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Ion-acoustic Gardner Solitons in electron-positron-ion plasma with two-electron temperature distributions

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The ion–acoustic solitons in collisionless plasma consisting of warm adiabatic ions, isothermal positrons, and two temperature distribution of electrons have been studied. Using reductive perturbation method, Korteweg-de Vries (K-dV), the modified K-dV (m-KdV), and Gardner equations are derived for the system. The soliton solution of the Gardner equation is discussed in detail. It is found that for a given set of parameter values, there exists a critical value of $\beta = T_c/T_h$, (ratio of cold to hot electron temperature) below which only rarefactive KdV solitons exist and above it compressive KdV solitons exist. At the critical value of β, both compressive and rarefactive m-KdV solitons co-exist. We have also investigated the soliton in the parametric regime where the KdV equation is not valid to study soliton solution. In this region, it is found that below the critical concentration the system supports rarefactive Gardner solitons and above it compressive Gardner solitons are found. The effects of temperature ratio of two-electron species, cold electron concentration, positron concentration on the characteristics of solitons are also discussed. © *2016 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4939802]

I. INTRODUCTION

Recently, the study of linear and nonlinear wave phenomena in electron-positron-ion plasma is a field of current research investigation. The electron-positron plasmas are generated naturally in space environments such as the pulsar magnetosphere,[1,2] early universe[3,4] or neutron stars, active galactic nuclei,[5] and in sun atmosphere.[6] The electron positron plasma has also been created in the laboratory.[7–9] Most of the astrophysical[6,10] and laboratory plasmas[11–13] are admixture of electrons, positrons, and small fraction of ions. The study of the electron-positron-ion (EPI) plasmas is important to understand the behavior of both space scenarios and laboratory.

The physics of EPI plasmas has received a great deal of attention in investigating the nonlinear structures, e.g., solitons, double layers, and modulational instability. The envelope solitons associated with the electromagnetic waves in EPI plasmas have been studied by Rizzato[14] and Berezhiani *et al.*[15] Ion-acoustic solitons in EPI plasmas have been reported by Popel *et al.*[16] Nejoh[17] studied the effects of the ion temperature on large amplitude ion-acoustic waves in an EPI plasma. Ion-acoustic compressive and rarefactive double layers in a warm multicomponent plasma have been studied by Mishra *et al.*[18] and Jain and Mishra.[19] The obliquely propagating ion-acoustic double layers in magnetized electron-positron-ion plasma have been studied by Chawla and Mishra.[20] Large amplitude solitary electromagnetic waves in EPI plasmas have been studied by Verheest and Cattaert.[21] The double layers associated with the kinetic Alfven wave in a magnetized electron-positron-ion plasma have been studied by Kakati and Goswami.[22] Ion-acoustic envelope solitons in electron-positron-ion plasmas have been studied by Salahuddin *et al.*[23] Chawla *et al.*[24] have studied the modulation instability of ion-acoustic waves in EPI plasma. Tiwari *et al.*[25] have studied the effects of positron density and temperature on ion-acoustic dressed solitons in electron-positron-ion plasma. Recently, Jain and Mishra[26] have studied the arbitrary amplitude ion-acoustic solitons in electron-positron-ion plasma having warm ions. Ion-acoustic periodic waves have been studied in electron-positron plasma using reductive perturbation method by Chawla and Mishra.[27]

Two electron temperature distributions are very common in the laboratory[28–30] as well as in space plasmas.[31] Earlier missions GEOTAIL and POLAR (MacFadden *et al.*[32]) in the magnetosphere have also reported the coexistence of such electron populations. Ion–acoustic waves in two electron temperature plasma have been studied by Jones *et al.*[33] and Nishihara and Tajiri.[34]

Ion-acoustic soliton in a multi-species plasma consisting of positive ions, electrons, and negative ions has been studied by Das and Tagare,[35] Tagare,[36] and Mishra *et al.*[37] These studies show that owing to the presence of negative ions, the system also supports rarefactive solitons above the critical concentration. The KdV equation describing the solitary waves (SWs) in a multicomponent plasma is not valid for parametric regime corresponding to: (i) $A = 0$ and (ii) $A \sim 0$, where A is coefficient of the nonlinear term of the KdV equation. In the first case (i.e., $A = 0$) for given set of fixed parameter values if we increase β, then at a critical value of β, A becomes zero (i.e., $A = 0$) and in this case KdV equation is no longer valid to study soliton solution. To discuss the soliton solution at this critical value, one must consider higher-order nonlinearity to derive modified K-dV (m-KdV) equation. In the second case, near the critical value of β where the coefficient A is of the order of ε (very small), the

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KdV equation gives infinitely large amplitude structure, which is physically not valid. Therefore, in this parametric regime also, the KdV equation is physically not valid to study soliton solution. To study the finite amplitude ion-acoustic solitary waves beyond this KdV limit, one must consider the other type of nonlinear dynamic equation that can be valid for this case. In this case, Gardner equation is used to study soliton solution.

Recently, there has been a great deal of interest in the study of Gardner solitons (GS) in plasma. Many researchers started studying the Gardner (mixed modified KdV equation) or modified Gardner (MG) soliton structures and solutions on different plasma systems by Wazwaz,[38] Vassilev *et al.*,[39] Hossain *et al.*,[40] and Ghosh *et al.*[41] The dust-ion-acoustic Gardner solitons in dusty plasma have been investigated by Masud *et al.*[42] and Mamun *et al.*[43] The dust ion-acoustic (DIA) waves were first reported by Shukla and Silin.[44] Small amplitude DIA SWs in a dusty plasma by reductive perturbation method have been studied by Alinejad and Mamun.[45] Large amplitude DIA solitary waves in a multicomponent dusty plasma whose constituents are warm ions, two-temperature trapped electrons, and negatively charged dust grains have been investigated by Alinejad.[46]

The aim of the present paper is to study ion–acoustic solitons in a multicomponent electron-positron-ion plasma in the parametric regime where KdV equation is not valid to study soliton solution. It is found that there exist two parametric regimes in which KdV equation is not applicable to study soliton solution. Using reductive perturbation method, we have derived KdV, m-KdV, and Gardner equations[47] to study ion-acoustic solitons in the different parametric regimes. It is found that for a given set of parametric values, there exists a critical value of β, below which only rarefactive KdV solitons exist and above it compressive solitons exist. At the critical value of β, both compressive and rarefactive m-KdV solitons co-exist. It is also investigated that the near the critical value of $\beta, A \sim 0(\varepsilon)$, below the critical value of β rarefactive Gardner solitons exist, and above it compressive Gardner solitons are found.

