PHYSICS OF PLASMAS **25**, 122302 (2018)

Ion acoustic super solitary waves in a magnetized plasma

T. Kamalam[a] and S. S. Ghosh[b]
Indian Institute of Geomagnetism, Navi Mumbai 410218, India

(Received 3 October 2018; accepted 13 November 2018; published online 7 December 2018)

Ion acoustic super solitary waves have been derived for a three component magnetized plasma consisting of warm fluid ions and two different temperature electrons having Boltzmann distributions by using the Sagdeev pseudopotential technique. The result exhibits similar traits of the recently reported electron acoustic super solitary wave in a magnetized plasma [Kamalam *et al.*, J. Plasma Phys. **84**, 905840406 (2018)]. This is the first report of super solitary waves for a low frequency wave in a magnetized plasma. This prediction might be useful to analyze the non-conventional structures in the low altitude auroral regions in the Earth's magnetosphere. *Published by AIP Publishing.* https://doi.org/10.1063/1.5063955

I. INTRODUCTION

Super Solitary Waves (SSWs) are nonlinear structures having a bipolar electric field with an extra wiggle. The concept of super solitary waves (SSWs) was introduced by Dubinov and Kolotkov in 2012.[1] Right after the discovery, SSWs have been investigated widely for different plasma models.[2] It was, however, difficult to obtain them for a magnetized plasma due to the imminent singularity. The first attempt to study a SSW in a magnetized plasma was done by Rufai *et al.* in 2014 for a three component plasma model consisting of cold fluid ions, cooler electrons obeying the Boltzmann distribution, and hotter electrons having Cairn's distribution.[3] Nevertheless, the reported Sagdeev pseudopotential profile had a spiky and narrow subwell. Later, it was identified as a singularity rather than the SSW.[4] The difficulty in obtaining the SSW in a magnetized plasma and the reported spiky, narrow subwell led us to explore it. We have investigated an electron acoustic SSW in a magnetized plasma for a four component model comprising beam, bulk electrons, and two ions obeying Boltzmann distributions.[5] We have reported that there is a lateral inversion of subwells of SSW between the magnetized and unmagnetized cases. We have incorporated the theory to identify the physically viable solutions, verified the structures and its characteristics, and discussed their transition types. However the model has several limitations. One major issue is about the assumptions of fluid electrons. Comparison between the phase velocity of the wave and the thermal velocities of different species revealed that all the four species have high thermal velocity than the wave phase velocity which invokes the ambiguity of the application of the model and the credibility of the results. In spite of these flaws, the model has provided a SSW without any singularity and aids to fill the lacuna. This raises the question whether the results are physically significant and qualitatively relevant within its own limit. Taking this into account, we have considered a different plasma system which is free from any such ambiguity, and we have succeeded to obtain similar results in accordance with our previous findings in Ref. 5. The current model is adapted from Ghosh and Lakhina (2004) with an exclusion of the beam ions.[6]

II. THE ANALYTICAL SOLUTION

We have considered a three component plasma comprising warm fluid ions and two temperature electrons obeying Boltzmann distributions. The wave is propagating in the $y-z$ plane making an angle θ with the magnetic field while the magnetic field is assumed to be in the z direction. The normalized fluid equations are as follows:

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{V}_i) = 0, \quad (1)$$

$$\frac{\partial \mathbf{V}_i}{\partial t} + (\mathbf{V}_i \cdot \nabla)\mathbf{V}_i = -\nabla\Phi + \alpha_i(\mathbf{V}_i \times \hat{\mathbf{b}}) - 3\sigma_i n_i \nabla n_i. \quad (2)$$

