Electrostatic shock wave in dusty plasmas

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The present investigation reports the existence of a one-dimensional (1-D) shock wave profile for a weakly nonlinear modified acoustic-like/dust-acoustic waves. Here, unlike in the conventional plasma systems, the dissipative effect to balance the nonlinear steepening arises self-consistently due to dust charge fluctuation dynamics in response to the collective plasma oscillations. The possibility of other localized coherent structures like dissipative/dispersive solitons in higher-order solutions has been highlighted. Its implications in laboratory and space plasmas are discussed. © *1995 American Institute of Physics.*

I. INTRODUCTION

Recently, dusty plasmas have become a subject of much current interest[1-12] all over the world due to their natural occurrence in various situations of practical importance. The presence of the highly charged ($Q_D \sim 10^3 e - 10^4 e$) dust grains with variable sizes (10 nm$-$100 μm) and masses in two component plasmas brings about a new physical effect, such as dust charge variation, which leads to interesting consequences on low-frequency wave propagation and instability phenomena. Dust particles immersed in a plasma with collective perturbations can exhibit self-consistent charge fluctuations in response to the oscillations in the plasma currents flowing into them. The dust electric charge thereby becomes a time-dependent dynamical variable which is coupled self-consistently to other dynamical variables like density, potential, and currents, etc. Without this novel effect, the dusty plasma is just like a conventional multicomponent plasma system. The main difference between dust particles and heavy ions is that the charges on dust particles are not fixed and the collective waves are waves in dust charges.[13] The detailed discussions about essential distinguishing features of the dusty plasmas from multicomponent plasmas are included in other publication by Tsytovich.[14] In conventional dusty plasmas, Rao *et al.*[15] have reported the existence of a dust-acoustic wave with the inertial role played by the heavy dust grains and that of the restoring force by the combined effort of the Boltzmann electrons and ions. However, under the approximations as described later, the dust-acoustic mode (DAM) and acoustic-like mode (ALM) as discussed by Dwivedi *et al.*[16,17] represent the same mode of perturbations with only a difference of the nomenclature. In essence, the DAM/ALM is the same collective mode of a three component plasma system which is likely to exist as a normal electrostatic mode in dusty plasmas.

Very recently Varma *et al.*,[18] Melandso *et al.*,[19] Jana *et al.*,[20] Bhatt and Pandey,[21] and Pandey *et al.*[22] have considered the dynamical behavior of the nonconventional role of the electron and ion fluctuations through the dust charge dynamics and analyzed the various low-frequency electrostatic waves and their stabilities under different possible situations. For example, Varma *et al.*[18] conclude that the dust charge fluctuation causes the damping of the high-frequency response of the plasma. Jana *et al.*[20] and Melandso *et al.*[19] study the damping of the dust-acoustic mode. Bhatt and Pandey[21] have considered the sink terms in electron and ion continuity equations and thus they have developed a set of self-consistent dynamical equations to reinvestigate the dust-acoustic mode and streaming instability in nonconventional dusty plasmas. The dissipative response of the dust to the plasma perturbations arises due to nonsteady dust charge fluctuation dynamics. Physically, the dust charge fluctuation due to collective plasma perturbations modifies the dust floating potential which in turn self-consistently opposes the buildup of the plasma currents at the surface of the dust grains and consequently causes the nonconventional collisionless damping of the plasma waves.

Acoustic-like mode (ALM) forms a natural mode of the low-frequency and short wavelength ($\lambda_{De} \gg \lambda \gg \lambda_{Dih}$) plasma fluctuations with finite but small Landau damping[16,17] in a multicomponent plasma system wherein the electrons are assumed to form a dynamic equilibrium background without participating in the perturbational dynamics. One of the ion components (hot ions) is Boltzmann distributed whereas the other one (cold ions) follows a full inertial dynamical response. The validity of these approximations[16] restricts the equilibrium plasma parameter values for the normal mode behavior of the ALM in multicomponent plasma systems:

$$\text{(a)} \quad \frac{T_e}{T_i} \gg 1, \quad \frac{T_e}{T_{ih}} \gg 1, \quad \frac{T_{ih}}{T_{ic}} \gg \frac{n_{ih0}}{n_{ic0}} \gg 1$$

$$\text{for} \quad \frac{Z_{ih}}{Z_{ic}}, \quad \frac{M_{ih}}{M_{ic}} \gg 1,$$

where T_μ, n_μ, Z_μ, and M_μ denote the temperature, density, charge multiplicity, and mass, respectively, of the μth species

