Time evolution of ion-acoustic double layers in an unmagnetized plasma

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Ion-acoustic double layers are examined in an unmagnetized, three-component plasma consisting of cold ions and two temperature electrons. Both of the electrons are considered to be Boltzmann distributed and the ions follow the usual fluid dynamical equations. Using the method of characteristics, a time-dependent solution for ion-acoustic double layers is obtained. Results of the findings may have important consequences for the real time satellite observations in the space environment. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2967901]

I. INTRODUCTION

Double layers are nonlinear structures found in a wide variety of plasmas, e.g., laboratory plasmas, Earth's magnetosphere, and dusty plasmas, and they are especially common in current-carrying plasmas. They can accelerate, decelerate, or reflect the plasma particles. A great deal of interest has been generated in understanding the formation of double layers. Sato and Okuda[1] carried out numerical simulations of an ion-acoustic instability. It was found that anomalous resistivity generated by the instability causes the buildup of a dc potential, which in turn accelerates electrons. The acceleration of electrons enhances the original instability and consequently leads to the formation of double layers. The existence and stability of double layers using various descriptions of plasma species has been theoretically investigated.[2–4] Bharuthram and Shukla[5] examined the theory of large-amplitude ion-acoustic double layers in a stationary frame in an unmagnetized plasma with cold ions and Boltzmann distributed two temperature electrons. A large-amplitude fluid theory of electrostatic ion-acoustic double layers[6] has been carried out and shown to be adaptable to stationary double layers in a two-ion plasma with kinetically determined ions. Baboolal *et al.*[7] examined kinetic double layers in two electron temperature multi-ion plasmas. The effect of various parameters such as ion temperature and light-ion concentration on the double-layer structure has been examined and compared with fluid theory.

Stationary ion-acoustic double layers have been investigated in an electron beam plasma system.[8] The parametric study showed the condition of existence of double layers sensitive to parameters such as the electron beam temperature, the ion temperature, the beam density, and the trapped electrons. Nonlinear theory of ion-acoustic waves in two-electron temperature plasma with isothermal electrons and cold ions has been developed.[9] Both compressive and rarefactive solitons, as well as double layers (depending on the concentration of low-temperature electrons), can be generated. Mishra *et al.*[10] studied ion-acoustic double layers in a plasma consisting of warm positive- and negative-ion species and two-electron temperature distributions. It was found that both compressive as well as rarefactive double layers exist depending upon negative ion concentration being below or above the critical concentration, respectively. Verheest *et al.*[11] examined the existence regime of ion-acoustic double layers in two-electron temperature plasmas using a fluid-dynamical approach, with polytropic equations of state indices γ_j. It was found that no double layers can be formed for $\gamma_j \geqslant 3/2$, due to total rarefaction of the cooler electrons or infinite compression of the ions. For the $\gamma_j < 3/2$ case, the rarefactive double layers do occur, but at unrealistically small cool electron densities or large Mach numbers. Das *et al.*[12] have investigated the existence and stability of the alternative solitary-wave solution of the ion-acoustic wave in magnetized plasma consisting of warm adiabatic ions and nonthermal electrons. The existence and stability of ion-acoustic double layers in magnetized plasma consisting of warm adiabatic ions and nonthermal electrons have also been studied.[13–15] Mishra *et al.*[16] examined small-amplitude ion-acoustic double layers in multicomponent plasma with positrons.

Space observations in near-Earth environments have revealed the presence of large-amplitude electrostatic structures, associated with high-frequency disturbances. Several theoretical studies were initiated based on these observations. Sutradhar and Bujarbarua[17] studied the ion acoustic double layer in the presence of negative ions and showed that the rarefactive double layers exist when the negative ion concentration is less than a limiting value. The results of the findings were found to be in agreement with the observations in the Earth's magnetosphere. Compressive and rarefactive ion-acoustic double layers have been studied in multicomponent plasmas using Sagdeev potential technique. Weak ion-acoustic double layers are described by a modified Korteweg–de Vries equation. Their relevance to auroral and experimental plasmas has been discussed.[18] Reddy *et al.*[19] investigated ion-acoustic double layers in a multispecies auroral beam plasma system. The findings of the model were in good agreement with the observations of double layers by S3-3 and Viking satellites. Compressive and rarefactive electron-acoustic solitons and double layers in space plasmas have also been studied.[20,21] High temporal and spatial resolution measurements by the Swedish satellite Viking have shown the presence of double-layer structures in the auroral regions of the magnetosphere.[22] These structures are negatively charged and have scale sizes of about 100 m. It has been very well established by satellite observations that parallel electric fields of both upward and downward current regions of the aurora are supported by strong double layers. It has been shown by direct observations of the ionospheric boundary of the auroral cavity region that a stationary, oblique double layer carries a substantial fraction of the auroral potential. Amplitude of oblique double layers greater than 100 mV/m has been found to occur on auroral cavity crossings. Double layers have also been established inside of the auroral cavity. The existence of these structures suggests a possibility of a midcavity or high-altitude acceleration mechanism.[23]