The Boltzmann distribution of electrons is as follows:

$$n_e = \mu \exp\left(\frac{\Phi}{\mu+\nu\beta}\right) + \nu \exp\left(\frac{\beta\Phi}{\mu+\nu\beta}\right), \quad (3)$$

where i (e) represents ions (electrons), $\alpha_i = \frac{\Omega_i}{\omega_{pi}}$ represents the ratio of the ion cyclotron frequency to the ion plasma frequency, $\sigma_i = \frac{T_i}{T_{eff}}$ represents the ratio of the ion temperature to the effective temperature of the electrons, $T_{eff} = \frac{T_{ec}T_{eh}}{\mu T_{eh}+\nu T_{ec}}$ being the effective electron temperature, $\hat{\mathbf{b}}$ is the unit vector along the magnetic field, μ (ν) are the ambient densities of cooler (hotter) electrons, and $\beta = \frac{T_{ec}}{T_{eh}}$ is the temperature ratio of cooler to hotter electrons.

The normalization scheme used is as follows: densities are normalized by the ambient plasma density n_0, time with respect to the inverse of the ion plasma frequency ω_{pi}^{-1}, length is by the effective electron Debye length, pressure is by the ion equilibrium pressure $P_0 = n_0 T_i$, temperature to the T_{eff}, magnetic field by the ambient magnetic field B_0, and potential is normalized by $\frac{T_{eff}}{e}$. The velocity and Mach number should be normalized with respect to the linear acoustic speed of the corresponding model.[7] However, for the sake of convenience, we have normalized the velocity and effective Mach number M_{eff} by the effective ion acoustic speed $\frac{T_{eff}}{m_i}$.

[a]Electronic mail: kamalam.unom@gmail.com
[b]Electronic mail: sukti@iigs.iigm.res.in

The derivation of Sagdeev pseudopotential in a magnetized plasma requires restrictions on the number of "magnetized" species.[8] This prompts us to use the neutrality condition instead of the Poisson's equation

$$n_i \approx n_e. \tag{4}$$

The following boundary conditions are considered:

$$\text{at} \quad |\mathbf{r}| \to \infty, \quad \Phi \to 0, \quad V_i \to 0, \tag{5}$$

$$n_i \to 1, \quad \text{and} \quad P_i \to n_0 T_i, \tag{6}$$

where $\mathbf{r} = \mathbf{k}_y y + \mathbf{k}_z z$ is the corresponding position vector, and $\mathbf{k}_y$ and $\mathbf{k}_z$ are the direction cosines along the y and z directions, respectively.

The stationary state solution is assumed as

$$\eta = k_y y + k_z z - M_{eff} t, \tag{7}$$

where M_{eff} is the effective Mach number.[7]

The corresponding Sagdeev pseudopotential is as follows:

$$\Psi(\Phi) = \frac{\alpha_i^2 L_i}{F^2}. \tag{8}$$

The numerator of the Sagdeev pseudopotential is

$$L_i(\Phi) = \left[k_y^2 \Phi + \frac{1}{2} M_{eff}^2 \left(\frac{S}{n_i}\right)^2 + w - 3h\right] + k_z^2 \left[h - \frac{1}{n_i}(\sigma S + S') + \frac{1}{2}\left(\frac{w}{M_{eff}}\right)^2\right], \tag{9}$$

and the denominator of the Sagdeev pseudopotential is

$$F(\Phi) = \frac{df}{d\Phi} = 1 + \left(3\sigma n_i - \frac{M_{eff}^2}{n_i^3}\right)\frac{dn_i}{d\Phi}, \tag{10}$$

where

$$f = \Phi + \frac{1}{2}\left(\frac{M_{eff}^2}{n_i^2} + 3\sigma_i n_i^2\right); \quad S = 1 - n_i; \quad S' = \delta - n_i' \quad \text{and} \quad \delta = n_i'|_{\Phi=0}, \tag{11}$$

$$h = \frac{\sigma(1 - n_i^2)}{2}; \quad t = \sigma(1 - n_i^3) \quad \text{and} \quad w = S' + t. \tag{12}$$

