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Eigen modes of a hot plasma discontinuity in magnetosphere*

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It is shown that the surface with finite width between two regions of a plasma medium having different Alfven velocity can sustain a discrete set of hydromagnetic waves. One set of eigen values depends upon β (ratio of the kinetic pressure to the magnetic pressure) value of the "surface" region and is weakly dispersive. The other set is similar to the familiar fast mode and is controlled by the width of the hot surface. The latter mode, therefore, has a much higher frequency than the weakly dispersive mode.

1 Introduction

It is already known that a sharp discontinuity in a plasma is capable of exciting and maintaining a host of hydromagnetic waves[1-4] with a spectrum basically controlled by the degree of sharpness of the discontinuity. With respect to the plasmapause in the magnetosphere such surface waves have been shown by Chen and Hasegawa[3] to be the sources of the PC-5 micropulsations. Similar problem was also dealt with by Uberoi[5] to explain the observed compressional waves excited by an ideal surface between two cold plasmas with a sharp change in the plasma density. Although Chen and Hasegawa[3] treated the surface between the two regions of plasma densities to have finite width, their restriction to the high k_x (azimuthal wave vector) reduced the problem to that of the cold surface.

In this paper we shall consider all the possible k_x values with respect to a region with finite β (ratio of the kinetic pressure to the magnetic pressure) and with finite width between two cold plasma regions with Alfven velocities V_{A1} and V_{A3} respectively. The purpose is to investigate the possible eigen modes supported by such a system.

2 Wave equation

Here we consider a model as described in Fig. 1, where medium II with Alfven Velocity V_A and finite β is bounded on both sides by media I and III with Alfven velocities V_{A1} and V_{A3} respectively. The width of the layer is a. The unperturbed magnetic field **B** in all the three regions are along the Z direction.

Let us consider a perturbation of the form $\exp\{-i(xk_x + zk_z - \omega t)\}$ with parallel wave vector k_z, the azimuthal wave vector k_x and the wave frequency ω respectively. The amplitudes of the perturbations of various plasma quantities are functions of y. Since the total pressure $(B^2 + \gamma P_0)$, where P_0 is the thermal pressure, must be continuous across the boundaries normal to the field line, we must have

$$B_1^2 = B_2^2(1+\beta) = B_3^2 \qquad \dots (1)$$

where $\beta = \gamma P_0/B_2^2$

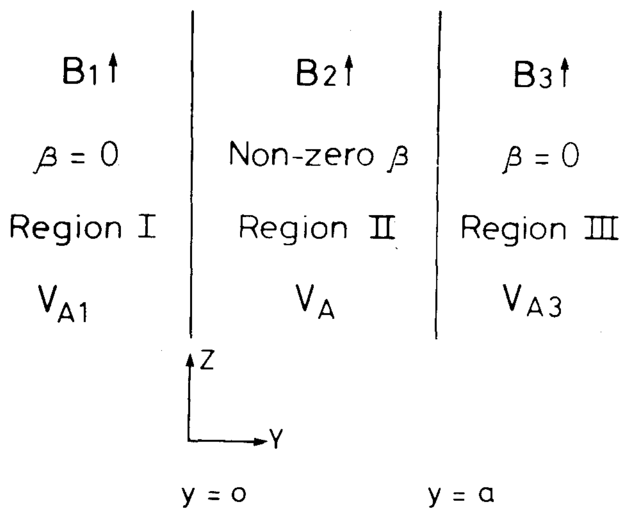

Fig. 1—Model of a hot surface layer of thickness a between two cold media.

