A study of non-linear characteristics of the dielectric between a Maxwell horizontal long wire antenna and Earth's surface

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In this paper the authors considered the horizontal long-wire antenna (Maxwell's antenna) and the earth surface to form a non-linear capacitor and subsequently a mathematical analysis is carried out on that basis. The circuit is energized by a harmonic potential say $E_m(\sin\omega t+\theta)$ due to tropical lightning activity. An upper atmospheric biasing potential E_0 from static electric cloud (thunder cloud) generator, account in the circuit as DC biasing potential. The effect of the bias potential in increasing the effective impedance of the circuit is clearly evident. It is apparent (theoretically) that an increase of the magnitude of the DC component produces a corresponding decrease in the amplitude of the air-earth current at tropical region.

1. Introduction

The capacitor described in this work is commonly used in atmospheric electricity experiments for the measurement of Maxwell's current. It is assumed that this capacitor consists of two parallel plates (viz., the long wire antenna and the earth's surface) of dynamic area "A" separated by a height "h". The capacitor is expected to be charged by the AC component of a tropical thunder storm. Let us assume that the surface charge density on upper plate is σ C/m².

Then $E = \frac{\sigma}{\varepsilon_0}\hat{n}$

The total charge on upper plate of area A is

$Q = \sigma A$

So the potential difference between the plates is

$$V = -\int_0^d E.dh$$

$$= \frac{\sigma}{\varepsilon_0} h$$

The capacitance is given by the expression

$$C = \frac{Q}{V} = \frac{\sigma A}{\sigma h/\varepsilon_0} = \frac{\varepsilon_0 A}{h} F.$$

If the wire is connected to the earth, the positive charges on the upper wire are neutralized due to the flow of electrons. The capacitor is separated by air at a height of 1m. The specific inductivity of dry air varies with aerosol loading and water vapour content, necessitating the system to be as non-linear capacitor.

In the steady state, there is no flow of electric charge towards the lower plate; the plates have a fixed potential difference. As Maxwell's current, which is a combination of convection, conduction and displacement current increases, the number of negative charges on the other plate also increases. On other hand an increase in the positive charge on one plate produces an equal amount of increase of negative charge on the opposite plate. Thus the variation of potential on one side of the capacitor is transmitted as an equal variation of potential on the other side of the capacitor, constituting a flow of alternating current in the external circuit.

The circuit is energized by a harmonic potential say $E_m \sin(\omega t+\theta)$ due to tropical thunderstorm activity. An upper atmospheric biasing potential E_0 from a thunder cell generator in the circuit acts as the DC biasing potential.

It will be assumed that the saturation curve of the non-linear air-earth capacitor may be approximated by the following cubic polynomial (1). The potential across the nonlinear capacitor is given by

$$E_c = \frac{q}{C_o} + aq^3 \qquad (1)$$

where 'C' is the initial capacitance of the system, 'q' is the charge separation on the antenna and ground and 'a' is the positive constant that depends on the characteristics of the nonlinear dielectric of the capacitor. From a physical perspective with change in potential the capacitor charge q and the circuit current 'i' satisfy the differential equation

$$\frac{q}{C_o} + aq^3 + i\int_0^H \omega_h dh + \frac{di}{dt} = E_0 + E_m \sin(\omega t+\theta)........(2)$$

For simplicity we denote the columnar resistance is

$\int_0^H \omega_h dh = {'C_R'}$ hereforth.

