 MeV $7/2_1^-$ states to the ground state are known. The experimental B(E2) values as reported by Stoquert *et al* (1973) for the transitions $5/2_1^- \rightarrow 3/2_1^-$ and $7/2_1^- \rightarrow 3/2_1^-$ are 0·8 W.U. and 6·61 W.U. respectively. We get 1·8 W.U. and 4·73 W.U. respectively. The experimental B(M1) value for the transition $5/2_1^- \rightarrow 3/2_1^-$ is $2 \cdot 7 \times 10^{-3}$ W.U. We get the same value. Thus one notes that our results for the B (E2), B (M1) agree remarkably well with the experimenta. results. It should be mentioned that previous theoretical attempts by Carola and Ohnuma (1971) give (5·5) W.U. and (10·4) W.U. for the B(E2) values for the transitions $5/2_1^- \rightarrow 3/2_1^-$ and $7/2_1^- \rightarrow 3/2_1^-$ respectively. Their B(M1) value for the transition $5/2_1^- \rightarrow 3/2_1^-$ is $(6 \cdot 0 \times 10^{-3})$ W.U. Thus they give much higher values for the transition rates.

Table 3 contains calculated reduced transition probabilities for various states in ^{55}Fe. In the absence of experimental results, we compare our results with those obtained by Carola and Ohnuma (1971). Some of the B(E2), B(M1) values differ by an order of magnitude. Comparison of their wavefunctions with those of ours indicates that although the structure is similar, some of the components differ in phase. Such a difference will not be evident in the results for spectroscopic factors.

Table 3. Transition rates, branching ratios and mean life times in ^{55}Fe

Transition		$B(E2)$ ($e^2 \cdot fm^4$)		$B(M1)$ (n.m.)^2		Branching ratio (%)		Mean life-time	
J_i	J_f	Theory[a]	Theory[b]	Theory[a]	Theory[b]	Expt[c]	Theory[a]	Expt[d]	Theory[a]
$1/2_1$	$3/2_1$	56	130	0·37	1·9			9^{+8}_{-5} ps[f]	1·32 ps
$5/2_1$	$3/2_1$	22·4	68	0·003	0·01	98±1	9	13·4±4 ps[e]	11·4 ps
	$1/2_1$	21·8				2±1	1	11±4 ps[f]	
$7/2_1$	$3/2_1$	58·8	129			96±1	49	$0{\cdot}91^{+0{\cdot}91}_{-0{\cdot}31}$ ps	1·77 ps
	$5/2_1$	0·45	0·29	0·18	1·1	4±1	51	14±3 ps[f]	
$1/2_2$	$3/2_1$	10·1	18	0·007	0·029	68±3	23	25±9 fs	140 fs
	$1/2_1$			0·054	0·051	32±3	77		
$3/2_2$	$3/2_1$	20·75	0·02	0·029	0·0008	24±1	44	31±8 fs	53 fs
	$1/2_1$	0·07	83	0·065	0·105	71±1	44		
	$5/2_1$	2·73		0·056		5±2	12		
$5/2_2$	$3/2_1$	25·7	34	0·024	0·039	18±2	81	55±14 fs	98 fs
	$1/2_1$	1·68	47			3±1	1		
	$5/2_1$	4·84	21	≃0·0	0·35	43±4	0		
	$7/2_1$	0·06	51	0·11	0·04	36±5	18		
$9/2_1$	$5/2_1$	51·5	180			75±5	86	$0{\cdot}9^{+0{\cdot}7}_{-0{\cdot}2}$ ps	2·8 ps
	$7/2_1$	2·44	21	0·002		15±5	14		

(*a*) Present work; (*b*) Carola and Ohnuma (1971); (*c*) Pilt *et al* (1970); Robertson *et al* (1971); (*d*) Robertson *et al* (1971); (*e*) Stoquert *et al* (1973); (*f*) Donahue and Hershberger (1971).

In table 3, we also report the results for branching ratios. Comparison with experimental values shows that the agreement is not very satisfactory. This may be due to the inherent limitations of the model itself, for in many transitions it fails to give correct B(M1) rates. These in turn may adversely affect the branching ratios. This point is further discussed in the conclusion.