The following conditions should be satisfied to obtain a solitary wave solution:

$$\Psi(\Phi)|_{\Phi=0} = \left.\frac{\partial \Psi(\Phi)}{\partial \Phi}\right|_{\Phi=0} = 0 \quad \text{and} \quad \left.\frac{\partial^2 \Psi(\Phi)}{\partial \Phi^2}\right|_{\Phi=0} < 0, \tag{13a}$$

$$\Psi(\Phi) < 0 \quad \text{for} \quad \begin{cases} 0 < \Phi < \Phi_0 & \text{when} \quad \Phi_0 > 0 \\ \Phi_0 < \Phi < 0 & \text{when} \quad \Phi_0 < 0, \end{cases}$$

$$\Psi(\Phi_0) = 0 \quad \text{and} \quad \left.\frac{\partial \Psi}{\partial \Phi}\right|_{\Phi=\Phi_0} \neq 0. \tag{13b}$$

In addition to the above conditions, the first derivative of the pseudopotential should have four roots for a SSW.

III. NUMERICAL RESULTS

Figure 1(a) shows the Sagdeev pseudopotential profiles of the three extra nonlinear structures, viz., gVSW (generalized Variable Solitary Waves), n-CoI (Curve of Inflection), and SSW[5,9] while Fig. 1(b) gives the subsequent phase portraits. The parameters are as follows: $\mu = 0.5$, $\beta = 1/40$, $\sigma_i = 1/50$, $\alpha_i = 0.5$, $\theta = 40°$, $M_{eff} = 0.79$, 0.8142, and 0.83, respectively. It is clearly evident from the figure that the auxiliary subwell [marked as (i) in Fig. 1(a)] is near zero, i.e., near the low potential. This marks the lateral inversion of the two subwells in a magnetized plasma vis-á-vis an unmagnetized case. A similar trend has been reported in our previous work on electron acoustic waves.[5] There we have mentioned that there is an association between the singularity and the SSW. The solution always terminates with the onset of the former.[5] The commencement of singularity invokes the difficulty in achieving SSWs in a magnetized plasma. This association instigated us to check the onset of singularity for the chosen parametric regime. We have incorporated an F-analysis (F being the denominator of the Sagdeev pseudopotential) to detect the onset of the singularity and to identify the various nonlinear structures. Figure 2(a) shows the F variation for the Sagdeev pseudopotentials with M_{eff} ranging from 0.79 to 0.9. The curve which corresponds to $M_{eff} = 0.89$ is

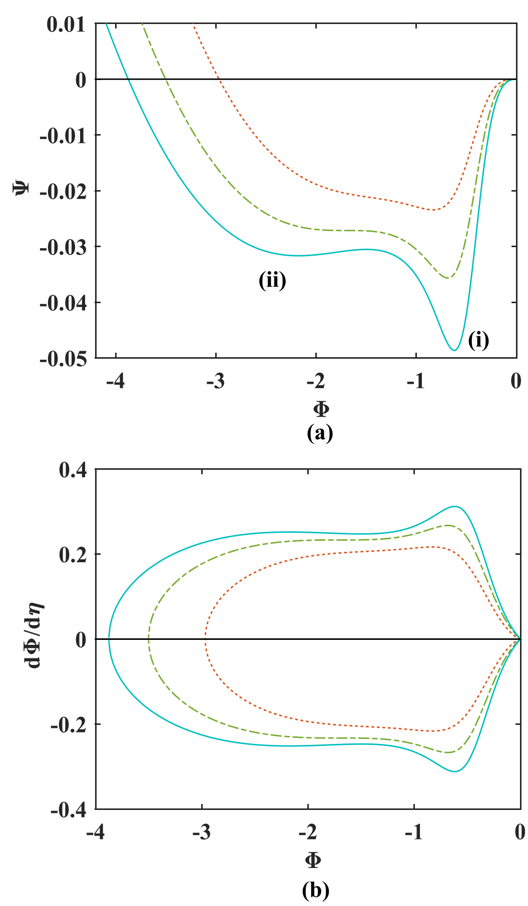

FIG. 1. (a) Sagdeev pseudopotential profile and (b) phase portrait of extra nonlinear structures.

