Effect of pressure pulse on geomagnetic field oscillations

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Abstract. The effect of solar wind pressure pulse on geomagnetic field oscillations has been computed by using Green's function technique. The dominance of toroidal oscillations during dawn/dusk sectors appears to be natural consequences of solar wind pressure pulse and may not be attributed to K-H instability at the magnetopause boundary caused by velocity shear. Pressure pulse generates surface waves at the magnetopause boundary and couples to the field oscillations to give rise such effects. The paper adopts the 3-dimensional approach to explain the phenomena.

Index Terms. Field line response, geomagnetic pulsations, solar-wind impulse, standing waves.

1. Introduction

The idea of explaining geomagnetic pulsations (10-600s) in terms of standing Alfvén waves excited on geomagnetic field lines was floated by Dungey (1954). With the dawning of satellite era space scientists have been actively involved in studying the phenomena from theoretical as well as observational points of view (Cummings et al., 1969; Singer et al., 1981; Anderson et al., 1990; Sinha and Rajaram, 1997; Sinha et al., 2005 and references therein). Field lines behave as loaded strings with variable loading of plasma along the field line. In this paper we make an attempt to provide a methodology to compute the field line response to an arbitrary external forcing at the magnetopause boundary. The force exerted causes re-adjustment at the magnetopause boundary and the corresponding surface current required for pressure balance. We discuss the methodology in general and use it to compute the field line response to sudden change in solar wind dynamic pressure to demonstrate its utility. A primitive model (Mead 1964) , which neglects the plasma currents within the magnetosphere and computes the magnetic field due to surface currents for a given magnetopause stand-off distance caused by sudden change in solar wind dynamic pressure, has been used for the purpose. The model has been effectively used to estimate the magnetic field associated with storm time sudden commencement and sudden impulse (cf. Schulz and Lanzerotti 1974) to understand the role of these transient variations in cross field diffusion and drift echoes. In the present context the model will be helpful in understanding the excitation of field line resonant oscillations by sudden change in solar wind pressure.

Chen and Hasegawa (1974) for the first time made an attempt to study the eigen mode excitation by any external force. They studied the problem in Cartesian geometry using Green's function technique to address the radial structure of oscillations induced by surface eigen modes, but the paper did not throw any light on mode structure along the field line as regards their response to external forcing. Leonovich and Mazur (1997) did the most significant work in this direction to study the excitation of field lines by the E-layer ionospheric currents in dipole geometry. They used natural eigen modes for the construction of Green's function. In the present work a similar methodology using spectral representation of Green's function has been applied to compute the response of magnetospheric transverse mode oscillations to the time varying oscillations in current flowing at the magnetopause boundary due to sudden change in solar wind dynamic pressure. This is a simplified model for simulating the oscillations associated with the solar wind dynamic pressure. Response of the field lines is computed taking the ionosphere as perfectly reflecting. The perfectly reflecting ionosphere can be attained in two extreme cases viz. (a) the ionosphere acts as perfect conductor that amounts to assuming that field lines are rigidly held at their feet in the ionosphere, and (b) the ionosphere acts as perfect insulator meaning that the field lines are free to move at their end in the ionosphere. Sudden change in solar wind dynamic pressure are known to excite field line oscillations along select field lines (Baumjohann et al., 1984) and hence the problem is important from theoretical point of view to look into the excitation mechanism of such oscillations. The problem has been solved for both the 'rigid' and 'free' end cases. The layout of the paper is as follows.

Section 2 describes the theory and analysis as to how the theory could be applied to bring out the response of transverse field line oscillations to the current variations in magnetopause associated with sudden change in solar wind dynamic pressure. Highlights of the results brought out by the present investigations and the main conclusions are stated in section 3.

2. Theory and analysis

We start with the second order wave equation (Singer et al., 1981) in the normalized form describing low frequency transverse waves in infinitely conducting stationary, magnetized plasma with negligible plasma pressure in dipolar field geometry imparting time dependence of the type $\exp(i\omega t)$ to all the perturbed quantities and taking the density distribution due to Cummings et al. (1969) of the type $\rho=\rho_o(r_o/r)^m$; ρ_o, r and m being proton mass density at r_o (the geocentric distance to the equatorial crossing point of the field line), the geocentric distance to the point of interest on the field line and density index respectively. In the original derivation of the equation by Singer et al. (1981) compression of the magnetic field was completely neglected. If we account for the compression the system can be described in its modified form by the following equation as:

$$[L-\lambda]X(S)=f(S) \qquad (1)$$

where,

$$L=-\frac{B.\sin^{2m}\theta}{h_\alpha^{\,2}}\frac{d}{dS}\left[h_\alpha^{\,2}.B\frac{d}{dS}\right], f(S)=-\frac{B.\sin^{2m}\theta}{h_\alpha^{\,2}}\frac{\partial b_s^{'}}{\partial\alpha}, \lambda=\Omega^2 \qquad (2)$$

and the quantities are normalized as

$$S=\frac{s}{R_e}, \Omega=\frac{\omega R_e}{V_{Aeq}}, X=\frac{\xi_\alpha}{R_e.h_\alpha}, B=\frac{B_o}{B_{eq}}, b_s^{'}=\frac{b_s}{b_{eq}} \qquad (3)$$

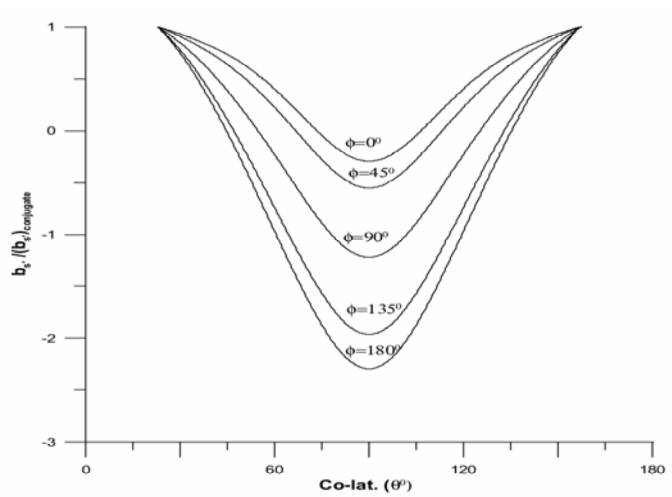

Fig. 1. The compressional component of geomagnetic field variation due to the oscillations of current at the magnetopause based on Mead's model (1964) along a field line (L_d=6.6) at different latitudes.

In the above equations s is the distance measured along the field line from the equator, θ is the co-latitude, ω is the mode frequency, ξ_α is the plasma displacement, B_o is the ambient magnetic field, b_s is the change in the magnetic field strength due to compression, R_e is the radius of the Earth, V_{Aeq} is the Alfvén velocity at the equator, B_{eq} is the ambient field at the equator, the parameter α defines the mode of oscillation and determines the direction of field line displacement and h_α is scale factor for the normal separation between the field lines in the direction of displacement and is determined by the ambient magnetic field structure. The derivative with respect to α in the right hand side means that the derivative has to be taken with respect to the longitude ϕ for the toroidal mode and with respect to ν for the poloidal mode, where form a $(\vec{\nabla}s, \vec{\nabla}\phi, \vec{\nabla}\nu)$ right handed orthogonal system.

Any fluctuation in the magnetopause current associated with pressure change will produce a field b_s at the field line position. It can then provide the impulse that can in turn generate a response in the field line oscillations (Lee, 1996). Thus, we can identify b_s that appears on the right hand side with external forcing and solve for response defined by the operator on the left hand side of the equation. Switching off the forcing term by putting rhs of Eq. (1) as zero, we get

$$[L-\lambda_n]u_n(S)=0 \qquad (4)$$

This is identical to the equation of Singer et al. (1981) derived by completely neglecting the compression in the magnetic field. The equation represents the normal modes of the system and can reproduce spatial and temporal structures of field line oscillations if solved under appropriate boundary conditions at the foot of field lines in the ionosphere. It should be noted that the function $f(S)$ in right hand side of Eq. (1) depends only on the change in the magnetic field due to compression caused by external forcing at magnetopause boundary.