Double layers have also been examined in the laboratory plasmas.[24–29] Sekar and Saxena[30] observed the weak ion-acoustic double layers resembling an asymmetric ion hole in a double plasma device. More recently,[31] double layers have been observed to propagate from the source region to the diffusion chamber of a helicon-type reactor filled up with a low-pressure mixture of Ar/SF6.

All the theoretical investigations discussed above have been restricted to a stationary, steady-state solution. However, in real situations, these nonlinear structures are time-dependent. To the authors' knowledge, there has not been any theoretical attempt to study the time-dependent double-layer solutions. In this paper, we attempt to obtain a time-dependent solution of ion-acoustic double layers in an unmagnetized plasma. The model considered here is an adaptation of that of Bharuthram and Shukla[5] with cold ions and two temperature electrons. The electrons are considered to be Boltzmann distributed and the ions follow the usual fluid dynamical equations. The paper is organized as follows. In the next section, our theoretical model is presented. In Sec. III, numerical results are discussed and conclusions drawn.

II. THEORETICAL MODEL

We consider an unmagnetized plasma consisting of cold, fluid ions (density n) and two species of electrons with densities, n_{ec}, n_{eh} and temperatures T_c and T_h, respectively. The electrons are considered to be Boltzmann distributed and ions follow the usual fluid dynamical equations. The complete set of normalized equations can be written as follows.

Ions:

Continuity equation:

$$\frac{\partial n}{\partial t} + v\frac{\partial n}{\partial x} = -n\frac{\partial v}{\partial x}. \tag{1}$$

Momentum equation:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{\partial \Phi}{\partial x}. \tag{2}$$

Electrons:

$$n_{ec,h} = N_{c,h}e^{\alpha_{c,h}\Phi}. \tag{3}$$

Poisson equation:

$$\frac{\partial^2 \Phi}{\partial x^2} = N_c e^{\alpha_c \Phi} + N_h e^{\alpha_h \Phi} - n(x,t), \tag{4}$$

where $\Phi = e\phi/T_{ef}$, $\alpha_{c,h}=T_{ef}/T_{c,h}$; $T_{ef}=N_0T_cT_h/(N_{0c}T_h + N_{0h}T_c)$, $N_0=N_{0c}+N_{0h}$, $N_{c,h}=N_{0c,h}/N_0$. In Eqs. (1)–(4), densities have been normalized by total equilibrium electron density, N_0, velocities by $c_{ef}=\sqrt{T_{ef}/m_i}$, distance by $\lambda_D=\sqrt{T_{ef}/4\pi N_0 e^2}$, time is normalized with respect to the inverse of ion plasma frequency, $\omega_{pi}^{-1}=\sqrt{m_i/4\pi N_0 e^2}$, m_i is the ion mass, and e is the electronic charge. Subscript "0" refers to equilibrium quantities.

To solve the time-dependent equations (1)–(4), we use the method of characteristics[32] and rewrite these equations as

$$\frac{dt}{ds} = 1, \tag{5}$$

$$\frac{dx}{ds} = v(s), \tag{6}$$

$$\frac{dn}{ds} = \frac{n(s)}{v^2(s)}\frac{d\Phi}{ds}, \tag{7}$$

$$\frac{dv}{ds} = -\frac{1}{v}\frac{d\Phi}{ds}, \tag{8}$$

$$\frac{d^2\Phi}{ds^2} = -\frac{1}{v^2(s)}\left(\frac{d\Phi}{ds}\right)^2 + v^2(s)[N_c e^{\alpha_c\Phi} + N_h e^{\alpha_h\Phi} - n(s)]. \tag{9}$$