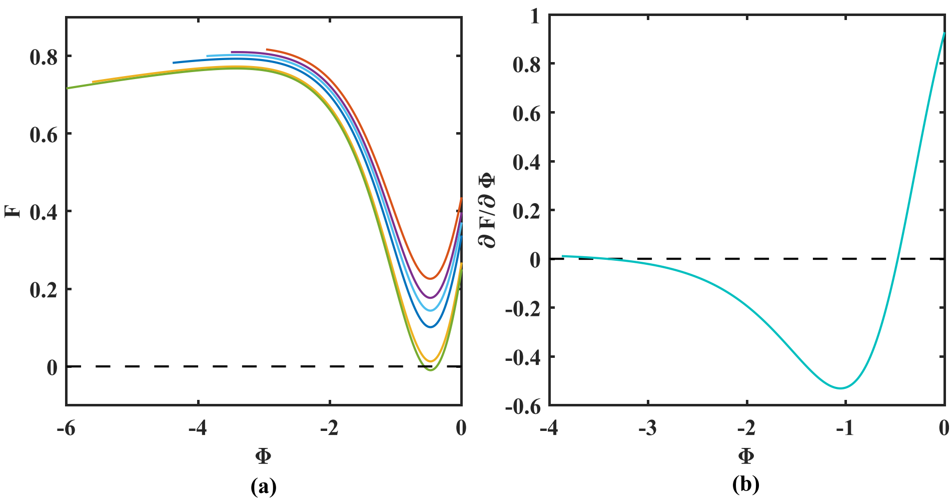

FIG. 2. (a) F variation for different effective Mach numbers $M_{\rm eff} = 0.79$, 0.8142, 0.83, 0.85, 0.89 and 0.9 and (b) derivative of F for SSW ($M_{\rm eff} = 0.83$).

proximal to zero, but never crossed the zero axis. However, $M_{eff} = 0.9$ crossed the zero axis which indicates the presence of singularity. It means that the F variation profile for an ion acoustic SSW always remains positive except for the case of singularity. The variation pattern is analogous to that reported electron acoustic SSW[5] though it looks inverted (upside-down) compared to the former where the F-profile was always found to be negative. The change in the sign further confirms our previous conjecture that the F-variation in the magnetized plasma is a useful proxy for the charge separation variation for the unmagnetized case. Consequently, Fig. 2(b) shows the variation of the derivative of F with Φ. The "fluctuation" in the profile confirms the solution to be an SSW. This also emphasizes the general trend for the F variation for a magnetized plasma. Analogous to our previous findings, Figs. 2(a) and 2(b), together, confirm the following conditions:

$$F \neq 0 \quad \text{for} \quad 0 \geq \Phi \geq \Phi_0, \tag{14}$$

$$\left.\frac{\partial F}{\partial \Phi}\right|_{\Phi=\Phi_a} \geq 0 \quad \text{for} \quad 0 > \Phi_a > \Phi_0. \tag{15}$$

Equation (15) ensures that there is an auxiliary subwell at $\Phi = \Phi_a$, lying within 0 and Φ_0, which in turn confirms the presence of the SSW. The condition in Eq. (14) ensures a non-zero F for the corresponding range. For a positive amplitude electron acoustic solitary wave, however, the conditions in Eqs. (14) and (15) get modified as

$$F \neq 0 \quad \text{for} \quad 0 \leq \Phi \leq \Phi_0, \tag{16}$$

$$\left.\frac{\partial F}{\partial \Phi}\right|_{\Phi=\Phi_a} \geq 0 \quad \text{for} \quad 0 < \Phi_a < \Phi_0, \tag{17}$$

where the potential becomes positive due to the change in the polarity of the amplitude. Equations (15) and (17) can, thus, be combined to have a more general condition as

$$\left.\frac{\partial F}{\partial \Phi}\right|_{\Phi=\Phi_a} \geq 0 \quad \text{for} \quad \begin{cases} 0 < \Phi_a < \Phi_0, \ \Phi_0 > 0 \\ 0 > \Phi_a > \Phi_0, \ \Phi_0 < 0. \end{cases} \tag{18}$$

Expectedly, like in Ref. 5, the potential profile in Fig. 3(a) has a bump near zero rather than near the maximum amplitude (i.e., near Φ_0). The subsequent electric field profile of the SSW in Fig. 3(b) confirms the lateral inversion of the auxiliary and main subwell. This also confirms that the observed lateral conversion is more akin to be a generic characteristic for a magnetized plasma and does not depend on the type of the oscillating species.