We proceed to solve Eq. (1) by spectral representation of the Green's function technique (Dudley, 1994). As a first step we have to solve the equation with right hand side zero (Eq. (4)) with the specified boundary conditions. It is assumed that $X(S)$ satisfies the same boundary conditions as that of $u_n(S)$. We solve the problem in perfectly ionospheric conditions viz. (a) ionospheric conductivity is infinitely large: fixed-end scenario and (b) ionospheric conductivity is infinitesimally small: free-end scenario. In terms of boundary conditions it amounts to saying that the eigen function $u_n(S)$ in the first case and its derivative in the second case are vanishing at the foot of the field line in the ionosphere for both the hemispheres. Eq. (4) was solved numerically for both toroidal and poloidal oscillations using second order Runge-Kutta method and λ_n was obtained by shooting method using the specified boundary conditions at the conjugate points. Proton density at the equator was taken to be 1/cm^3 and density index m is taken to be 5. Eigen functions are symmetric (or asymmetric) about the equator for the odd (or even) harmonics for the fixed-end case (Cummings et al., 1969), while for the free-end case the even (or odd) harmonics are symmetric (or asymmetric) about the equator (Newton et al., 1978). The eigen function thus obtained is normalized in the domain and hence the normalized eigen function u_n corresponding to a particular eigen value λ_n is obtained as a function of S. The orthonormality of solutions thus obtained is ensured by subjecting them to the following criteria:

$$\int_D u_n^*(S)u_n(S)dS = 1, \int_D u_m^*(S)u_n(S)dS = 0, m \neq n \qquad (5)$$

As the operator L given by Eq. (2) is real in the domain specified by the boundaries at the ionosphere, it is also self-adjoint which is an essential condition for this analysis. Having confirmed that the operator L is self-adjoint, a complete orthonormal set of solutions of Eq. (4) can be used to obtain the solution of Eq. (1) by constructing the Green's function $G(S, S')$ and the solution can be written as

$$\mathrm{X(S)} = \int_D \mathrm{G(S,S')f(S')dS'} \qquad (6)$$

where, the integration is carried out in the domain D of the eigen function and $G(S,S')$ is given as

$$G(S,S') = \sum_{n=1}^{\infty} \frac{u_n(S)u_n^*(S')}{\lambda_n - \lambda} \qquad (7)$$

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Here, the summation runs over the harmonics.

Finally, the solution of equation (1) can be written by combining Eqs (6) and (7) as

$$X(S) = \sum_{n=1}^{\infty} \left(\frac{u_n(S)}{\lambda_n - \lambda}\right) \int_D u_n^*(S')f(S')dS' \qquad (8)$$

Response $X(S)$ of a given mode depends on its contribution in the expansion of the forcing term and also on the proximity of λ to λ_n. The response maximizes when the frequency of the wave is in the vicinity of one of the natural frequencies of field line oscillations and u_n closely reproduces the latitudinal structure of the function $f(S)$. For numerical computation, we impart a small imaginary part γ to the wave frequency ω in order to avoid singularity at the resonant condition i.e. when $\lambda = \lambda_n$. If we write $\omega = \omega_r + i\gamma$ then in resonant condition it can be seen that $|\lambda_n - \lambda| \sim 2\gamma\omega_r$. Thus, greater responses are expected for the lower harmonics. For the present analysis the value of γ is chosen to be 0.005. The ratio of γ to the fundamental normal mode angular frequency is of the order of 10^{-4} and the ratio will still be smaller for higher mode frequencies. Here, a finite γ has been taken just to avoid singularity and to get physically meaningful response.

2. 1 Response to sudden change in solar –wind dynamic pressure

It is obvious from the integral in equation (8) that the magnitude of response of the field line oscillations is sensitive to the latitudinal structure of the impulse term. It is, therefore important to look for a working model that at least qualitatively represents the latitudinal structure of impulse. Mead's model (1964) provides an estimate of magnetic impulse along a field line caused by distribution of magnetopause currents associated with the sudden impulse of solar wind dynamic pressure. In the model interplanetary magnetic field and plasma pressure within the magnetosphere have been neglected and the Earth's dipole is assumed to be normal to the direction of undeflected solar wind. It can be seen that the compressional component of geomagnetic field due to the oscillations of currents at the magnetopause based on Mead's model is given as

$$b_s = \frac{1}{\sqrt{1+3\cos^2\theta}}\left[-3\frac{B_1}{R_e}\left(\frac{R_e}{a}\right)^4(1-3\cos^2\theta) + 4\frac{B_2 L_d \sin^3\theta\cos\phi}{R_e}\left(\frac{R_e}{a}\right)^5(1+2\cos^2\theta)\right]\Delta a \qquad (9)$$

where, $B_1 \sim 0.25$ Gauss, $B_2 \sim 0.21$ Gauss, a is the equatorial stand-off distance from the point dipole to the magnetopause in the noon meridian and Δa is the displacement of the magnetopause in the direction perpendicular to its surface. The variation of b_s along a particular field line ($L_d = 6.6$) has been shown in Fig. 1.