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of the plasma. In general the normal mode behavior of the ALM in the dusty plasma model with multiply ionized ions requires the following conditions:

$$\text{(b)} \quad \frac{\epsilon_{nD}\epsilon_D^2}{\alpha_m} \gg 1$$

to be satisfied, where $\epsilon_{nD}=n_{i0}/n_{D0}$, $\epsilon_D=eZ_i/Q_{D0}$, $\alpha_m=m_i/m_D$. It is remarkable to note that the inequality defined in (b) provides a more general condition for the existence of the normal mode of ALM in dusty plasmas. In fact, this condition could have been derived even in an earlier paper.[16] However, the practical feasibility of the plasma system with wide range variability of mass ratio α_m and charge ratio ϵ_D was doubtful and hence the analysis was restricted to these ratios ≈ 1. Later, in another paper,[17] this point was addressed. The essential point to bring to the notice of the readers is that within the validity of the restriction (b), the dispersion relation of the low-frequency electrostatic modes in dusty plasma is that of the acoustic-like mode derived in a previous publication[16] and it is the same as reported later by Rao *et al.*[15]

This type of three component theoretical plasma model is very likely to be realized in dusty plasmas. Here the charged dust grains and the plasma ions represent the cold and hot ion components of the proposed plasma model.[16,17] The charged dust grains being heavier than the other plasma components, maintain the nonisothermal behavior of the proposed plasma system. The stationary state response of the electrons limits the size and charge of the dust grains. For example, based on the simple charging model[6] of the dust grains due to differential thermal motions of the electrons and ions in hydrogen plasma, a rough estimation shows that the above conditions may be produced by dispersing the moderately charged dust grains ($Q_{D0}\geqslant 10e$) of the size $a\geqslant 10$ nm in a hydrogen plasma with $\epsilon_T(T_i/T_D)\geqslant 10$, $T_e\approx 100$ eV, $T_i\approx 1$ eV, $T_D\approx 0.1$ eV and $\epsilon_{nD}\geqslant 10$. Further restriction on the dust-plasma parameters arises due to collective dust behavior which appears when the intergrain distance $a_g\approx[(3/4\pi)n_{D0}^{-1}]^{1/3}$ becomes smaller or comparable to the plasma Debye lengths. This is to highlight that a few more interesting modes in dusty plasma have been discussed by other workers with[23,24] and without $(m_D\rightarrow\infty)$[25] grain dynamics within fluid and kinetic[26] models.

The present paper deals with the possible effect of the dynamics of the dust charge fluctuation on the linear and nonlinear wave propagation of the ALM/DAM. It forms a new effort to look into the possibility of nonlinear coherent structures in nonconventional dusty plasmas. The well-known reductive perturbation method (RPM) has been applied for the analytical analysis. Section II deals with the basic equations and derives the linear dispersion relations of the desired electrostatic modes. In Sec. III, a nonlinear perturbational analysis has been carried out to derive the Burgers equation which describes the properties of a weak shock profile. Section IV involves the results and discussions along with concluding remarks.

II. BASIC EQUATIONS AND LINEAR ANALYSIS

Let us consider monodisperse spherical charged dust grains of the same size and charge with surface potential ϕ_f distributed in a plasma reservoir at an average plasma potential V_P. The difference (ϕ_f-V_P) determines the ion and the electron currents to the grain surface. We assume that an overall charge neutrality of the equilibrium dust plasma holds good. The charge neutrality means that each grain is surrounded by an equal and opposite space charge and the total average charge on the grains $n_{D0}Q_{D0}$ equals the net average plasma space charge arising due to deviation from the plasma neutrality $[(Z_ien_{i0}-en_{e0})\neq 0]$, i.e., $n_{D0}Q_{D0}+Z_ien_{i0}=en_{e0}$.