Experimentally, the transition $7/2_1^- \rightarrow 5/2_1^-$ is highly hindered. It has been a consistent feature of the ICM to fragment out the γ-ray intensity for the transitions from the $7/2_1^-$ state. Similarly all the ICM calculations fail to favour the transition $3/2_2^- \rightarrow 1/2_1^-$ as much as observed experimentally. For the above mentioned transitions the shell model results as reported by Carola and Ohnuma (1971) are qualitatively similar. Our results for branching ratios for transitions from $1/2_2^-$ and $5/2_2^-$ state disagree with experimental results, whereas those by Carola and Ohnuma (1971) agree to some extent. We have found that these ratios very much depend on the values of the parameters g_l and g_s and much less on e_{eff}. This is so because the B(M1) values are found to bevery sensitive to the variations in these parameters. For instance, when we take $e_{eff} = 0{\cdot}5$ e and $g_l = 0{\cdot}4$ n.m., the branching ratios for the transitions $1/2_2^- \rightarrow 3/2_1^-$ and $1/2_2^- \rightarrow 1/2_1^-$ change to 80% and 20% respectively. Using the same value, the branching ratios for the transitions from the $5/2_2^-$ state change to 52%, 1%, 34% and 13% respectively. The M1 transition between $5/2_2^-$ and $1/2_1^-$ is not allowed, so the change of parameters has effectively redistributed the transition strength between the transitions $5/2_2^- \rightarrow 3/2_1^-$ and $5/2_2^- \rightarrow 5/2_1^-$.

One sees from table 3 that our results for life times of various states agree well with the experimental results. According to Carola and Ohnuma (1971), the state $1/2_1^-$ is very short lived. Among the available theoretical results, only our calculation predicts correct lifetime of the $5/2_1^-$ state at 0·93 MeV. The shell model predicts very high life time for this state. Similarly all the ICM calculations result in predicting very high life time of the $1/2_2^-$ state at 1·93 MeV. Experimentally the lower limit to the life time of the $11/2^-$ state at 2·54 MeV is 0·66 ps (Robertson *et al* 1971). We get 5·2 ps for the same. In the same paper, the life time of the $5/2^-$ state at 2·58 MeV is reported as (67 ± 9) fs, we get 60 fs. It should be mentioned at this stage that using the parameters $e_{eff} = 0{\cdot}5$e and $g_l = 0{\cdot}4$ n.m. which improve branching ratios for transitions from some of the states, give much more enhanced life times for the states.

3.2 *Nucleus* ^{57}*Fe*

Here ^{56}Fe is considered as a vibrating core. There are some experimental evidences of vibrational characteristics of ^{56}Fe (Sengupta *et al* 1971, Mani 1971). In the theoretical investigations by (i) Dayras and Čujec (1972) for ^{57}Co and (ii) Robertson *et al* (1972) for the hole states in ^{55}Fe, referred to in section 3.1, the nucleus ^{56}Fe is considered as a vibrating core. However, in our calculations, the neutron occupancy of single particle states $p_{3/2}$, $f_{5/2}$ and $p_{1/2}$ is fragmented. In this sense a quasineutron is coupled to the core.

Energy spectrum: The parameters are shown in table 1. The quasiparticle amplitudes and the energies were not used as free parameters. Instead, the values obtained from pairing theory and reported by Sengupta *et al* (1971) were used.

The quasiparticle amplitudes were, however, normalized to give neutron occupancy 1·31, 0·62 and 0·07 for single particle states $p_{3/2}$, $f_{5/2}$ and $p_{1/2}$ respectively in the core. The excitation energy 0·85 MeV of 2_1^+ state in ^{56}Fe is used as $\hbar\omega$. Thus the number of free parameters is quite reduced. The best fit to the ^{57}Fe spectrum is obtained for $\xi = 2\cdot 5$ which is again within the reasonable range. These values alongwith the value of stiffness parameter C, obtained from electromagnetic data, give for the coupling strength k, the value (22·8) MeV which is a slightly smaller value.

As in case of ^{55}Fe, the quadrupole moment of 2_1^+ state in ^{56}Fe is used as a free parameter. From the study, we predict the quadrupole moment of the 2_1^+ state in ^{56}Fe to be close to (− 0·18) barn. The experimental value, as compiled by Christy and Häusser (1972) is (− 0·24) barn and the values reported by Cameron *et al* (1972) is (—0·12 ± 0·16) barn or (+ 0·02 ± 0·16) barn. It is obvious that the value we used for the calculation for this quadrupole moment agrees with the measured values.