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From the general hydromagnetic equation of a compressible fluid[3], one can write the wave equation for the y-component of the fluid displacement $\xi_y(y)$ as

$$\frac{d}{dy}\left[\frac{\alpha\varepsilon B^2}{\varepsilon-\alpha k_x^2 B^2}\frac{d\xi_y}{dy}\right]+\varepsilon\xi_y=0 \quad \ldots (2)$$

where

$$\varepsilon=k_z^2 B^2\left(\frac{\omega^2}{k_z^2 V_A^2}-1\right) \quad \ldots (2a)$$

and

$$\alpha=1+\beta\left[1-\frac{\beta k_z^2 V_A^2}{\omega^2}\right]^{-1} \quad \ldots (2c)$$

In our model, ε's are separately constant in the three regions. Therefore, we can write the solutions of (2) in these regions as

$$\begin{aligned}\xi_y &= \exp(y\lambda_1) && \text{for } y\leqslant 0\\ &= A\exp(y\lambda)+D\exp(-y\lambda) && \text{for } 0\leqslant y\leqslant a\\ &= C\exp(-y\lambda_3) && \text{for } y\geqslant a \quad \ldots (3a)\end{aligned}$$

Here

$$\lambda^2=\frac{\psi^2}{\psi(\beta+1)-\beta}-k^2 \quad \ldots (3b)$$

$$\psi=\frac{\omega^2}{k_z^2 V_A^2}\bigg|, \quad k^2=k_x^2+k_z^2 \quad \ldots (3c)$$

$$\lambda_{1,3}^2=k^2-\psi_{1,3}$$

and

$$\psi_{1,3}=\frac{\omega^2}{k_z^2 V_{A1,3}^2} \quad \ldots (3d)$$

Here we measure the lengths in unit of k_z^{-1}. The solutions are chosen so that $\xi_y\to 0$ as $|y|\to\infty$. The constants A, D, C are determined from the boundary conditions, which are that the normal displacement ξ_y and the total pressure $\tilde{p}=\alpha B^2\varepsilon\xi_y'/(\varepsilon-\alpha B^2 k_x^2)$ are both continuous across the boundaries $y=0$ and $y=a$. Since we are looking for normal modes, ω is assumed to be real.

In order to satisfy these boundary conditions, it can be shown by simple algebra that one must have

$$\cos 2\lambda a=\frac{(\lambda^2-S_1^2)(\lambda^2-S_3^2)-4\lambda^2 S_1 S_3}{(\lambda^2+S_1^2)(\lambda^2+S_3^2)} \cdot \quad \ldots (4a)$$

$$\sin 2\lambda a=\frac{2\lambda(S_1+S_3)(\lambda^2-S_1S_3)}{(\lambda^2+S_1^2)(\lambda^2+S_3^2)} \quad \ldots (4b)$$

where

$$S_{1,3}=\frac{\psi-1}{(\beta+1)}\cdot\frac{\lambda_{1,3}}{\psi_{1,3}-1} \quad \ldots (4c)$$

Eqs (4a) and (4b) will define the necessary eigen modes.

3 Search for the eigen modes

One set of possible solution is achieved by the condition $(S_1+S_3)=0$ and $2\lambda a=2n\pi$, n being an integer.

For the lowest mode,

$$\lambda=\frac{k_b}{2}, \quad k_b=2\pi/a \quad \ldots (5)$$

For this value of λ, $\cos 2\lambda a=1$.

From Eqs (3b) and (5), one gets for the eigen modes

$$\psi_\pm=\frac{k_0^2(\beta+1)}{2}\left[1\pm\left\{1-\frac{4\beta}{(\beta+1)^2 k_0^2}\right\}^{1/2}\right] \quad \ldots (6a)$$

where

$$k_0^2=k^2+k_b^2/4 \quad \ldots (6b)$$

The lower value ψ_- is actually almost independent of k_x and is given by

$$\psi_-=\frac{\beta}{\beta+1}\left[1+\frac{\beta}{(\beta+1)^2 k_0^2}+\cdots\right] \quad \ldots (6c)$$

whereas

$$\psi_+=k_0^2(\beta+1)-\psi_- \quad \ldots (6d)$$

and is virtually proportional to k_0^2.