Electrons and ions are assumed to follow Boltzmann distributions:

$$n_e=n_{e0}\exp(e\phi/T_e), \tag{1}$$

$$n_i=n_{i0}\exp(-eZ_i\phi/T_i), \tag{2}$$

where n_{e0} and n_{i0} are the average equilibrium density of the electron and ion fluids, respectively, well beyond the grain vicinity. Dust grains being heavier than both the electrons and ions, their dynamical response is governed by the full inertial cold fluid equations:

$$\left(\frac{\partial}{\partial t}+\mathbf{v}_D\cdot\nabla\right)\mathbf{v}_D=-\frac{Q_D}{m_D}\nabla\phi, \tag{3}$$

$$\frac{\partial n_D}{\partial t}+\nabla\cdot(n_D\mathbf{v}_D)=0. \tag{4}$$

The Poisson's equation reads

$$\nabla^2\phi=4\pi e\left(n_e-Z_in_i-\frac{n_DQ_D}{e}\right). \tag{5}$$

Finally the dust charge equation[1] as given below closes the set of equations for investigating the effect of the dust charge fluctuation dynamics on the plasma waves and oscillations,

$$\left(\frac{\partial}{\partial t}+\mathbf{v}_D\cdot\nabla\right)Q_D=I_e+I_i,$$

where Q_d is dust charge and I_e and I_i are the electron and ion currents collected by the dust grains. The dust charging model neglects the photoemission and other charging processes such as secondary emission, etc., and this may be justified in laboratory plasmas. However, in general these processes should be included to deal with physical phenomena in dusty plasmas especially in the space plasmas.[27,28] These dust grains act as a plasma source and/or sink changing thereby the density, temperature, and the flow field of the plasma environment.[29] The real dusty plasmas consist of dust grains with size and charge distributions which introduce different charging times for the grains of different sizes[30] and thus may lead to a significant alteration in the Maxwellian distribution of the plasma. Consequently the validity of the dust charging equation (6) becomes questionable. To avoid these complications, the dust grains have been modeled as a collection of equal radius spherical grains carrying identical charges. Their effect on the plasma distribution has also been neglected for the mathematical convenience. However, the analysis of the dusty plasmas is incomplete without incorporating the above-mentioned effects which is beyond the scope of the present work.

For the cold dust grains, the expressions for the electron and ion currents are given by[1]

$$I_e = -\pi a^2 e\left(\frac{8T_e}{\pi m_e}\right)^{1/2} n_e \exp\left(\frac{e(\phi_f - V_p)}{T_e}\right), \tag{7}$$

$$I_i = \pi a^2 e\left(\frac{8T_i}{\pi m_i}\right)^{1/2} Z_i n_i \left(1 - \frac{eZ_i(\phi_f - V_p)}{T_i}\right), \tag{8}$$

where a is the dust grain radius, $T_{e,i}$ the electron (ion) temperature, $(\phi_f - V_p)$ the potential difference between the dust grain surface potential, ϕ_f, and the ambient plasma potential, V_P. These expressions for the electron and ion currents at the grain surface are useful only when the dust is at rest or, otherwise, if the dust grains are moving with speed w, it is smaller than the thermal speeds of the electrons and ions. The effect of the moving dust grains on their charging process has been discussed in detail by Al'pert in Ref. 31 and also by Whipple.[32] However, our interest lies in the former.

Let us linearize Eqs. (1)–(8) around the equilibrium values of the physical variables: $n_e = n_{e0} + \tilde{n}_e$, $n_i = n_{i0} + \tilde{n}_i$, $n_D = n_{D0} + \tilde{n}_D$, $\mathbf{v}_D = \tilde{\mathbf{v}}_D$, $\phi = \tilde{\phi}$, $Q_D = Q_{D0} + \tilde{Q}_D$, $\phi_f = \phi_{f0} + \tilde{\phi}_f$ with $\phi_f = Q_D/C$, C being the capacitance of the dust grain defined as $C = a(1 + a/\lambda_D)$ (Ref. 33) with λ_D the plasma Debye length, and $\phi_{f0} = Q_{D0}/C$ is the equilibrium floating potential of the grain surface. Now the linear perturbed equations are given as