The manuscript is organized as follows: The basic set of equations are given in Sec. II. The KdV, m-KdV, and standard Gardner (sG) equations are derived in Secs. III, IV, and V, respectively. The solitary wave solution of sG equation is given in Sec. VI. The brief discussion is given in Sec. VII and conclusions have been summarized in Sec. VIII.

II. BASIC EQUATIONS

We consider a plasma consisting of warm adiabatic ions, isothermal positrons, and two-temperature electrons. The nonlinear behavior of ion-acoustic waves may be described by the following set of normalized fluid equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x} - \frac{5}{3}\delta {n_i}^{-1/3}\frac{\partial n_i}{\partial x}, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} = -\rho = n_h + n_c - (1-\alpha)n_i - \alpha n_p, \tag{3}$$

$$n_c = \mu \exp\left[\frac{1}{\mu + \nu\beta}\phi\right], \tag{4}$$

$$n_h = \nu \exp\left[\frac{\beta}{\mu + \nu\beta}\phi\right], \tag{5}$$

$$n_p = \exp\{-\gamma\phi\}, \tag{6}$$

$$n_e = n_h + n_c = 1 + \phi + \frac{(\mu + \nu\beta^2)}{2(\mu+\nu\beta)^2}\phi^2 + \frac{(\mu+\nu\beta^3)}{6(\mu+\nu\beta)^3} + ... \tag{7}$$

Here

$$\beta = \frac{T_c}{T_h}; \quad \alpha = \frac{n_{po}}{n_{eo}}; \quad \mu = \frac{n_{co}}{n_{eo}}; \quad \nu = \frac{n_{ho}}{n_{eo}}; \quad \delta = \frac{T_i}{T_{eff}};$$
$$\gamma = \frac{T_{eff}}{T_p}; \quad \text{and} \quad T_{eff} = \frac{T_h T_c}{(\mu T_h + \nu T_c)}.$$

In the above equations, n_i, u_i are the density and fluid velocity of the ion-species, respectively. n_{co}, n_{ho}, and n_{io}, are the equilibrium densities of two electron components and of the ion component, respectively, ϕ is the electrostatic potential, α is the equilibrium density ratio of positron to electron species, μ is the equilibrium density ratio of cold electron to electron species, and δ is the ratio between temperature of ion and electron effective temperature.

In Eqs. (4) and (5), the electron density distributions are considered to be Maxwell-Boltzmann. In the above equations, n_i is the ion density normalized by its equilibrium value n_{i0}; u_i is the ion fluid speed normalized by ion acoustic speed $C_i = (T_{eff}/m_i)^{1/2}$; potential (ϕ), time (t), and space coordinate (x) have been normalized with respect to thermal potential $\frac{T_{eff}}{e}$, inverse of the ion-plasma frequency in the mixture ω_{pi}^{-1}, and Debye length λ_D, respectively; whereas electron densities n_h and n_c are normalized by n_{eo}. In the mixture, the ion-acoustic speed C_i, the ion plasma frequency ω_{pi}, and the Debye length λ_D are given by

$$C_i = \left[\frac{T_{eff}}{m_i}\right]^{\frac{1}{2}}, \quad \omega_{pi}^{-1} = \left[\frac{4\pi n_{eo} e^2}{m_i}\right]^{-\frac{1}{2}}, \quad \text{and}$$
$$\lambda_D = \left[\frac{T_{eff}}{4\pi n_{eo} e^2}\right]^{\frac{1}{2}}.$$

The charge-neutrality condition is expressed as $\mu + \nu = \alpha + \frac{n_{io}}{n_{eo}}$.

III. DERIVATION OF K-dV EQUATION

We first derive the K-dV equation from the basic set of equations, viz., Eqs. (1)–(7). For KdV equation, we introduce the following stretching of coordinates (ξ) and (τ) as:

$$\xi = \varepsilon^{1/2}(x - V_p t) \tag{8}$$

and

$$\tau = \varepsilon^{3/2} t, \tag{9}$$

where ε is a small parameter and V_p is the phase velocity of the wave to be determined later. Now we expand the field quantities in Eqs. (1)–(7) around the unperturbed uniform state in powers of ε in the following form:

$$\begin{bmatrix} n_i \\ u_i \\ \phi \\ \rho \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \varepsilon \begin{bmatrix} n_1^{(1)} \\ u_i^{(1)} \\ \phi^{(1)} \\ \rho^{(1)} \end{bmatrix} + \varepsilon^2 \begin{bmatrix} n_1^{(2)} \\ u_i^{(2)} \\ \phi^{(2)} \\ \rho^{(2)} \end{bmatrix} + \varepsilon^3 \begin{bmatrix} n_1^{(3)} \\ u_i^{(3)} \\ \phi^{(3)} \\ \rho^{(3)} \end{bmatrix} + \dots \tag{10}$$