In this case the properties of the media I and II must satisfy the condition $(S_1+S_3)=0$

or

$$\psi_3=1+\frac{k_x^2(1-\psi_1)}{k^2-\psi_1} \quad \ldots (7)$$

We have plotted ψ_3 against ψ_1 for various values of k_x in Fig. 2. This relationship appears to be similar to the cold plasma discontinuity discussed by Uberoi[5]. But the basic difference here is that the solution is not really continuous. Only certain

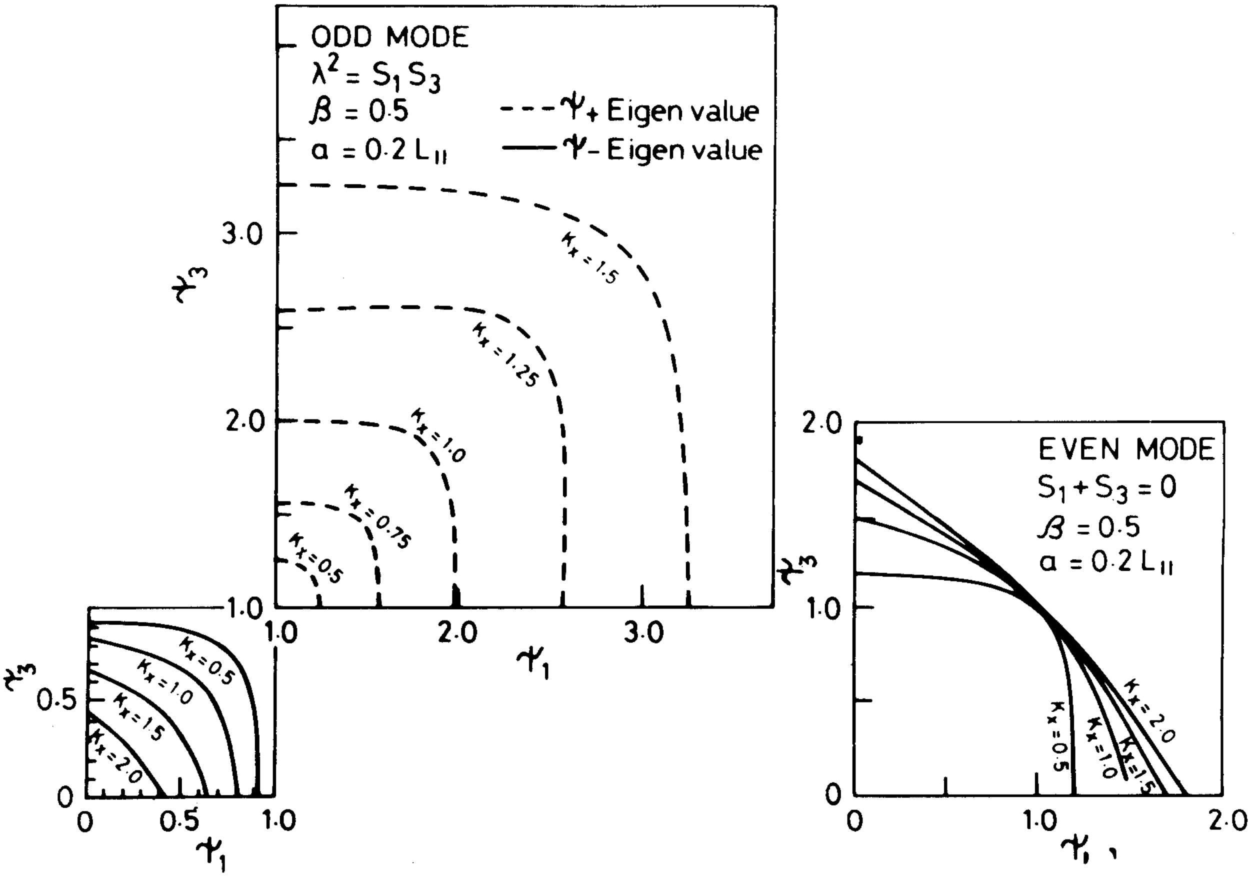

Fig. 2—ψ_3 versus ψ_1 for different values of k_x. For the case where $S_1+S_3=0$, there is only one set of graphs. But there are two sets of graphs corresponding to ψ_+ and ψ_- when $\lambda^2=S_1S_3$.

values of k_x will satisfy Eq. (7) for a given ratio of ψ_3/ψ_1. For the ψ_- mode, which is virtually independent of k_x, there will always be some k_x for which Eq. (7) is satisfied.