$$\frac{\tilde{n}_e}{n_{e0}} \approx \frac{e\tilde{\phi}}{T_e}, \quad \frac{\tilde{n}_i}{n_{i0}} \approx -\frac{eZ_i\tilde{\phi}}{T_i}, \tag{9}$$

$$\frac{\partial \tilde{\mathbf{v}}_D}{\partial t} = -\frac{Q_{D0}}{m_D} \nabla \tilde{\phi}, \tag{10}$$

$$\frac{\partial \tilde{n}_D}{\partial t} + n_{D0} \nabla \cdot \tilde{\mathbf{v}}_D = 0, \tag{11}$$

$$\frac{\partial \tilde{Q}_D}{\partial t} + \eta \tilde{Q}_D = |I_{e0}|\left(\frac{\tilde{n}_e}{n_{e0}} - \frac{\tilde{n}_i}{n_{i0}}\right), \tag{12}$$

$$\nabla^2 \tilde{\phi} = 4\pi e\left(\tilde{n}_e - Z_i \tilde{n}_i - \frac{n_{D0}\tilde{Q}_D + Q_{D0}\tilde{n}_D}{e}\right). \tag{13}$$

Here $\eta = (e|I_{e0}|/C)(1/T_e + 1/w_0)$ with I_{e0} the equilibrium electron current and $w_0 = T_i - Z_i e(\phi_{f0} - V_P)$.

Now assuming the fluctuations to vary as $\exp(-i\omega t + kx)$ and carrying out the Fourier analysis of the linearized equations, one can derive (under the approximation $|\partial/\partial t| \ll \eta$) the following dispersion relation:

$$\omega^2 = \frac{\eta}{\eta + \beta_2} k^2 C_{\mathrm{ALM}}^2, \quad \text{for modified ALM (m-ALM)},$$

$$\omega^2 = \frac{\eta}{\eta + \beta_2} k^2 C_{\mathrm{DAM}}^2, \quad \text{for modified DAM (m-DAM)}, \tag{14}$$

where

$$\beta_2 = \beta_1 \left(1 + \frac{n_{e0}}{n_{i0}} \frac{\lambda_{De}^2}{\lambda_{Di}^2}\right)\left(1 + \frac{\lambda_{De}^2}{\lambda_{Di}^2}\right)^{-1} \approx \beta_1 \frac{n_{e0}}{n_{i0}},$$

$$\beta_1 = \left(\frac{|I_{e0}|}{e}\right) \frac{n_{D0}}{n_{e0}},$$

$$C_{\mathrm{ALM}}^2 \approx \frac{T_i}{m_D} \left(\frac{Q_{D0}}{eZ_i}\right)^2 \frac{n_{D0}}{n_{i0}}$$

and

$$C_{\mathrm{DAM}}^2 \approx \frac{T_e}{T_i} \frac{n_{i0}}{n_{e0}} \frac{1}{(1 + \lambda_{De}^2/\lambda_{Di}^2)} C_{\mathrm{ALM}}^2.$$

This is to emphasize that the interdependence of the dispersion property of the m-ALM/m-DAM and the nonconventional role of the electrons and ions is reflected through η and β_2. It is a typical behavior of the real dusty plasma. We notice from the above dispersion relation that for $T_e \gg T_i$, $\lambda_{De} \gg \lambda_{Di}$ and hence the dispersion relation of the m-DAM is exactly reduced to that of the m-ALM. Further, an additional comment deserves to highlight that for $\lambda_{De} \gg \lambda_{Di}$, both the shorter wavelength ($\lambda_{De} \gg \lambda \gg \lambda_{Di}$) and the longer wavelength ($\lambda \gg \lambda_{De}, \lambda_{Di}$) electrostatic modes are characterized by the same dispersion relation as described elsewhere.[16,17] Since these approximations are more likely in dusty plasmas, ALM appears to be a more natural mode of the dusty plasmas. Thus it appears the acoustic-like mode is a more appropriate nomenclature for the natural electrostatic mode of the unmagnetized dusty plasma until the above approximation is violated which seems less likely in dust–plasma system.