On substituting the expansion (10) into Eqs. (1)–(7), using Eqs. (8) and (9), and equating terms with the same powers of ε, we obtain a set of equations for each order in ε. The set of Eqs. (1)–(3) at the lowest order, i.e., o (ε), give

$$u_i^{(1)} = \frac{3V_p}{\left(3V_p{}^2 - 5\delta\right)}\psi, \tag{11}$$

$$n_i^{(1)} = \frac{3}{\left(3V_p{}^2 - 5\delta\right)}\psi, \tag{12}$$

$$V_p = \sqrt{\frac{(1-\alpha)}{(1+\alpha\gamma)} + \frac{5\delta}{3}}, \tag{13}$$

where $\psi = \phi^{(1)}$. Eq. (13) gives the dispersion relation for ion-acoustic solitary waves. For next higher order, i.e., o (ε^2), we obtain a set of equations, which, after using Eqs. (11)–(13), can be simplified as

$$\frac{\partial n_i{}^{(1)}}{\partial \tau} - V_p\frac{\partial n_i{}^{(2)}}{\partial \xi} + \frac{\partial u_i{}^{(2)}}{\partial \xi} + \frac{\partial\left(n_i{}^{(1)}u_i{}^{(1)}\right)}{\partial \xi} = 0, \tag{14}$$

$$\frac{\partial u_i{}^{(1)}}{\partial \tau} - V_p\frac{\partial u_i{}^{(2)}}{\partial \xi} + u_i{}^{(1)}\frac{\partial u_i{}^{(1)}}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \xi} - \frac{5}{9}\delta n_i{}^{(1)}\frac{\partial n_i{}^{(1)}}{\partial \xi} + \frac{5}{3}\delta\frac{\partial n_i{}^{(2)}}{\partial \xi} = 0, \tag{15}$$

$$\frac{\partial^2\psi}{\partial \xi^2} - \alpha\gamma\phi^{(2)} + \frac{1}{2}\alpha\gamma^2\psi^2 - \phi^{(2)} - \frac{1}{2}\frac{\left(\mu+\nu\beta^2\right)}{\left(\mu+\nu\beta\right)^2}\psi^2 + (1-\alpha)n_i{}^{(2)} = 0. \tag{16}$$

Now combining Eqs. (14)–(16), we get the following KdV equation:

$$\frac{\partial\psi}{\partial\tau} + A\psi\frac{\partial\psi}{\partial\xi} + B\frac{\partial^3\psi}{\partial\xi^3} = 0, \tag{17}$$

where

$$A = \left[\frac{\left(3V_p{}^2 - 5\delta\right)^2}{18(1-\alpha)V_p}\right]\left[\frac{81(1-\alpha)V_p{}^2}{\left(3V_p{}^2 - 5\delta\right)^3} + \alpha\gamma^2 - \frac{\left(\mu+\nu\beta^2\right)}{\left(\mu+\nu\beta\right)^2} - \frac{15(1-\alpha)\delta}{\left(3V_p{}^2 - 5\delta\right)^3}\right] \tag{18}$$

$$\text{and } B = \frac{\left(3V_p{}^2 - 5\delta\right)^2}{18(1-\alpha)V_p}. \tag{19}$$

The soliton solution of KdV equation is given by

$$\psi = \psi_m \sec h^2[\zeta/\delta_1], \tag{20}$$

where $\zeta = \xi - U_o\tau$. The amplitude ψ_m and width δ_1 are given by $\psi_m = 3U_0/A$ and $\delta_1 = \sqrt{4B/U_0}$.

For the typical plasma parameters $\mu = 0.15$, $\delta = 0.01$, $\gamma = 0.1$, $\alpha = 0.1$, and $U_0 = 0.01$, Eq. (20) indicates that (i) hump shape (positive potential) soliton exists if $\beta \gg 0.07215$, (ii) small amplitude solitary waves with $\psi < 0$ dip shaped (negative potential) soliton exists if $\beta \ll 0.07215$, and (iii) no solitons can exists at $\beta = 0.07215$. It is found that KdV soliton exists with positive potential far above the critical value (shown in Fig. 2) and negative potential far below the critical value (shown in Fig. 3).

IV. DERIVATION OF THE m-KdV EQUATION

The nonlinear coefficient of the KdV equation (i.e., A) vanishes at the critical value of β (i.e., β_c). The vanishing of the nonlinear coefficient in the KdV equation (i.e., $A = 0$ in (18)) determines the critical value of β

$$\beta = \beta_c = \frac{-L\mu\nu \pm \sqrt{L\mu\nu^2 + L\mu^2\nu - \mu\nu}}{\nu(L\nu - 1)}, \tag{21}$$

$$\text{where } L = \frac{3(1+\alpha\gamma)^2}{(1-\alpha)} + 5\delta\frac{(1+\alpha\gamma)^3}{(1-\alpha)^2} + \alpha\gamma^2 - \frac{5}{9}\delta\frac{(1+\alpha\gamma)^3}{(1-\alpha)^2}. \tag{22}$$

To discuss the soliton solution at this critical value β_c, one has to consider higher order nonlinearity. Accordingly, we use different stretching of co-ordinates and time to derive the appropriate equation. To derive the m-KdV equation, we adopt the same expansion (Eq. (10)) of all the dependent quantities and we introduce the following stretching of coordinates (ξ) and (τ) as:

$$\xi = \varepsilon(x - V_p t) \tag{23}$$

and

$$\tau = \varepsilon^3 t. \tag{24}$$

By using Eqs. (23) and (24) in Eqs. (1)–(4) and (10), we will get the same values of $n_i{}^{(1)}, u_i{}^{(1)}$, and V_p as in the case of KdV equation. For the next higher order, i.e., o(ε^2), we get a set of equations, after using the values of $n_i{}^{(1)}, u_i{}^{(1)}$, and V_p can be simplified as

$$u_i{}^{(2)} = \frac{81V_p{}^3\psi^2}{2(3Vp^2 - 5\delta)^3} + \frac{3\phi^{(2)}V_p}{(3Vp^2 - 5\delta)} - \frac{15\delta\psi^2 V_p}{2(3Vp^2 - 5\delta)^3} - \frac{9V_p\psi^2}{(3Vp^2 - 5\delta)^2}, \tag{25a}$$

$$n_i{}^{(2)} = \frac{81V_p{}^2\psi^2}{2(3Vp^2 - 5\delta)^3} + \frac{3\phi^{(2)}}{(3Vp^2 - 5\delta)} - \frac{15\delta\psi^2}{2(3Vp^2 - 5\delta)^3}, \tag{25b}$$

$$\rho^{(2)} = -\frac{1}{2}A\psi^2 = 0. \tag{26}$$

It should be noted that the above equation is satisfied identically owing to the criticality condition, i.e., $A = 0$.