The resulting energy spectrum is shown in figure 2. From figure 2 one sees that the same general remarks may be applied as in the case of ^{55}Fe. It is only in our theoretical spectrum, the first excited $3/2^-$ state is correctly reproduced. Also, the overall agreement with the experimental spectrum is fairly good. The model's inadequacy to take into account the Pauli principle may be regarded as responsible for occurrence of many low spin states.

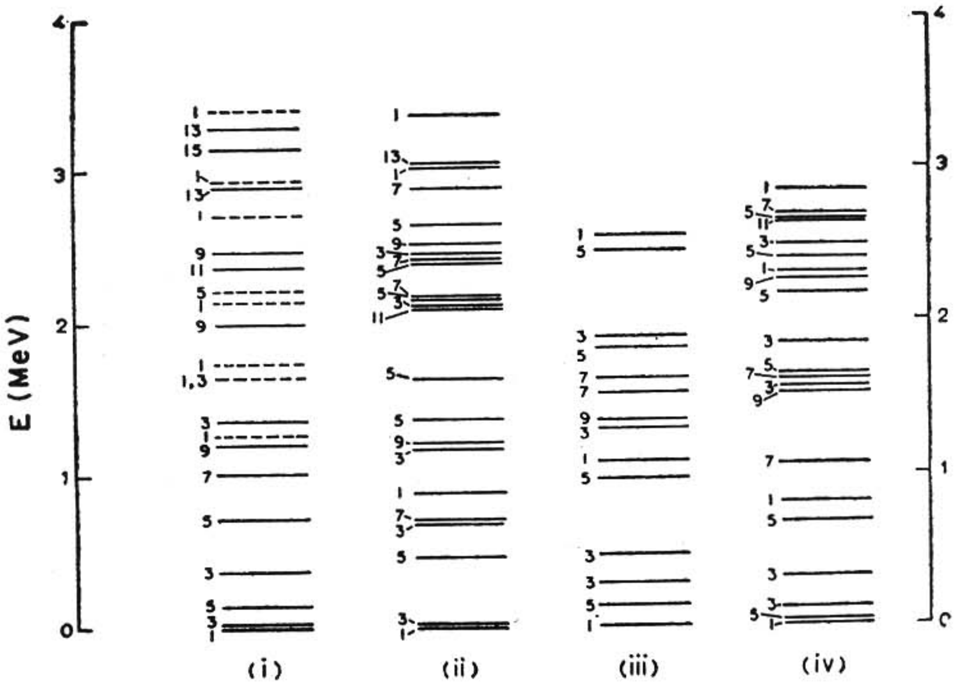

Figure 2. Experimental and theoretical level schemes for ^{57}Fe. All spin values are multiplied by two and refer to negative parity states. (i) Experimental spectrum (Sawa 1972); dotted lines are dominant states observed in stripping reaction by Cohen *et al* (1962). (ii) Results of present calculations. (iii) Strong coupling calculations by Comfort *et al* (1971). (iv) ICM calculations by Paar (1972).

Table 4. Spectroscopic factors $(2J + 1)\,S$ for the ^{56}Fe (d, p) ^{57}Fe reaction (All states have negative parity)

Energy (MeV)	Experiment *a* J	Experiment *a* $(2J+1)\,S$	Experiment *b* J	Experiment *b* $(2J+1)\,S$	Theory *c* J	Theory *c* $(2J+1)\,S$	Theory *d* $(2J+1)\,S$
0·0	1/2	0·304			$1/2_1$	0·146	0·06
0·014	3/2	0·914	3/2	2·20	$3/2_1$	2·15	1·29
0·14	5/2	2·45			$5/2_1$	2·01	3·48
0·37	3/2	0·755	3/2	0·99	$3/2_2$	0·36	0·52
0·70	5/2	0·172			$5/2_2$	2·37	0·29
1·26	3/2	0·525	1/2	0·68	$1/2_2$	0·01	0·66
					$3/2_3$	1·81	0·002
1·36	5/2	0·301			$5/2_3$	0·564	
1·63	3/2	0·09	1/2	0·08	$3/2_4$	0·0	0·006
2·21	5/2	0·405		0·371	$5/2_4$	0·40	
3·39	1/2	0·115	1/2	0·21	$1/2_4$	0·19	

(*a*) Sengupta *et al* (1971); (*b*) Cohen *et al* (1962); (*c*) Present work; (*d*) Comfort *et al* (1971).