The modification in the linear dispersion relation of the ALM/DAM is a consequence of the approximation $|\partial/\partial t| \ll \eta$. This approximation is an essential requirement to find out a self-consistent stretched coordinate system for carrying out the nonlinear perturbational analysis. This, in the lowest order of the fluctuation level, enforces an equality of the electron and ion currents at the surface of the dust grains to obey the approximation and the steady-state charge fluctuation helps to maintain it. However, in the higher order of the fluctuation level (nonlinear regime of the fluctuation amplitude), this equality is violated, thereby leading to a nonsteady dust charge fluctuation which then provides an effective damping mechanism to oppose the continuous buildup of the higher harmonics due to nonlinear steepening. Under certain conditions, these two may balance each other to form a coherent shock structure. Analytical exploration of the possibility of such a structure in real dusty plasma is the focal point of the subsequent sections.

III. NONLINEAR ANALYSIS: SHOCK SOLUTION

Based on the above background, let us develop a mathematical framework for the nonlinear shock wave propagation of the m-ALM/m-DAM in dusty plasmas. Assuming the Boltzmann distributed electrons and ions with inertial charged dust grains, the basic set of nonlinear equations (1)–(8) are normalized and rewritten as:

$$N_e=\alpha_{ne}\exp(\psi),$$
$$N_i=\exp[-\alpha_T Z_i(\psi)]; \tag{15}$$

dust dynamical equations:

$$\frac{\partial \mathbf{V}_D}{\partial \tau}+\alpha(\mathbf{V}_D\cdot\nabla)\mathbf{V}_D=-\beta Q_D\nabla\psi, \tag{16}$$

$$\frac{\partial N_D}{\partial \tau}+\alpha\nabla\cdot(N_D\mathbf{V}_D)=0, \tag{17}$$

$$\frac{\partial \tilde{Q}_D}{\partial \tau}=\frac{I_{e0}}{\alpha_{ne}}N_e\exp(X\tilde{Q}_D)+I_{i0}N_i(1-Y\tilde{Q}_D); \tag{18}$$

Poisson's equation:

$$\nabla^2\psi=[N_e-Z_iN_i-(Q_{D0}/e)Q_DN_D]/\alpha_{ne}, \tag{19}$$

where

$$I_{e0}=-A\alpha_{ne}\exp(\psi_{f0}-\psi_P),$$

$$I_{i0}=AZ_i(\alpha_m/\alpha_T)^{1/2}[1-\alpha_T Z_i(\psi_{f0}-\psi_p)],$$

$$A=\pi a^2 e\left(\frac{8T_e}{\pi m_e}\right)^{1/2}\frac{n_{i0}}{Q_{D0}\omega_{pD}},$$

$$\alpha=\frac{1}{\sqrt{(1+b)}},$$

$$\beta=(eZ_i/Q_{D0})\alpha_{nD}\sqrt{(1+b)},$$

$$b=\frac{n_{i0}T_e}{n_{e0}T_i}=\frac{\alpha_T}{\alpha_{ne}},$$

$$\alpha_T=T_e/T_i,$$

$$\alpha_{ne}=n_{e0}/n_{i0},\quad \alpha_{nD}=n_{e0}/n_{D0},$$

$$X=Q_{D0}e/T_eC,$$

$$Y=\frac{\alpha_T X}{[1-\alpha_T Z_i(\psi_{f0}-\psi_P)]},$$

ψ_{f0} and ψ_p are the normalized steady-state equilibrium dust floating potential and plasma potential, respectively. Normalizations are defined as follows: $\psi=e\phi/T_e$, $N_\mu=n_\mu/n_{i0}$, μ denoting the electrons, ions and dust grains, $\mathbf{V}_D=\mathbf{v}_D/C_D$, $C_D=C_{rs}/\sqrt{(1+b)}$, $C_{rs}=\omega_{pD}/\lambda_{De}$, $\tau=\omega_{pD}t$, $\xi=x/\lambda_{De}$, $Q_D=Q_D/Q_{D0}$, $\omega_{pD}^2=4\pi n_{D0}Q_{D0}^2/m_D$, $\psi_{f,P}=e\phi_{f,P}/T_e$.