To the next higher order of ε, we get a set of equations

$$\frac{\partial n_i{}^{(1)}}{\partial \tau} - V_p \frac{\partial n_i{}^{(3)}}{\partial \xi} + \frac{\partial}{\partial \xi}\left(u_i{}^{(2)} n_i{}^{(1)}\right) + \frac{\partial}{\partial \xi}\left(n_i{}^{(2)} u_i{}^{(1)}\right) + \frac{\partial u_i{}^{(3)}}{\partial \xi} = 0, \tag{27}$$

$$\frac{\partial u_i{}^{(1)}}{\partial \tau} - V_p \frac{\partial u_i{}^{(3)}}{\partial \xi} + u_i{}^{(1)} \frac{\partial u_i{}^{(2)}}{\partial \xi} + u_i{}^{(2)} \frac{\partial u_i{}^{(1)}}{\partial \xi} + \frac{\partial \phi^{(3)}}{\partial \xi} - \frac{5}{9}\delta n_i{}^{(2)} \frac{\partial n_i{}^{(1)}}{\partial \xi} + \frac{5}{3}\delta \frac{\partial n_i{}^{(3)}}{\partial \xi} - \frac{5}{9}\delta n_i{}^{(1)} \frac{\partial n_i{}^{(2)}}{\partial \xi} + \frac{10}{27}\delta n_i{}^{(1)2} \frac{\partial n_i{}^{(1)}}{\partial \xi} = 0, \tag{28}$$

$$\frac{\partial^2 \psi}{\partial \xi^2} + (1-\alpha) n_i{}^{(3)} - \alpha\gamma\phi^{(3)} - \frac{1}{6}\alpha\gamma^3\psi^3 + \alpha\gamma^2\psi\phi^{(2)} - \phi^{(3)} - \frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}\psi\phi^{(2)} - \frac{1(\mu+\nu\beta^3)}{6(\mu+\nu\beta)^3}\psi^3 = 0. \tag{29}$$

Differentiating Eq. (29) with respect to ξ and using Eqs. (27) and (28), we get the following m-KdV equation:

$$\frac{\partial \psi}{\partial \tau} + BC\psi^2 \frac{\partial \psi}{\partial \xi} + B\frac{\partial^3 \psi}{\partial \xi^3} = 0, \tag{30}$$

where

$$C = \left[\begin{array}{l} \dfrac{6561(1-\alpha)V_p{}^4}{2\left(3V_p{}^2-5\delta\right)^5} - \dfrac{2565(1-\alpha)\delta V_p{}^2}{2\left(3V_p{}^2-5\delta\right)^5} - \dfrac{486(1-\alpha)V_p{}^2}{\left(3V_p{}^2-5\delta\right)^4} + \dfrac{225\delta^2(1-\alpha)}{2\left(3V_p{}^2-5\delta\right)^5} \\ + \dfrac{30(1-\alpha)\delta}{\left(3V_p{}^2-5\delta\right)^4} - \dfrac{\alpha\gamma 3}{2} - \dfrac{1}{2}\dfrac{\left(\mu+\nu\beta^3\right)}{(\mu+\nu\beta)^3} \end{array}\right]. \tag{31}$$

It may be noted that the coefficient B is the same as in the case of the KdV equation.The soliton solution of m-KdV equation is given by

$$\psi = \psi_m \sec h[\zeta/\delta_2], \tag{32}$$

where the amplitude ψ_m and width δ_2 are given by

$$\psi_m = \pm\sqrt{6U_0/BC}, \tag{33}$$

$$\delta_2 = \sqrt{\frac{B}{U_0}}. \tag{34}$$

The two values of the amplitude ψ_m show that for the modified KdV equation (30), both compressive and rarefactive solitons exist in the plasma at the critical value of β_c. In the case of negative ion plasma, this has been confirmed experimentally by Nakamura and Tsukabayashi.[48] The m-KdV solitons are valid at critical value of β_c whose amplitude and width are given by Eqs. (33) and (34), respectively. Here m-KdV solitons have finite value at the critical value ($\beta = 0.07214899$). Here we found m-KdV hump shape soliton at the critical value with positive amplitude (i.e., $\psi_m = +\sqrt{6U_0/BC}$) shown in Fig. 4 and m-KdV negative dip type soliton with negative potential (i.e., $\psi_m = -\sqrt{6U_0/BC}$) shown in Fig. 5.

V. DERIVATION OF GARDNER (sG) EQUATION

Near the critical value of β_c, for a parametric regime corresponding to the coefficient of nonlinear term of KdV equation, i.e., $A \sim 0(\varepsilon)$, if we use the KdV equation to study soliton solution, then it is found that it gives rise to infinitely large amplitude soliton. This breaks down the validity of the reductive perturbation method. Therefore, to study the finite amplitude solitons beyond this KdV limit, one must consider the other types of nonlinear dynamical equation, which can be valid for $\beta \sim \beta$ c. We use Gardner equation which is valid for $A \sim 0(\varepsilon)$, where ε is a smallness parameter.