If we take the time variation of Δa as exp $(i\omega t)$ then $f(S)$ in Eq. (1) for toroidal and poloidal modes are of the forms:

$$f(S) = \frac{4BB_2'\sin\phi\sin^{2m}\theta(1+2\cos^2\theta)}{L_d\sin^3\theta . a'^5\sqrt{1+3\cos^2\theta}}\Delta a' \text{ for toroidal} \qquad (10a)$$

$$f(S) = -\frac{54BB_1'\cos^2\theta(1+\cos^2\theta)\sin^{2m}\theta}{h_\nu L_d\sin\theta . a'^4(1+3\cos^2\theta)^2}\Delta a' \text{ for poloidal} \qquad (10b)$$

where, B_1 and B_2 have been normalized wrt B_{eq}, a and Δa are normalized wrt R_e.

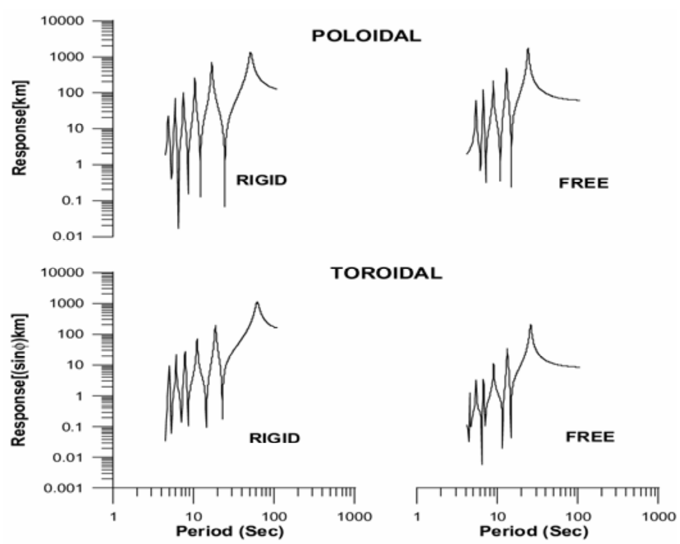

Fig. 2. The equatorial response of a magnetic shell (L_d=6.6) as a function of wave-periods. Peaks in decreasing sequence of wave-periods for rigid-end case correspond to odd harmonics i.e. harmonic numbers 1, 3, . .and those in case of free-end correspond to even harmonics i.e. harmonic numbers 2, 4. . . The toroidal response at a given longitude can be obtained in kilometers by multiplying the ordinate with a factor sin φ.

Eqs. (10a) and (10b) show that for both the toroidal and poloidal oscillations $f(S)$ is symmetric about the equator. The longitudinal variation of $f(S)$ due to the factor sin φ in case of toroidal oscillations and its axially symmetric behaviour in case of poloidal mode is to be noted. It follows straight away that only symmetric modes will respond to the forcing. In case of fixed-end the odd harmonics are symmetric and in the case of free-end the even harmonics are symmetric. Also, *f(S)* depends on the amplitude of the magnetopause displacement $\Delta a'$ and the stand-off distance a' of the magnetopause. In the present analysis we have taken the value of $\Delta a'$ as 0.1 and that of a' as 10. Moreover, *f(S)* will be different for different magnetic shells characterized by their L_d values. In fact *f(S)* decreases with increase in L_d. The presence of a' in the denominator with one power less and the factor $\cos^2\theta$ in the numerator in the expression of *f(S)* for the poloidal mode as compared to that of toroidal mode indicate the greater response for poloidal mode as compared to that for toroidal mode. All these characteristics should be reflected in the response *X(S)* of magnetic field lines.

Constructing the Green's function using the normal mode solutions of field line oscillations by equation (7) and plugging it along with *f(S)* in equation (6), solution *X(S)* of equation (1) could be obtained. In other words, the response of toroidal and poloidal oscillations to the oscillations of current at the magnetopause associated with the solar wind dynamic pressure is computed using Mead's model in perfectly reflecting ionospheric conditions. The response were computed for fixed L_d taking different periods of driving oscillations (i.e. for different corresponding $\lambda = \Omega^2$) for both types of boundary conditions (i.e. 'rigid-end' and 'free-end') (Fig. 2). The field line structures of response for dominant modes (i.e. fundamental in case of rigid-end and second harmonic in case of free-end) have also been computed taking L_d = 6.6 for both the toroidal and poloidal oscillations (Fig. 3). Longitudinal variation of response can be obtained by multiplying with $\sin\varphi$ for various driving periods only in case of toroidal oscillations as such longitude dependence is not that obvious for poloidal oscillations. This could be possible because we took the three dimensional distribution of compressional source. It is not possible to get the longitudinal variation of response in the simple box geometry. The results and their various implications have been discussed in the following section.