From Eq. (4b) we can see there is another set of solutions given by

$$\lambda^2 = S_1 S_3 \qquad \ldots (8a)$$

This implies that $\cos 2\lambda a = -1$. The lowest value of λ in this case is given by

$$\lambda = k_b/4 \qquad \ldots (8b)$$

These we define as odd eigen modes and are given by the same set of Eqs (6a) to (6d), but here

$$k_0^2 = k^2 + k_b^2/16 \qquad \ldots (8c)$$

In this case also the ψ_- mode is virtually dispersionless whereas ψ_+ mode is similar to a fast mode with much higher frequency. However, media I and III parameters must satisfy a different condition now in order to sustain these modes This new condition $\lambda^2 = S_1 S_3$ can be expressed as

$$\frac{[(k^2-\psi_1)(k^2-\psi_3)]^{1/2}}{(\psi_1-1)(\psi_3-1)} = \frac{k_b^2(\beta+1)^2}{16(\psi_\pm-1)^2} \qquad \ldots (8d)$$

The possible values of ψ_3 for a given ψ_1 are represented also in Fig. 2 for both the $\psi_\pm$ sets. It must be noted that both ψ_1 and ψ_3 must be simultaneously greater than one or less than one, whereas in the previous case $(S_1+S_3=0)$ if ψ_1 is less (greater) than one then ψ_3 must be greater (less) than one.

4 Relevance to geomagnetic micropulsations

It is well known that the storm time plasmapause is populated by hot plasma. The thickness of the region can be of the order of $1R_E$ to $2R_E$, where R_E is the earth's radius. This is also the region where the density of the cold plasma has a sharp change, dropping off with increasing distance away from the earth. Thus Alfven velocity generally increases from magnetopause towards the plasmapause and then towards the earth. But near the region of finite temperature at the plasmapause, V_A actually comes down and then increases. In this situation both ψ_+ and ψ_- modes can be sustained by this hot plasma layer.

The nearly dispersionless low frequency wave is always lower than the local Alfven mode $k_z V_A$ which is generally excited by various types of plasma instabilities like drift mirror instability[6] or by the coupling between the Alfven and the unstable drift mirror waves[7]. In all these cases, $\psi \sim 1.0$ in our notation. However, the present model predicts waves with frequencies considerably lower than these local Alfven frequencies.

The second group represented by ψ_+ is strongly dispersive and generally has high frequencies since k_b is definitely of the order of 2 to 5. Although the frequencies of the ψ_+ waves are high they are not necessarily localized in longitude, i.e. k_x does not have to be very large. It should be noted that the coupled mode excited by the hot plasmapause according to the theory of Lin and Parks[7] or surface waves of Chen and Hasegawa[3] all have very large k_x but lower frequency (i.e. lower than ψ_+ mode). The present model, however, predicts that the hot plasmapause will be able to sustain both high and low freequency fluctuations simultaneously.

Thus if ψ_- represents waves in the PC-5 range, ψ_+ will be in the PC-2 or PC-3 pulstion type. The maximum field-aligned wavelength at about $4R_E$ location may be of the order of $10R_E$. Thus k_b for a $2R_E$ thick plasmapause will be of the order of 5. For such a situation, $(\psi_+/\psi_-)^{1/2} \simeq 8$, $k_x = 2$ and $\beta = 0.25$. Hence a 10 s PC-1 pulsation and a 80 s PC-3 pulsation can be simultaneously sustained by such a model. In fact, simultaneous occurrence of low and high frequency waves are quite common. For example, the ground magnetometer data presented by Wedeken *et al.*[8] show periodicities ranging from 60 s to 600 s. Since $\psi_- \simeq [\beta/(\beta+1)]$, the corresponding PC-5 wave will indicate the temperature of the hot layer, but the ψ_+ mode will give a measure of the thickness of the hot region

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