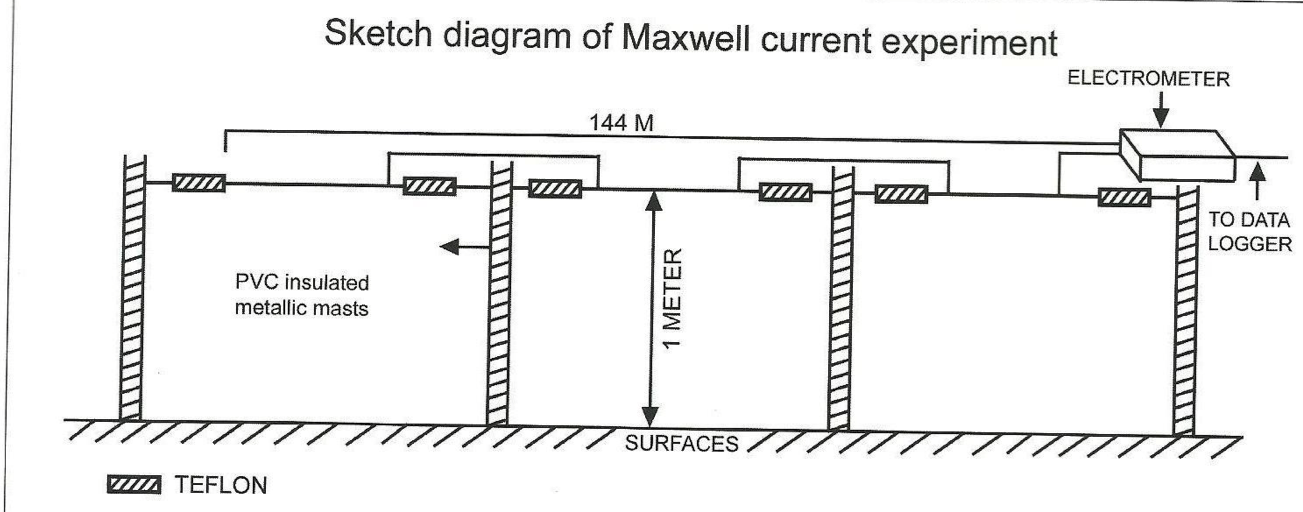

Fig. 1. *Sketch of the Maxwell current experiment*

Let us consider a steady state, we assume i = I_m sin(ωt), and the charge $q = \frac{Q_0 - I_m cos(\omega t)}{\omega}$ where I_m is the maximum amplitude of the AC current and Q_0 is the constant of charge accumulation on the wire of the capacitor.

Substitute the field values of 'i' and 'q' in (1), we get

$$F(t) = \left[\frac{di}{dt} + C_{R_i} + \frac{q}{C_0} + aq^3 \right]_{i = I_m \sin\omega t}$$

$$= \frac{d}{dt}[I_m \sin\omega t] + C_R I_m \sin\omega t + \frac{1}{C_0}\left[Q_o - \frac{I_m}{\omega}\cos\omega t\right]$$

$$+ a\left[Q_o - \frac{I_m}{\omega}\cos\omega t\right]^3$$

$$= \frac{Q_0}{C_0} + aQ_0^3 + \frac{3aQ_0 I_m^{\ 2}}{2^{\ 2}} + RI_m \sin\omega t + \left(\omega I_m - \frac{I\omega}{C_0\omega} - \frac{3aI_m^{\ 3}}{4\omega^3} - \frac{3aQ_0^2 I_m}{\omega}\right)$$

$$\cos\omega t - \frac{aI_m^{\ 3}}{4\omega^3}\cos^3\omega t + \frac{3aQ_o I_m^{\ 2}\cos^2\omega t}{2\omega^2} \text{....(3)}$$

Equating the similar components of sin(ωt) and cos(ωt) and numbering that

$$E(t) = E_{(0)} + E_m \sin(\omega t + \theta)$$

$$= E_{(0)} + E_m \cos\theta \sin\omega t + E_m \sin\theta \cos\omega t \text{.....(4)}$$

We get

$$E_0 = \frac{Q_0}{C_0} + aQ_0\left(Q_0^2 + \frac{3I_m^{\ 2}}{2\omega^2}\right) \text{.............(5)}$$

$$E_m \cos\theta = C_R I_m \text{..............................(6)}$$

$$E_m \sin\theta = \omega I_m - I_m\left(\frac{1}{C_0\omega}\right) - 3\frac{a}{\omega} I_m\left(Q_0^2 + \frac{I_m^{\ 2}}{4\omega^2}\right) \text{...(7)}$$