For the validity of the dynamic electromagnetic equations to describe the macroscopic phenomena in dusty plasmas, the mathematical idealization of the dust grains as the point charged particles should be obeyed. This limits the scale length λ of the mode to be larger than the intergrain distance a_g and the grain size a. Transforming the basic set of governing equations (15)–(19) into a stretched coordinate system defined by $\zeta=\epsilon(\xi-\alpha M\tau)$, $\overline{\tau}=\epsilon^2\tau$ with M (Mach number)$=V/C_D$, V being the phase velocity of the m-ALM/m-DAM and then carrying out the usual perturbational analysis, the nonlinear evolution of the lowest-order fluctuation can be described by the Burgers equation as given below:

$$\frac{\partial\tilde{\psi}^{(1)}}{\partial\overline{\tau}}+S\tilde{\psi}^{(1)}\frac{\partial\tilde{\psi}^{(1)}}{\partial\zeta}=T\frac{\partial^2\tilde{\psi}^{(1)}}{\partial\zeta^2}, \tag{20}$$

where

$$S=-0.5\epsilon_D^{-1}\frac{\sqrt{(1+b)}}{M}\left[\left(\frac{\epsilon_D}{\alpha_{nD}}(\alpha_{ne}-\alpha_T^2Z_i^2)+X^{-1}\frac{\gamma(1+\alpha_T)^2}{(1+\gamma)^3}\right)M^4+3[X^{-1}\epsilon_D(1+\alpha_T)\times(1+b)/(1+\gamma)]M^2-3[\epsilon_D(1+b)]^2\right],$$

$$T=-X^{-2}\epsilon_D^{-1}\frac{1}{\alpha_{nD}I_{e0}}\frac{(1+\alpha_T)M^4}{[(1+\alpha_T/\alpha_{ne})(1+\gamma)]^2},$$

$$\gamma=\frac{Y}{X}.$$

While deriving Eq. (20), all the variables have bee[illegible] expanded perturbatively around their equilibrium values like:

$$N_e=\alpha_{ne}+\epsilon\tilde{N}_e^{(1)}+\epsilon^2\tilde{N}_e^{(2)}+\cdots,$$

$$N_i=1+\epsilon\tilde{N}_i^{(1)}+\epsilon^2\tilde{N}_i^{(2)}+\cdots,$$

$$N_{nD}=\alpha_{nD}+\epsilon\tilde{N}_{nD}^{(1)}+\epsilon^2\tilde{N}_{nD}^{(2)}+\cdots,$$

$$V_D=\epsilon\tilde{V}_D^{(1)}+\epsilon^2\tilde{V}_D^{(2)}+\cdots,$$

$$\psi=\epsilon\tilde{\psi}^{(1)}+\epsilon^2\tilde{\psi}^{(2)}+\cdots,$$

$$\psi_f=\psi_{f0}+\epsilon\tilde{\psi}_f^{(1)}+\epsilon^2\tilde{\psi}_f^{(2)}+\cdots,$$

$$Q_D=1+\epsilon\tilde{Q}_D^{(1)}+\epsilon^2\tilde{Q}_D^{(2)}+\cdots.$$

It would be desirable to add a few words regarding the choice of the stretched coordinate transformation for the nonlinear perturbational analysis of the shock. The assumption that the nonsteady response of the dust charge flu[illegible]ation dynamics and wave dispersions are nonlinear effects and arise, respectively, at second and third order of nonlinearity is related with the existence of a self-consistent choice of an appropriate stretching transformation to carry out the nonlinear perturbational analysis. This in turn facilitates finding out a self-consistent choice of the index values of the space and time scalings in the stretched coordinate system, i.e., the values of p and q in $\zeta=\epsilon^p(\xi-\alpha M\tau)$, $\overline{\tau}=\epsilon^2\tau$ which come out to be $p=1$, $q=2$. Under these approximations, the linear frequencies of ALM and DAM are modified due to the nonconventional role of the electrons and ions through the steady dust charge fluctuation. As a consequence undamped electrostatic modes namely; m-ALM and m-DAM are found to exist in lowest order of their fluctuation levels.