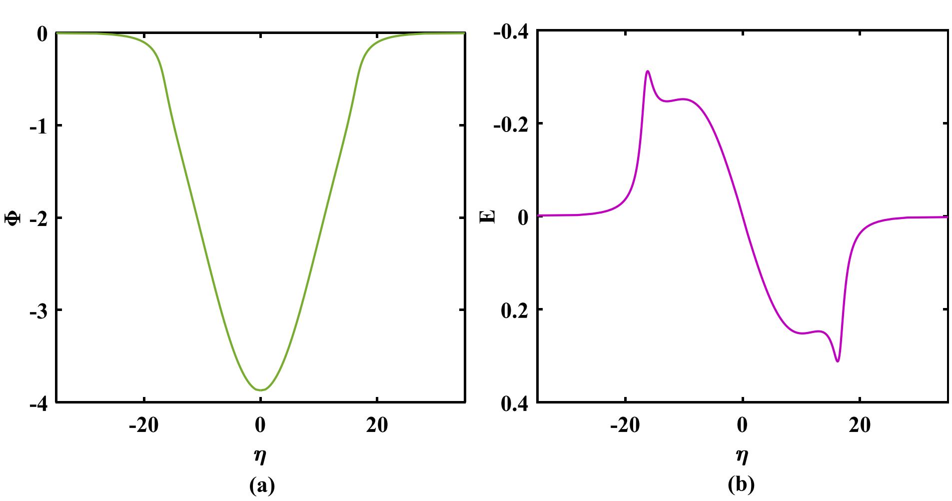

FIG. 3. (a) Potential and (b) electric field profile of SSW.

Using the derivative analysis incorporated by Varghese and Ghosh,[9] the transition route to the onset of the ion acoustic SSW is found to be as follows:

$$\text{gVSW} \rightarrow \text{n-CoI} \rightarrow \text{SSW} \rightarrow \textit{leading to the singularity}.$$

The above transition does not have any intermediate Double Layer (DL). The transitions are also clearly seen in the phase portrait in Fig. 1(b). The absence of the separatrix reveals that the current transition is not of Type I.[10] For an unmagnetized case, the SSW is known to remain sandwiched between the p-CoI and n-CoI as described in the following statement:

$$\begin{aligned}\text{RSW} &\rightarrow \text{gVSW} \rightarrow \text{p-CoI} \rightarrow \text{SSW} \rightarrow \text{n-CoI}\\ &\rightarrow \text{gVSW} \rightarrow \textit{leading to wave breaking}.\end{aligned}$$

For the present case, however, the onset of the SSW occurs after an n-CoI rather than a p-CoI. The lateral inversion is responsible for this new type of transition to occur in a magnetized plasma. This present modification is a new type of transition caused due to the lateral inversion of the auxiliary subwell and appears to be generic for a SSW in a magnetized plasma. Therefore, we have characterized it as type IIA where II ensures the absence of the intermediate DL and A denotes the presence of wiggles near zero. We have previously conjectured that the lateral inversion could be due to either the presence of magnetic field or rarefaction of the oscillating species.[5] The same has been validated for the current model as well. It further shows that the transition route remains the same for the ion and electron acoustic SSWs.