Table 5. Static moments of some negative parity states of ^{57}Fe

J	Magnetic moment (n.m.) Theory *a*	Theory *b*	Theory *c*	Expt *d*	Quadrupole moment Q (e.b.) Theory *a*	Theory *b*	Theory *c*	Expt *e*
1/2	0·43	0·34	0·09	0·09				
3/2	−0·86	−0·008	−0·22	−0·16	0·07	0·16	0·12	0·21
5/2	0·847	1·14	0·51	0·85	−0·24	−0·20	−0·28	
3/2	−0·44	−0·50	0·19	$\lvert<0\cdot6\rvert$	−0·10	−0·13	−0·10	
5/2	0·45	0·35	0·04		−0·14	−0·06	−0·05	
1/2	0·38	0·78	0·31					

(*a*) Present work; (*b*) Comfort *et al* (1971); (*c*) Paar (1972); (*d*) Fuller and Cohen (1969) and Sprouse and Hanna (1969); (*e*) Chappert *et al* (1969).

Table 6. Transition rates, mean life times and branching ratios in ^{57}Fe

Transition		$B(E2)$ ($e^2 \cdot fm^4$)				$B(M1)$ (n.m.^{2})				Branching ratio (%)		Mean life-time	
		Theory			Expt	Theory			Expt	Theory	Expt	Theory	Expt
J_i	J_f	*a*	*b*	*c*	*e*	*a*	*b*	*c*	*f*	*a*	*e*	*a*	*e*
$3/2_1$	$1/2_1$	114	0·5	5·3	4·8±0·8[d]	0·035	0·031	0·023	0·016			0·348 μs	0·141 μs[g]
$5/2_1$	$1/2_1$	210	105	100	133±20					14·3	11·2	10·23 ns	13·8±2·1 ns
	$3/2_1$	52		70		0·0014	0·0008	0·0003	0·002	85·7	88·8		
$3/2_2$	$1/2_1$	130	68	130	200±40	0·086		0·002		100	14	7·8 ps	10±2 ps
	$3/2_1$	82		140		0·001		0·027		0	76		
	$5/2_1$	30		50		0·0018		0·001		0	10		
$5/2_2$	$1/2_1$	25	11	90	43±10					1·6	3·7	2·93 ps	4±1 ps or 5·9±1·5 ps
	$3/2_1$	120		260		0·021		0·007		68·2	85·9		
	$5/2_1$	11		20		0·019		0·0003		29·9	7·8		
	$3/2_2$	4		0		0·007		0·036		0·3	2·6		

(*a*) Present work; (*b*) Comfort *et al* (1972); (*c*) Paar (1972); (*d*) Thomas and Grace (1964); (*e*) Sprouse and Hanna (1969), (1965); (*f*) Values deduced from the data on life-times, internal conversion coefficients and branching ratios (Comfort *et al* 1972); (g) Kistner and Sunyar (1965

Table 4 gives the spectroscopic factors for the reaction ^{56}Fe (d, p) ^{57}Fe. Except for the $5/2^-$ state at 0·703 MeV, our results agree fairly well with the experiment.

Electromagnetic properties: From the wavefunctions obtained, the electro magnetic properties are studied. Again no effective charge is attributed to the neutron and the core gyromagnetic ratio g_R has its hydrodynamical value, given by Z/A. Overall good fit to the experimental results is obtained by using orbital gyromagnetic ratio g_l along with spin gyromagnetic ratio g_s and the stiffness parameter C as free parameters. Whereas g_s has the same value as in case of ^{55}Fe the value of g_l is enhanced to (0·5) n.m. compared to $g_l = 0$ in ^{55}Fe. The stiffness parameter C has the value 80 MeV. The hydrodynamical value is 64 MeV and the value reported by Wong (1968) is $(40·3 \pm 4·0)$ MeV. Our value of C yields the deformation parameter β_2 equal to (0·16) for the ^{56}Fe core. The experimental value as given by Marchese *et al* (1972) is (0·22). Again the two have good agreement.