It is well known that the Burgers equation describes a weak shock wave profile. The basic criterion for the formation of the shock is that the coefficient of the dissipative term (T in our case) should be positive otherwise an unstable solution would be obtained which is beyond the scope of the present analysis. Accordingly, let us look for a steady-state solution of Eq. (20). Transforming this equation into a moving frame defined by $\eta' = \zeta - U\tau$, U being the velocity of the frame, its solution can be derived as

$$\tilde{\psi}^{(1)} = \frac{\tilde{\psi}_2^{(1)} + \tilde{\psi}_1^{(1)} \exp[(S/2T)(\tilde{\psi}_2^{(1)} - \tilde{\psi}_1^{(1)})\eta']}{1 + \exp[(S/2T)(\tilde{\psi}_2^{(1)} - \tilde{\psi}_1^{(1)})\eta']}. \quad (21)$$

In deriving the above expression, the boundary conditions for a shock wave profile; $\tilde{\psi}^{(1)} \to \tilde{\psi}_1^{(1)}(\tilde{\psi}_2^{(1)})$ at $\eta' \to +\infty(-\infty)$ and $d\tilde{\psi}^{(1)}/d\eta \to 0$ at $\eta' \to \pm\infty$ have been used. Implementation of these conditions leads to the derivation of self-consistent shock velocity U and shock width L_s as

$$U = 0.5S[\tilde{\psi}_1^{(1)} + \tilde{\psi}_2^{(1)}], \quad (22)$$

$$L_s = 2\frac{T}{S}\frac{1}{[\tilde{\psi}_2^{(1)} - \tilde{\psi}_1^{(1)}]}. \quad (23)$$

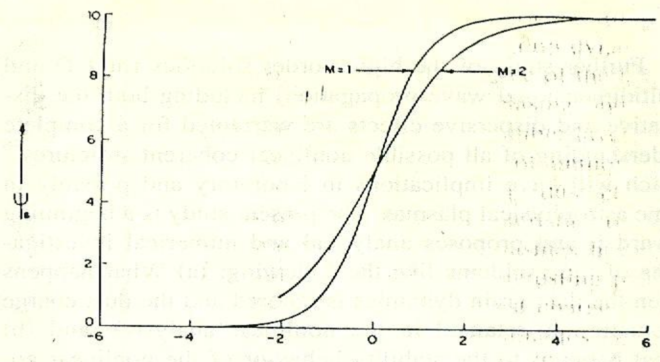
Description of the discontinuous shock structure within the continuum model of the charged dust grains is limited to larger shock widths such that $L_s > a_g$, a. This in turn restricts the required Mach number values for given plasma parameters or vice versa.

IV. RESULTS AND DISCUSSIONS

The conditions for the existence of steady-state shock solution are satisfied for both the positive and negative dust grains. However, the present analysis holds good only for the negative dust particles. The nature of the shock profile either with monotonically decreasing amplitude or the opposite is decided by the value of M through the nonlinear coefficient S for given plasma parameters. For the positive potential shocks with $\tilde{\psi}_2^{(1)} > \tilde{\psi}_1^{(1)}$, $S>0$ whereas with $\tilde{\psi}_2^{(1)} < \tilde{\psi}_1^{(1)}$, $S<0$ and the reverse is true for the negative potential shocks. These inequalities coupled with other approximations decide the range of the Mach number and plasma parameters for the existence of the m-ALM/m-DAM shock wave profiles in real dusty plasmas.

Analytical investigation of the shock formation in dusty plasmas signifies the physical effect of the nonsteady dust charge fluctuation dynamics which provides a nonlinear dissip[illegible]e mechanism to counteract the nonlinear compression of the m-ALM/m-DAM electrostatic modes. This is purely a dust induced physical effect to be addressed at least at theoretical level. As the experimental work in dusty plasmas has just started,[7,8,12] it is at the moment difficult to provide observational support for our findings. However, numerical estimates of physical parameters have been carried out to suggest a model experiment for nonlinear wave propagation in dusty plasmas. Graphical presentations of the shock solution given by Eq. (21) exhibit the possible potential profiles for a typical sets of RF dust–plasma parameters.