In order to understand the correct physical behavior, we solve the system of equations (5)–(9) numerically using the following initial conditions:

$$t(0) = 0, \quad x(0) = p, \quad n(0) = 1, \quad v(0) = 1, \tag{10}$$

and assuming the initial solution of Eqs. (5)–(9) as

$$\Phi(0) = 1 + 0.01\tanh(p - 2x_0), \quad \Phi'(0) = 0, \tag{11}$$

where we have $p \in [0,20]$ and x_0 is a shift that we choose as $x_0=5$. Because of the initial profile, the switching occurs at $x=2x_0$. For each p, we integrate the characteristic equations for $s=0$ to s_{final}, where s_{final} is the final time. In this way, we can follow each point in time and evaluate the system of Eqs. (5)–(9). It must be noted that the initial value of $\Phi(0)$ has been deduced from a small-amplitude study in a stationary frame.[5] We would like to describe the method of characteristics in brief for the better understanding of the results. It is a technique that reduces a first-order partial differential equation or a hyperbolic partial differential equation to a system of first-order ordinary differential equations which are much

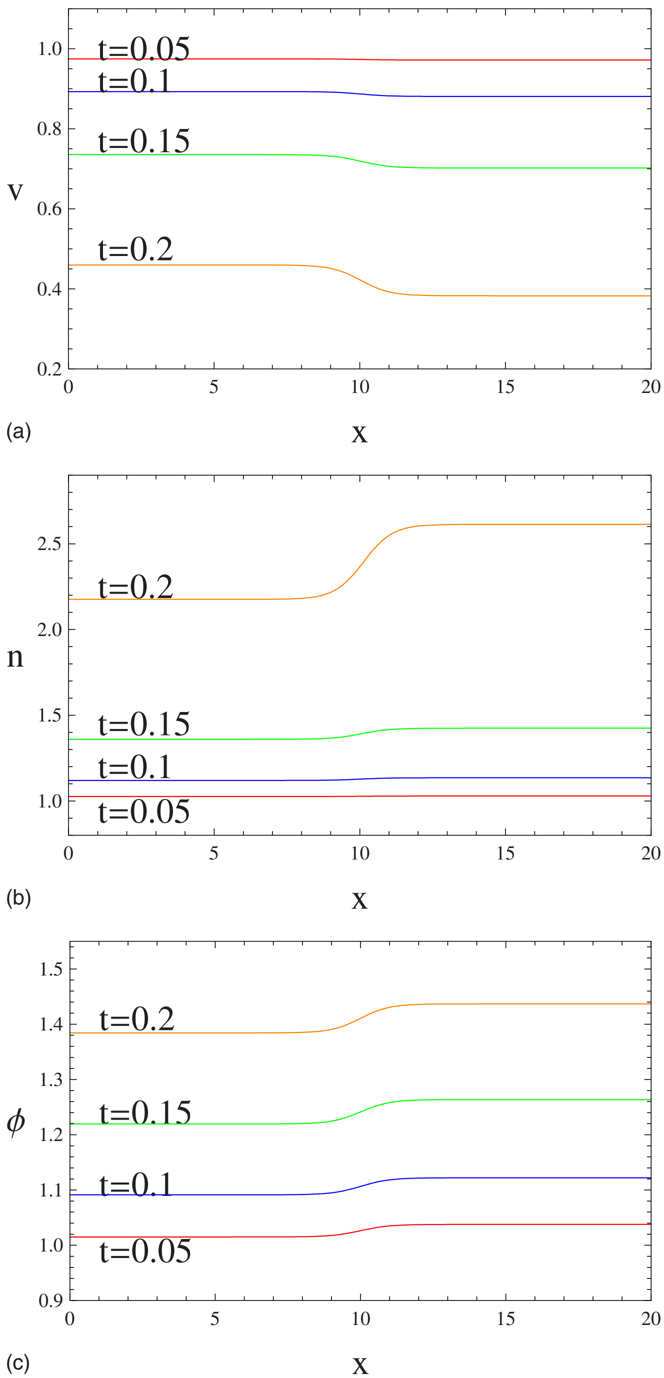

FIG. 1. (Color online) Time-dependent double-layer solution of Eqs. (5)–(9) for N_h=0.9, N_c=0.1, T_c/T_h=1/10. (a)–(c) represent the velocity, density, and potential vs x for different times as depicted on the curves.