In table 5 we report the results of various calculations, along with the experimental values, of static moments of various states in ^{57}Fe. Present calculations and those by Comfort *et al* (1971) give a higher value for the ground state magnetic moment. During the present study of the static moments of the low-lying states of ^{57}Fe, we found that the $3/2^-$ state calculated at 0·018 MeV energy appears to resemble the experimental $3/2^-$ state at 0·367 MeV and conversely, the higher calculated $3/2^-$ state at 0·68 MeV shows the properties similar to the experimental $3/2^-$ state at 0·014 MeV. Thus the first two $3/2^-$ states seem to have interchanged their roles. This was considered reasonable in view of the fact that their energy separation is not much.

The wavefunctions of levels $3/2_1^-$ and $3/2_2^-$ were interchanged in calculating the static moments and electromagnetic transition rates listed in table 5 and table 6 respectively. Table 5 shows that the results of various calculations for quadrupole moments of states in ^{57}Fe, do not differ much. However, the magnetic moment of $3/2_2^-$ state has different sign in case of results by Paar (1972) on the one hand and our results and results by Comfort *et al* (1971) on the other. It is suggested to revise the experimental investigations to ascertain the sign of the same.

Table 6 reports B(E2) and B(M1) values for transitions among various states in ^{55}Fe. The agreement of our results with experimental ones is, in general, good. Our B(M1) value for the transition $5/2_2^- \rightarrow 3/2_1^-$, is fairly close with the experimental value. Alaga model as applied by Paar (1972) gives better results than ours, however, they are not quite close to the experimental ones.

Table 6 also contains branching ratios of electromagnetic transitions among few low-lying states. Although the overall agreement is fairly good, our model predicts strong transition to the ground state $1/2^-$, rather than the $3/2^-$ state at 0·014 MeV from the $3/2^-$ state at 0·367 MeV.

One sees from table 6 that our results for life times of various states agree very well with the experimental results. Moreover the life times of some of the high spin states are also correctly predicted. Sawa (1972) reports upper limits to the life times of $11/2^-$ and $13/2^-$ states at 2·35 MeV and 2·88 MeV respectively. The limits are 0·69 ps and 0·67 ps respectively. In the present calculations, the life time of the $11/2^-$ state is 0·22 ps. Also the upper limit set by the model for the life time of the $13/2^-$ state is 0·18 ps. Thus, both of them are reasonable.

Conclusion

Our ICM calculations for ^{55}Fe differ from other ICM calculations in the assumption of core properties. We find that the coupling strength is reasonable which suggests that the interaction energy is not too large and that mixing of three-phonon states for low-lying levels will not be significant. This justifies our prescription of neglecting three phonon states. Moreover, the wavefunctions calculated by Carola and Ohnuma (1971) already exhibit that the components of three phonon states in them are very small.

Our studies for these isotopes have revealed also that it is important to incorporate anharmonicities of the core. By so doing, we have been able to take into account many core properties. This could be a reason why we do not require any effective charge. However, it would be interesting to study the Pauli principle effects, which is not so simple for the isotopes under study.

The simultaneous study of ^{55}Fe and ^{57}Fe has shown that the strength of dominant single particle states gets fragmented for ^{57}Fe whereas that of dominant collective states is not much affected. This is also seen in experiment.

As remarked by Häusser *et al* (1972), the present prescription which can describe quite well the E2 matrix elements, is not well suited for calculations of M1 matrix elements, because in the model the collective motion is coupled only to the orbital motion of the particle. This may be a reason for ICM's failure in accounting for branching ratios.

The general agreement of ^{54}Fe results is better than that of ^{57}Fe. This suggests that ^{57}Fe could be better described through coupling of three quasiparticle states to the anharmonic vibrations of the ^{55}Fe core. This will also explain the existence of some seniority three states, which the present formalism has missed. A calculation on this line is being planned.

Our results for the static quadrupole mements of the 2_1^+ states in ^{54}Fe and ^{56}Fe, listed in table I, agree qualitatively with the values (-0.073) barn and (-0.16) barn respectively obtaind by Sharma (ICTP preprint IC/74/62) through HFB formalism.

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