Figures 1(a) and 1(b) depict, respectively, the compressive and rarefactive shock profiles with potential jump $|\tilde{\psi}_2^{(1)} - \tilde{\psi}_1^{(1)}| \approx 10^{-3}$ for $n_{i0} \approx n_{e0} = 10^{10}$ cm^{-3}, $n_{D0} = 10^8$ cm^{-3}, $T_e \approx 100$ eV, $a = 10^{-5}$ cm, $m_D \approx 10^{-15}$ g for dust grain mass density ≈ 1 g/cm^3 and $Q_{D0} = 10e$. Here the y axis represents a potential defined by $\psi_s = (\tilde{\psi}^{(1)}/\tilde{\psi}_2^{(1)} - 1) \times 10^3$. This range of parameters may be attended in laboratory plasmas. It is emphasized to reveal that for this set of dust–plasma parameters, the dust charging frequency ν_q defined by $\nu_q \approx \omega_{pi} a/\lambda_D$ (Ref. 13) comes out to be $\approx 5 \times 10^{-4}$ s^{-1} and dust plasma oscillation frequency $\omega_{pd} \approx 10^{-4}$ s^{-1} with $a_g \sim 10^{-3}$ cm and $\lambda_{De} \sim 10^{-2}$ cm. These numerical estimates justify the simultaneous consideration of the grain dynamics and charge fluctuation dynamics. In the absence of the latter the system behaves as a multicomponent plasma and the likely nonlinear structures are solitons and double layers as discussed by other workers.[34,35] In the case of unmovable heavy dust grains ($m_D \to \infty$), the effect of the dust charge fluctuation dynamics on the nonlinear characteristics of the electrostatic modes in unmagnetized dusty plasmas has not been discussed earlier and hence forms an interesting problem to look into. Here the coupling of the dust charge fluctuation with the other plasma dynamic variables arises through the quasineutrality as discussed by Tsytovich.[13] However, this is beyond the scope of the present analysis.



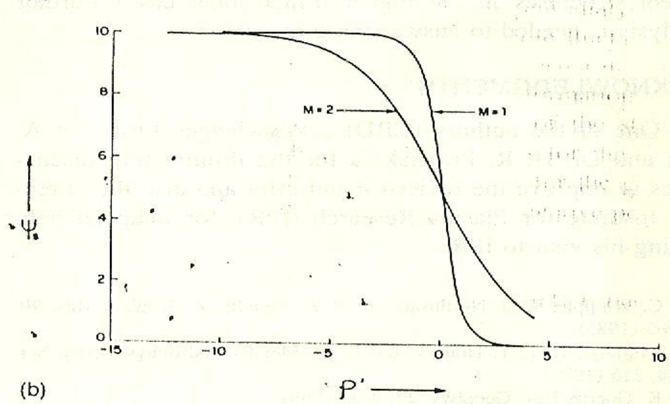


FIG. 1. Spatial potential profiles for compressive (ψ_s) (a) and rarefactive ($-\psi_s$) (b) shocks for typical RF dust–plasma parameters; $n_{i0} \approx n_{e0} = 10^{10}$ cm^{-3}, $n_{D0} = 10^8$ cm^{-3}, $T_e = 100$ eV, $a = 10^{-5}$ cm, $m_D \approx 10^{-15}$ gm and $Q_{D0} = 10e$ with potential jump $|\tilde{\psi}_2^{(1)} - \tilde{\psi}_1^{(1)}| \approx 10^{-3}$.

In brief it is concluded that the dust charge fluctuation coupled with other dynamic variables of the plasma fluctuations leads to the nonlinear shock wave formation for m-ALM/m-DAM in dusty plasmas. Validity of the shock conditions and those of the continuum model for the dust grains restrict the range of the Mach number for the given plasma parameters or vice versa for positive/negative potential shocks.

Further study of the higher-order solutions (in 1-D and multidimensional wave propagation) including both the dissipative and dispersive effects are warranted for a complete understanding of all possible nonlinear coherent structures[36] which will have implications in laboratory and possibly in some astrophysical plasmas. The present study is a beginning toward it and proposes analytical and numerical investigations of the problems like the following: (a) What happens when the dust grain dynamics is ignored and the dust charge fluctuation is retained in the nonlinear analysis?; and (b) What happens to the stability behavior of the nonlinear coherent structures in the higher dimensional case? Further analysis is needed to answer these questions.

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