We recall that the first ever pursuit of SSWs in a magnetized plasma was disrupted by the onset of the singularity.[3] As a second attempt, we found electron acoustic SSW for a magnetized plasma comprising energetic ions. However, the very low inertia of the electrons makes the validity of the fluid formalism itself often questionable. In spite of the lacuna in our previous reported paper, it provides a qualitatively consistent clue to detect the SSW in a magnetized plasma. In the present paper, we have used a well-established model and achieved, for the first time, an ion acoustic SSW for a magnetized plasma which is free from any ambiguities due to the imminent singularity. Through an F analysis, we have shown that the qualitative assessments in the present as well as the previously reported paper hold true. This paper has delivered a primitive idea of the properties of the SSW in a magnetized plasma. The compatibility between these results in turn validates the qualitative results of our previous paper.

IV. CONCLUSIONS

The ion acoustic SSW has been obtained for a three component magnetized plasma comprising warm fluid ions and two electrons having Boltzmann distributions. The obtained results are compatible with our previously reported electron acoustic SSW in a magnetized plasma. We have found that the onset of singularity is entangled with the probability of finding SSW in a magnetized plasma. We have also reported that there is an association between the singularity and the SSW in a magnetized plasma. The F analysis would provide a way to determine the parametric regime to detect SSW in a magnetized plasma and also to check the plausibility of the result. Varghese and Ghosh[9] reported that the fluctuation in the charge separation is responsible for the existence of the extra subwell. We have conjectured that the fluctuation of F in magnetized plasma is akin to the fluctuation in charge separation for the unmagnetized plasma. The magnetic field or the rarefaction of the oscillating species is accountable for the swapping of the position of the subwells. The presence of magnetic field and the rarefaction of the oscillating species broadens the probability of finding the SSW for a wide-range of parameters. The prediction of SSW in a magnetized plasma may provide a myriad possibilities to analyze the low altitude auroral regions in the Earth's magnetosphere, especially the non-conventional localized structures in the E-field data.

ACKNOWLEDGMENTS

The authors would like to thank Professor Frank Verheest (Sterrenkundig Observatorium, Universiteit Gent, Belgium) and G. S. Lakhina (Indian Institute of Geomagnetism) for their fruitful discussions.

[1] A. E. Dubinov and D. Y. Kolotkov, "Interpretation of ion-acoustic solitons of unusual form in experiments in sf6-ar plasma," High Energy Chem. **46**, 349–353 (2012).
[2] A. E. Dubinov and D. Y. Kolotkov, "Above the weak nonlinearity: Supernonlinear waves in astrophysical and laboratory plasmas," Rev. Mod. Plasma Phys. **2**, 2 (2018).
[3] O. R. Rufai, R. Bharuthram, S. V. Singh, and G. S. Lakhina, "Ion acoustic solitons and supersolitons in a magnetized plasma with nonthermal hot electrons and Boltzmann cool electrons," Phys. Plasmas **21**, 082304 (2014).
[4] F. Verheest and M. A. Hellberg, "Oblique propagation of solitary electrostatic waves in magnetized plasmas with cold ions and nonthermal electrons," Phys. Plasmas **24**, 022306 (2017).
[5] T. Kamalam, S. V. Steffy, and S. S. Ghosh, "Electron acoustic super solitary waves in a magnetized plasma," J. Plasma Phys. **84**, 905840406 (2018).
[6] S. S. Ghosh and G. S. Lakhina, "Anomalous width variation of rarefactive ion acoustic solitary waves in the context of auroral plasmas," Nonlinear Processes Geophys. **11**, 219–228 (2004).
[7] A. E. Dubinov, "On a widespread inaccuracy in defining the Mach number of solitons in a plasma," Plasma Phys. Rep. **35**, 991 (2009).
[8] F. Verheest, "Oblique propagation of solitary electrostatic waves in multispecies plasmas," J. Phys. A: Math. Theor. **42**, 285501 (2009).
[9] S. S. Varghese and S. S. Ghosh, "Transitional properties of supersolitons in a two electron temperature warm multi-ion plasma," Phys. Plasmas **23**, 082304 (2016).
[10] S. V. Steffy and S. S. Ghosh, "Existence domain of the compressive ion acoustic super solitary wave in a two electron temperature warm multi-ion plasma," Phys. Plasmas **24**, 102111 (2017).