We have the small amplitude solitary waves with a positive potential for $\beta > \beta_c$. So, for β around its critical value (β_c), $A = A_0$ can be written as

$$A_0 \cong s\left(\frac{\partial A}{\partial \beta}\right)|\beta - \beta_c| = c_1 s\varepsilon, \tag{35}$$

where c_1 is a constant depending the parameters β, μ, and ν. $|\beta - \beta_c|$ is the small and dimensionless parameter and can be given as expansion parameter ε, i.e., $|\beta - \beta_c| \cong \varepsilon$ and s = 1 for $\beta > \beta_c$ and s = − 1 for $\beta < \beta_c$. So, $\rho^{(2)}$ can be expressed as

$$\varepsilon^2\rho^{(2)} \cong -\varepsilon^3 \frac{1}{2} c_1 s\psi^2 \tag{36}$$

which, therefore, must be included in the third order Poisson's equation. To the next higher order in ε, we get the following equation:

$$\frac{\partial^2 \psi}{\partial \xi^2} + \frac{1}{2}c_1 s\psi^2 + (1-\alpha) n_i{}^{(3)} - \alpha\gamma\phi^{(3)} - \frac{1}{6}\alpha\gamma^3\phi^{(3)} - \phi^{(3)} - \frac{(\mu+\nu\beta^2)}{(\mu+\nu\beta)^2}\psi\phi^{(2)} - \frac{1(\mu+\nu\beta^3)}{6(\mu+\nu\beta)^3}\psi^3 = 0. \tag{37}$$

After simplification, we can write Eq. (37)

$$\frac{\partial \psi}{\partial \tau} + sc_1 B\psi \frac{\partial \psi}{\partial \xi} + BC\psi^2 \frac{\partial \psi}{\partial \xi} + B\frac{\partial^3 \psi}{\partial \xi^3} = 0. \tag{38}$$

Equation (38) is a sG equation. It contains both ψ-term of KdV and ψ^2 term of m-KdV equation. It may be noted that when $A = c_I s = 0$;, Eq. (38) reduces to mKdV equation (30).

This Gardner equation is valid for $\beta \sim \beta_c$. It may also be noted that the coefficients B and C are the same as in the cases of KdV and m-KdV equations.

VI. SOLITON SOLUTION OF sG EQUATION

Here we first analyze stationary GS solution of Gardner equation (38). For that, we first consider a transformation $\zeta = \xi - U_0\tau$, which allows us to write Eq. (38) under steady state condition, as

$$\frac{1}{2}\left(\frac{\partial\psi}{\partial\zeta}\right)^2 + V(\psi) = 0, \tag{39}$$

where the pseudo-potential V (ψ) is

$$V(\psi) = -\frac{U_0}{2B}\psi^2 + \frac{c_1 s}{6}\psi^3 + \frac{C}{12}\psi^4 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{40}$$

We note here that U_0 and α_2 are always positive. It is obvious from (39) that

$$V(\psi)|_{\psi=0} = \left.\frac{dV(\psi)}{d\psi}\right|_{\psi=0} = 0, \tag{41}$$

$$\left.\frac{d^2V(\psi)}{d\psi}\right|_{\psi=0} < 0. \tag{42}$$

Conditions (40) and (41) imply that soliton solutions of (39) exist if

$$V(\psi)|_{\psi=\psi_m} = 0. \tag{43}$$

The latter can be solved as

$$U_0 = \frac{c_1 sB}{3}\psi_{m_{1,2}} + \frac{BC}{6}\psi^2{}_{m_{1,2}}, \tag{44}$$

$$\psi_{m_{1,2}} = \psi_m\left[1 \pm \sqrt{1 + \frac{U_0}{V_0}}\right], \tag{45}$$

where $\psi_m = -c_1 s/C$ and $V_0 = c_1{}^2 s^2 B/6C$. Now using (40) and (45) in (39), we have

$$\left(\frac{d\psi}{d\xi}\right)^2 + p\psi^2(\psi - \psi_{m_1})(\psi - \psi_{m_2}) = 0, \tag{46}$$

where $p = \alpha_1/6$. The soliton solution of the sG equation can be given by

$$\psi = \left[\frac{1}{\psi_{m_2}} - \left(\frac{1}{\psi_{m_2}} - \frac{1}{\psi_{m1}}\right)\cosh^2\left\{\frac{\zeta}{W}\right\}\right], \tag{47}$$

$\psi_{m_{1,2}}$ are given by Eq. (45), and soliton width W is given by

$$W = \frac{2}{\sqrt{-p\psi_{m_1}\psi_{m_2}}}, \tag{48}$$

where $p = C/6$.

Eq. (47) represents the soliton solution of Gardner (Eq. (38)). Here we can see that the solitary structures represented by the soliton solution of Gardner, Eq. (47), are different from that represented by soliton solution of the KdV equation, Eq. (20). It may also be noted that for A $\sim$ 0 (i.e., $\beta \approx 0.07215$), KdV theory is not valid. The results, got from Gardner solution, are shown in Figures 6–9. From these figures, it can be noted that both positive and negative Gardner solitons exist around the critical value $\beta_c = 0.07215$.

We can see from Fig. 6 the compressive solitary waves above the critical value (say 0.0722) and rarefactive solitary waves below the critical value (say 0.0720). We have also found that the amplitude of compressive (rarefactive) Gardner soliton decreases (increases) with the increase of β (shown in Fig. 9). Fig. 9 also shows that when the temperature ratio between population of two electrons is increased, the amplitude of the soliton also decreases.