3. Results and conclusion

Main features of the results obtained through the present analysis can be stated as follows:

(a) The positions of peaks in Fig. 1 match with the time-periods of harmonics of natural modes of field line oscillations as given by Cummings et al. (1969). Peaks in rigid-end case correspond to the odd harmonics (left panel of Fig. 2) and peaks in free-end case correspond to the even harmonics (right panel of Fig. 2). Results suggest that the waves caused by external forcing at the magnetopause boundary may excite the field line oscillations as suggested by Lee (1996).

(b) For a given magnetic shell, poloidal mode shows larger response as compared to toroidal. (Fig. 2).

(c) It is revealed through the analysis that odd harmonics are excited in case of 'rigid-end' boundary and even harmonics are excited in case of 'free-end' boundary (Fig. 2) as expected from expression given in Eq. (6). Thus, it is predominantly fundamental mode that responds in the 'rigid-end' case and second harmonic that responds in the 'free-end' case.

(d) The field line structures of the dominant modes (viz. fundamental in rigid-end case and second harmonic in free-end case) have been computed and presented in Fig. 3. The displacement gets enhanced almost by the order of three in presence of compression. The factor $\sin\varphi$ in case of toroidal oscillations clearly indicates the longitudinal dependence of the mode and arises due to the three dimensional model of the compression at the magnetopause boundary.

The most significant result of the present analysis is the dominance of toroidal mode in dawn/dusk sector that is brought out by taking the 3-D model of compresional wave caused by solar wind dynamic pressure variation at the magnetopause boundary.

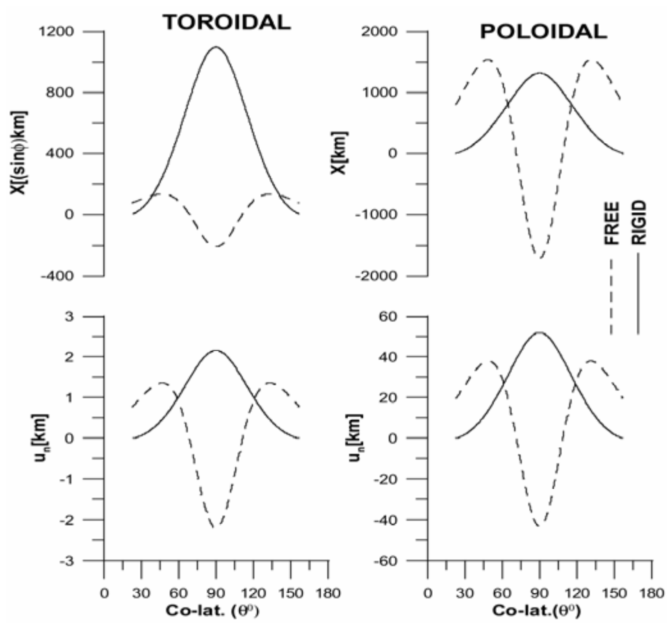

Fig. 3. The latitudinal response-structures of dominant modes (fundamental for rigid-end case and second harmonic for free-end case) along a field line (L_d = 6.6). The lower panel shows the mode structure when there is no compression whereas the upper panel represents the field line structure in presence of compression.

It is traditionally believed that the dominant nature of observed toroidal oscillation in the dawn/dusk sectors can be attributed to K-H instability (Anderson et al., 1990), but without much observational evidences. The present analysis does not account for the propagation aspect (and hence the shear in velocity) of the compressional waves generated by the solar wind pressure variation at the magnetopause boundary and still could reveal the dominance of toroidal oscillations in the dawn and the dusk regions. Parallel developments (Southwood and Kivelson, 1990; Chen and Hasegawa, 1974) have considered the radial propagation effect in simplified geomagnetic field geometry, thus ignoring the realistic mode structure along the field line and the latitudinal structure of the impulse forcing. The two approaches have to converge in future. We want to make a point about the methodology applied to solve the problem through spectral representation of Green's function that the method has immense potential in the sense that response to the coupling of complex forcing associated with K-H instability or tearing instability can also be evaluated given b_s the compression associated with the driving force.

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