In practice the constat "a" is negligibly small (5) can then be rewritten as,

$$E_0 = \frac{Q_o}{C_0} \ or \ Q_0 = E_0 C_0 \text{...........(8)}$$

The initial capacitance C_o can be directly measured. Since $Q_0^2 >> \mathrm{Im}^2/4w^2$ (7) reduces to

$$E_m \sin\theta = \omega I_m - I_m\left(\frac{1}{C_o\omega}\right) - 3\frac{a}{\omega} I_m (E_0 C_0)^2 \text{..........(9)}$$

To compute E_m we square and add (6) and (9), thus getting

$$E_m^2 = \frac{I_m^{\ 2}}{\omega 2}\left[R_c^2\omega^2 + \left(1/C_0 + 3a(E_0 C_0)^2 - \omega^2 \right)\right]$$

From which

$$I_m^2 = \frac{E_m^{\ 2}\omega^2}{\left\{ R_c^2\omega^2 + \left(\frac{\left(\frac{1}{C_0} + 3aE_0^2 C_0^2 - \omega^2\right)^2}{\omega^2} \right) \right\}}$$

Thus the amplitude of the alternating current is approximately

$$I_m = \frac{E_m}{\sqrt{R_c^2 + \frac{1}{\omega^2}\left\{ {1}/{C_0} + 3aE_0^2C_0^2 \right\}^2}} \quad \text{................ (10)}$$

But we have

$i = C\frac{dv}{dt}$ and thus

$$C = \frac{1}{\left[R_c^2 + \frac{1}{\omega^2}\left(\frac{1}{C_0} + 3aE_0^2C_0^2\right)^2\right]^{1/2}} \quad \text{................ (11)}$$

Again ε = Ch / A (12)

where $A = \frac{2\pi lh}{\log(2h/r_0)}$ is the effective area of the horizotal long – wire antenna collecting the atmospheric electric vertical current, l is the length of the wire, h is the height and r_0 is the diameter of the antenna. For the present experimental setup at Equatorial Geophysical Research Laboratory the computed effective area is 168m^2 corresponding to a length of 144 m. The initial capacitance C_0 = 1400 pF; and the height is 1m. From (12) we conclude that while A and h remain constant, the dielectric constant may vary. This implies that ε is proportional to

$\left[R_c^2 + \frac{1}{\omega^2}\left(\frac{1}{C_0} + 3aE_0^2C_0^2\right)^2\right]^{1/2}$ and thereby to the aerosol content of the air column between the long wire and the ground.

The effect of the biasing potential is to increase the effective impedance of the circuit as is clearly evident from (10). It is thus apparent that an increase of the magnitude of the DC component produces a corresponding decrease in the amplitude of the I_m.

This work focuses on the nonlinear characteristic of a dielectric between the Maxwell's long wire antenna and earth's surface. In the future we will study the Maxwell's current and electric potential at height of 1 m under different atmospheric conditions. Expressions for the current and the dielectric constant will be derived for various conditions. From our earlier observations, the oscillations of the electric field are most unstable under foggy and wet conditions, since fog and moisture enhance the aerosol loading.

As demonstrated by (9), the amount of current that a thunderstorm contributes to the G.E.C. is a function of the time averaged generator current. And the effect of ionospheric and magnetospheric generators increases the amplitude of the oscillation.

2. Conclusion

As has been demonstrated, an increase in the aerosol / moisture content, changes the dielectric constant of the air between a horizontal long wire antenna and earth's surface. The effect of the bias potential is to increase the effective impedance of the circuit. Also an increase of the magnitude of the DC component produces a corresponding decrease in the amplitude of the I_m. Thus atmospheric electric measurements below the planetary boundary layer may vary with respect to aerosol loading.

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