VII. DISCUSSION

The notable information from our current numerical study can be summarized as follows: In Fig. 1, we have plotted the variation of the β with respect to μ, for the different values of equilibrium density ratio of positron to electron species (α) = 0.09 (dotted green line), 0.10 (solid blue line), and 0.11 (solid red line). We also note from Fig. 1 as the equilibrium density ratio of positron to electron species (α) increases the critical value of β decreases. In Fig. 2, we have shown the profile of positive KdV soliton (ψ) with cold to hot electron temperature ratio (β), for the equilibrium density ratio of positron to electron species (α) = 0.1, the equilibrium density ratio of cold electron to ion species (μ) = 0.15, the ratio between temperature of ion and electron effective temperature (δ) = 0.01, and U_0 = 0.01. We also note from Fig. 2 that K-dV soliton exists with positive potential above the critical value (β_c). In Fig. 3, we have shown the profile of amplitude of negative KdV soliton (ψ) with cold to hot electron temperature ratio (β), for the equilibrium density ratio of positron to electron species (α) = 0.1, the equilibrium density ratio of cold electron to ion species (μ) = 0.15, the ratio between temperature of ion and electron effective

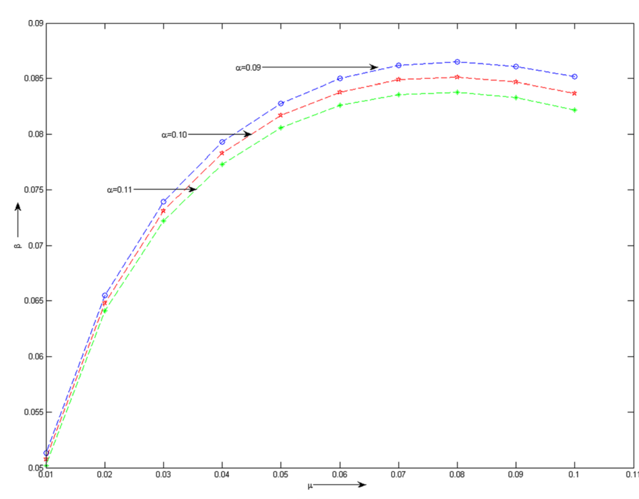

FIG. 1. The plot between the β (ratio of cold to hot temperature) and μ (equilibrium density ratio of cold to ion species) at the different values of α(the equilibrium density ratio of positron to ion species) = 0.09, 0.10, and 0.11.

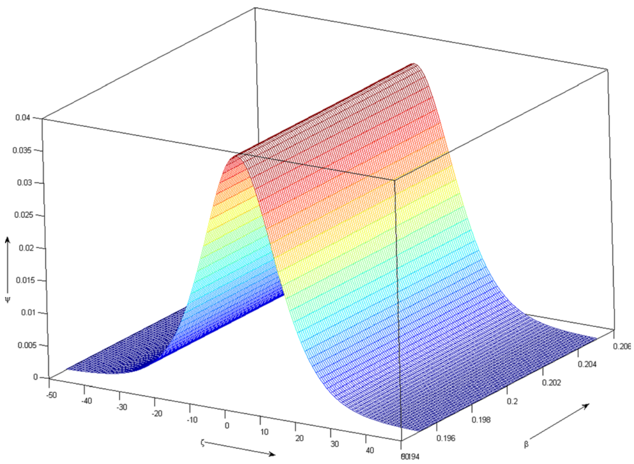

FIG. 2. Variation of the positive KdV soliton potential (ψ) with β for $\mu=0.15$, δ (the ratio between temperature of ion and electron effective temperature) $=0.01$, γ (the ratio between effective temperature of electron and positron) $=0.1$, α (the equilibrium density ratio of positron to ion species) $=0.1$, and $U_0=0.01$.

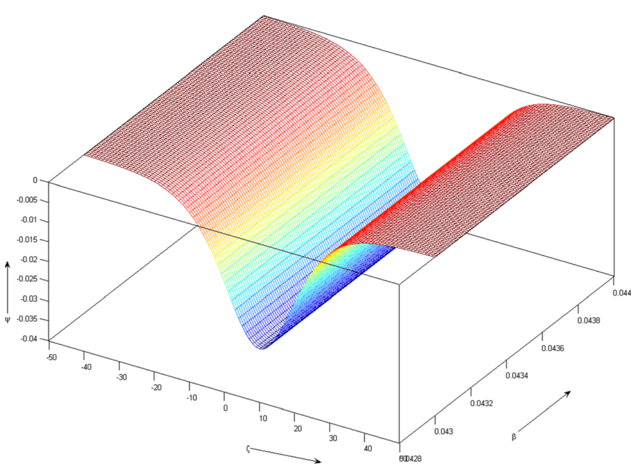

FIG. 3. Variation of the negative KdV soliton potential (ψ) with β for $\mu=0.15$, δ (the ratio between temperature of ion and electron effective temperature) $=0.01$, γ (the ratio between effective temperature of electron and positron) $=0.1$, α (the equilibrium density ratio of positron to ion species) $=0.1$, and $U_0=0.01$.

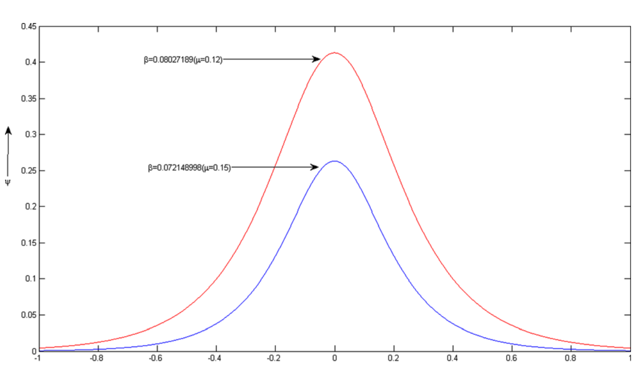

FIG. 4. Plot of compressive m-KdV soliton potential (ψ) at two different values of $\beta=0.07215$ ($\mu=0.15$) and 0.080272 ($\mu=0.12$), on positive amplitude ($\psi_m=\sqrt{6U_0/B*C}$), for $\delta=0.01$, $\gamma=0.1$, $\alpha=0.1$, and $U_0=0.01$.