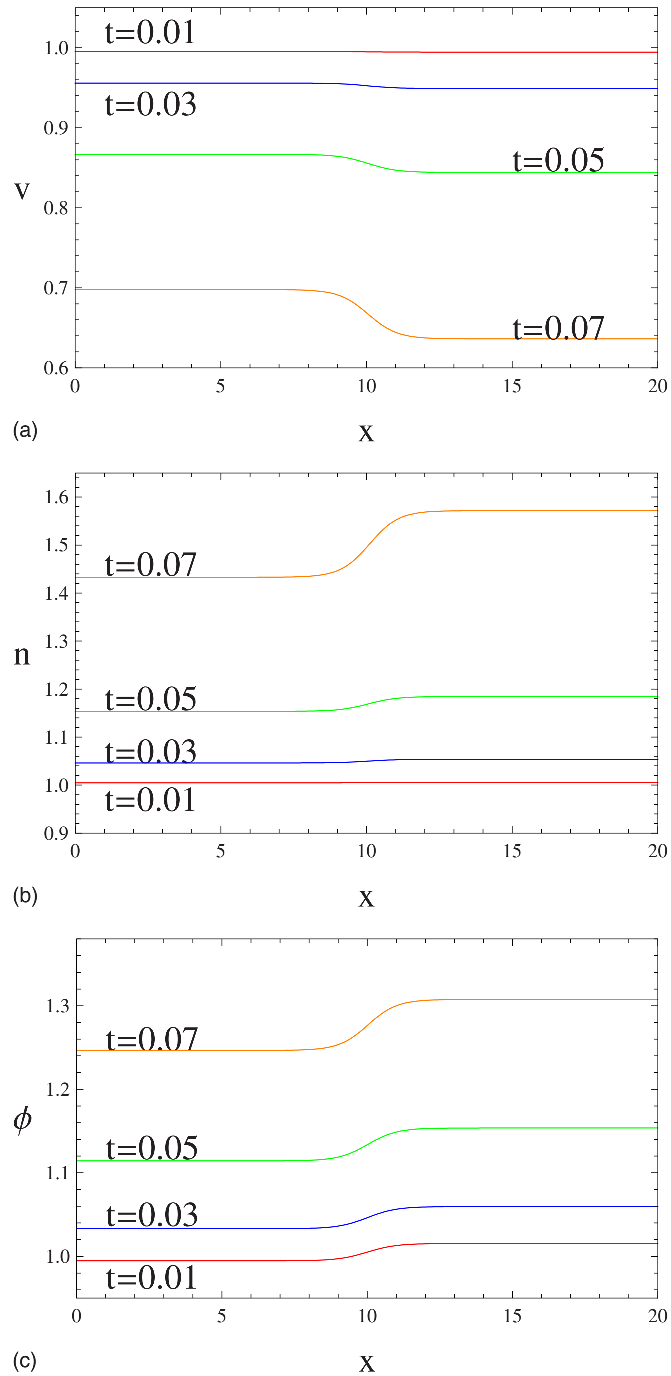

FIG. 2. (Color online) Time-dependent double-layer solution of Eqs. (5)–(9) for N_h=0.9, N_c=0.1, T_c/T_h=1/20. (a)–(c) represent the velocity, density, and potential vs x for different times as depicted on the curves.

easier to integrate along an appropriate initial surface. In our case, the appropriate initial surface is given by the boundary conditions (11). The interested reader may refer to Garabedian[32] for more details on method of characteristics.

III. RESULTS AND DISCUSSION

We numerically solve Eqs. (5)–(9) for the parameters N_h=0.9, N_c=0.1. Figures 1(a)–1(c) show the time evolution of ion velocity (v), ion density (n), and potential Φ versus x for the above parameters and T_c/T_h=0.1. It can be seen from the figures that starting from an initial perturbation, the double-layer structure grows stronger as time evolves. Numerical computations show that the evolving structures are quite stable.

Figures 2(a)–2(c) show the time evolution of the double layers for the parameters of Fig. 1 but at T_c/T_h=0.05. It is obvious from the examination of the two figures that the amplitude of the double layers increases with the decrease of the cold to hot electron temperature ratio, i.e., T_c/T_h. This behavior is in agreement with the double-layer solutions obtained in the stationary state condition for the same model by

Bharuthram and Shukla.[5] The figures show that the double-layer structures grow quite rapidly in time, reaching a stable structure in a period less than ω_{pi}^{-1}. Moreover, for small T_c/T_h (cf. Fig. 2), double layers form much faster in time compared to Fig. 1.

Earlier theories developed for the double layers used the stationary-state solutions, whereas we have obtained time-dependent solutions. Though we have not applied our results to explain any particular event or observation, it may be recalled that satellite observations in space environment, e.g., in Earth's magnetosphere, are made in space and time. Thus, our results may be useful in understanding the observations in these regions.

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