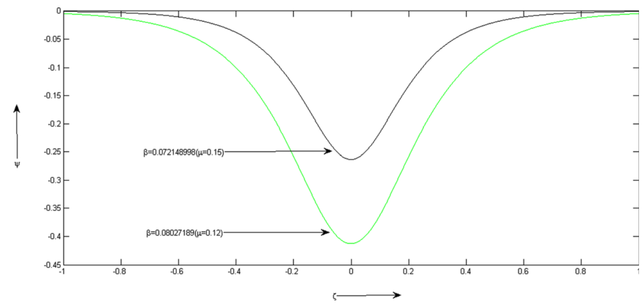

FIG. 5. Plot of rarefactive m-KdV soliton potential (ψ) at two different values of $\beta=0.07215$ ($\mu=0.15$) and 0.080272 ($\mu=0.12$), on negative amplitude ($\psi_m=-\sqrt{6U_0/B*C}$), for $\delta=0.01$, $\gamma=0.1$, $\alpha=0.1$, and $U_0=0.01$.

temperature (δ) $=0.01$, and $U_0=0.01$. We note from Fig. 3 that K-dV soliton exists with negative potential below the critical value (β_c). In Fig. 4, we have plotted the variation of positive potential m-KdV soliton for two critical values of β_c: (i) $\beta=0.07215$ ($\mu=0.15$) and (ii) $\beta=0.080272$ ($\mu=0.12$), for fixed values of $\delta=0.01$, $\gamma=0.1$, $\alpha=0.1$, and $U_0=0.01$. Here mKdV solitons have finite value at the critical value so we got compressive mKdV soliton at critical concentration.

In Fig. 5, we have plotted the variation of negative potential m-KdV soliton for two critical values of β_c: (i) $\beta=0.07215$ ($\mu=0.15$) and (ii) $\beta=0.080272$ ($\mu=0.15$), for fixed values of $\delta=0.01$, $\gamma=0.1$, $\alpha=0.1$, and $U_0=0.01$. We also note from Fig. 5 that at critical value with negative amplitude, rarefactive m-KdV soliton exists. In Fig. 6, we have plotted the variation of positive potential Gardner soliton (ψ) with (β), for $\mu=0.15$, $\delta=0.01$, $\gamma=0.1$, and $U_0=0.01$. We also note from Fig. 6 that the hump shape (positive potential) solitary waves exist just above the critical value 0.07215, say 0.0722. In Fig. 7, we have plotted the variation of negative potential Gardner soliton with cold to hot electron temperature ratio (β), for $\mu=0.15$, $\delta=0.01$, $\gamma=0.1$, and $U_0=0.01$. We also note from Fig. 7 that the dip type (negative potential) solitary waves exist just below the critical value 0.07215, say 0.0720. In Fig. 8, we have plotted the variation of positive potential Gardner soliton with μ, for $\delta=0.01$, $\gamma=0.1$, $\beta=0.07215$, and $U_0=0.01$. We found the

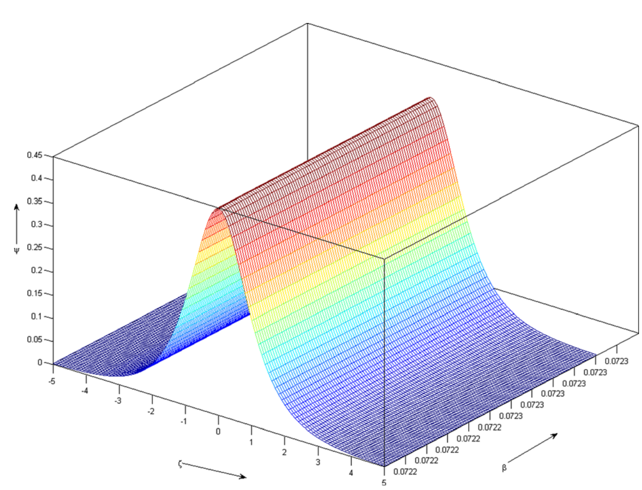

FIG. 6. Plot of positive Gardner soliton potential (ψ) with (β), for $\mu=0.15$, $\delta=0.01$, $\gamma=0.1$, $\alpha=0.1$, and $U_0=0.01$.

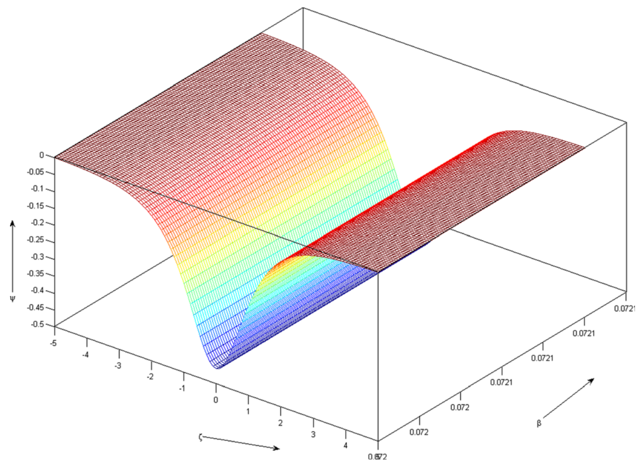

FIG. 7. Plot of negative Gardner soliton potential (ψ) with (β), for $\mu = 0.15$, $\delta = 0.01$, $\gamma = 0.1$, $\alpha = 0.1$, and $U_0 = 0.01$.

compressive (positive potential) solitary waves for $\mu = 0.15$ to 0.16. In Fig. 9, we have plotted the variation of positive potential Gardner soliton for $\mu = 0.15$, $\nu = 0.85$, $\delta = 0.01$, $\gamma = 0.1$, and $U_0 = 0.01$ at different values of β. We also note from this figure that when the temperature ratio between population of two electrons is increased, the amplitude of the soliton decreases. In Fig. 10, we have plotted the variation of positive potential Gardner soliton and KdV soliton for comparison. This figure clearly shows that for the set of parameters having $A \sim 0$, the amplitude of KdV equation becomes greater than one. Hence, KdV equation cannot be used for these set of parameter values.

We have derived the Gardner equation (which is valid beyond the KdV limit) in an unmagnetized plasma system consisting of an adiabatically warm ion with two electron temperature distribution in the presence of positrons. The reductive perturbation method has been used to derive the Gardner equation. We have shown the existence of both hump and dip type ion-acoustic Gardner solitons, which exist beyond the KdV limit, i.e., for $\beta = 0.07215$. The ion-acoustic Gardner solitons are completely different from KdV solitons. It is also investigated that the characteristics of finite amplitude Gardner solitons (polarity, amplitude, width,

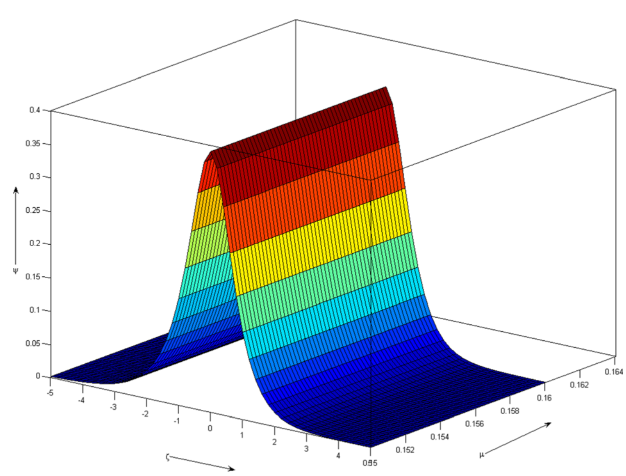

FIG. 8. Plot of positive Gardner soliton potential (ψ) with (β), for $\mu = 0.15$–0.16, $\delta = 0.01$, $\gamma = 0.1$, $\alpha = 0.1$, and $U_0 = 0.01$.

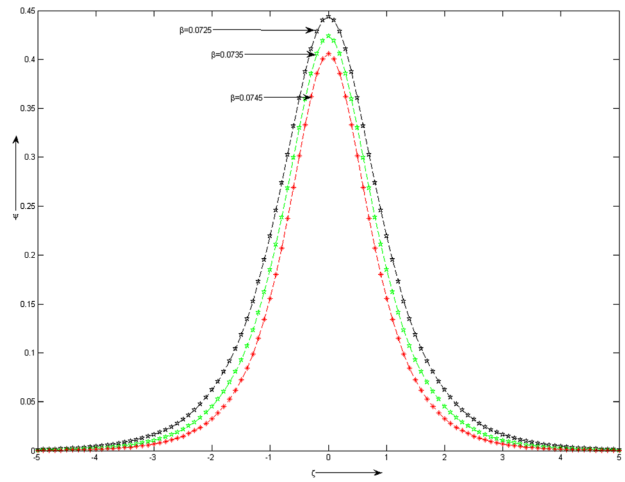

FIG. 9. Plot to show the effect of two electron temperatures on the amplitude of the ion acoustic positive Gardner solitons for different $\beta = 0.0725$ (upper), 0.0735 (middle), and 0.0745 (lower).

etc.) strongly depend on the temperature ratio of two electron species and density ratio. The implication of our results may be useful in understanding the electrostatic perturbations observed in laboratory and space plasmas.

VIII. CONCLUSIONS

Our main conclusion in electron-positron ion plasma with two electron temperature distribution is as follows:

(i) It is found that in the case of multicomponent plasma for the given set of parameter values (μ, ν, δ, α, and γ), there exists a critical value of β_C below which rarefactive ion-acoustic KdV solitons exist and above it compressive solitons exist. It is also found that KdV solitons exist only far above or far below the critical value (i.e., when $\beta \ll \beta_C$ or $\beta \gg \beta_C$).

(ii) It is also found that in the case of multicomponent plasma, the KdV equation does not support solitary wave for parametric regime corresponding to $A = 0$ and $A \sim 0(\varepsilon)$ where A is the coefficient of nonlinear term of the KdV equation.

(iii) For parametric regime corresponding to $A = 0$, it is found that at the critical value of β, both compressive and rarefactive m-KdV solitons co-exist.

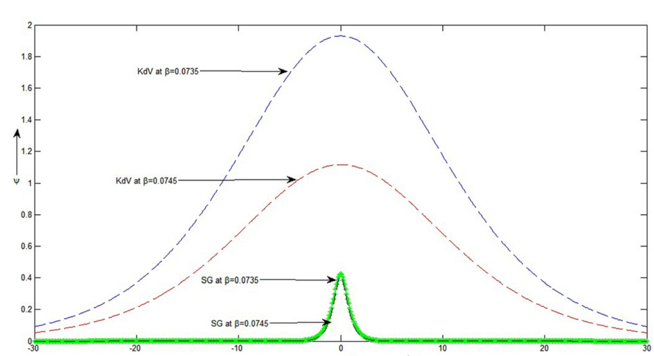

FIG. 10. Plot to show the variation of positive potential GS and KdV soliton for different $\beta = 0.0735$ and $= 0.0745$.

(iv) It is also found that near the critical value of β_C for parametric regime corresponding to $A \sim 0\ (\varepsilon)$, the system supports Gardner solitons. In this case, it is also found that for $\beta < \beta_C$, rarefactive Gardner soliton exists, whereas for $\beta > \beta_C$, compressive Gardner soliton exists.

(v) It is also investigated that the amplitude of compressive (rarefactive) Gardner soliton decreases (increases) with β.

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