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# Numerical techniques for proton magnetometers

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**Abstract**

The problem of estimation of the precession frequency of the proton magnetometer has been addressed here. The precession frequency has to be determined from the successive zero crossings of the sinusoidal output signal induced in the coil by the proton rich fluid immersed in the earth's magnetic field. The dependence of the accuracy of the signal to noise ratio and decay of the signal is discussed. Least square methods and the robust median fit methods are used to estimate the signal period. It is found that the signal processing method using a robust technique gives superior performance when higher sampling is needed or the environmental conditions degrade the available signal to noise ration. For a five second sampling under ideal observatory conditions, even the least square technique provides a 0.1 nT accuracy.

**Introduction**

The proton precession magnetometer (PPM) is being used for measuring the geomagnetic field for many years now (Wienert, 1970). It involves measuring the sinusoidal oscillation frequency induced in the sensor coil by the precession of protons in the ambient geomagnetic field. The typical frequency for the magnetic field of the earth ranges from 1 kHz to 4 kHz. The protons are first polarized in a chosen direction with the help of a relatively large polarizing current. When the polarization current is withdrawn the proton's precession around the earth's magnetic field is initiated. The signal amplitude decays with spin-phase memory time constant $T_2$. This time constant depends on the liquid in which the sensor is immersed and on the gradient of the magnetic field across the sensor. Even in a uniform field, this time rarely exceeds two seconds. Initial signal amplitude at the sensor output is a few microvolts peak-peak. The task of the electronic circuitry along with its accompanying signal processing algorithms is to derive the period of precession frequency accurately with a measuring time of less than one second.

First generation magnetometers achieved this by using phase lock loop techniques. A phase lock loop locks on to the incoming signal and at the same time multiplies its frequency by a suitable factor. This process filters out the noise in the proton signal apart from reducing the measuring time. This method has its limitations. Apart from the fact that the phase lock loop takes a finite time to lock on to the incoming signal, the multiplying factor in the phase lock loop cannot be increased indefinitely to achieve an increase in measurement accuracy.

An accuracy of 1 nT (nano Tesla) can be achieved for a typical requirement of sampling period of 5 seconds but to attain greater precision a different approach has to be adopted. Sophisticated methods also become essential even for 1 nT accuracy when the sampling rates are around 1 second or less and the number of cycles available becomes too small to effectively weed out the errors generated by the noise component. In what follows, we discuss numerical data processing techniques that can be used to achieve such an objective. Two independent algorithms that can be used with the proton magnetometer output signal are discussed in detail. The numerical results focus on the typical output of a standard proton magnetometer but the same techniques are equally applicable to the Overhauser magnetometer as well. These include the standard least square signal for linear fit to the time progression of the zero crossings technique and the robust median technique.

**Definition of the problem**

The relation between the geomagnetic field and proton precession frequency is given by

$$B = gF_0 \quad \textbf{(1)}$$

Where B – Geomagnetic field in nT

g – constant – 23.4874

$F_0$ – Proton Precession frequency

The period $T_0$, is estimated from the number of cycles of a higher frequency signal $F_r$ measured during N number of signal cycles $F_0$. This involves recording the times of zero crossings of the signal voltage. If C is the total count of high frequency cycles of $F_r$, the period $T_0$ is given by

$$T_0 = \frac{C}{NF_r} \quad \textbf{(2)}$$

Substituting this value in (1) gives

$$B = \frac{g}{C} NF_r \quad \textbf{(3)}$$

We have tacitly assumed that the signal is pure and free from noise so that the zero crossings can be identified unambiguously. In reality we do have a noise component n(t). We can describe the signal Voltage V as (Hancke, 1990)

$$V = S.Sin(\omega t) + n(t)$$

Where S is the signal amplitude and n(t) is the noise component already defined. In principle any threshold level Δv can be used to estimate the period $T_0$ but as we shall see soon, the zero crossings are the most appropriate points for such estimations.

Let the signal S crosses the threshold level Δv at time $t_1$. Then

$$\Delta v = S.Sin\,\omega t_1 + n(t_1)$$

and

$$t_1 = \frac{1}{\omega} Sin^{-1} \frac{\Delta v - n}{S}$$

Let us assume that the pure sine wave would have crossed the threshold Δv at time $t_2$.

$$\Delta v = S.Sin\,\omega t_2$$

$$t_2 = \frac{1}{\omega} Sin^{-1} \frac{\Delta v}{S}$$

The error ΔT introduced by the noise is then given by

$$\Delta T = t_1 - t_2 = \frac{1}{\omega}\left[Sin^{-1}\frac{\Delta v - n}{S} - Sin^{-1}\frac{\Delta v}{S}\right] = \frac{1}{\omega}\left[\frac{-n}{S} - \frac{(\Delta v)^2 n}{S^3} + \frac{\Delta v n^2}{S^3} - \frac{n^3}{S^3}.....\right]$$

We get a minimum value for ΔT when Δv = 0 i.e. at the zero crossing point. This is not surprising as the slope of the sine wave attains its largest value at its zero crossings.

Thus to the lowest order we get

$$\Delta T = \frac{1}{\omega R} \quad \textbf{(4)}$$

Where $R = \frac{S}{n}$ is the signal to noise ratio.

Thus measured period $T_0$ is accompanied by an error ΔT at every zero crossing of the signal. In addition to the error due to the noise, there is one more source error. The frequency Fr that is assumed to be a constant may not be so.

Taking into consideration these two sources of error we can use equation (3) to re-write the error in ΔB as

$$\Delta B = B\frac{\Delta F_r}{F_r} + B\frac{\Delta T}{T} \qquad (5)$$

The error caused by uncertainties in the reference frequency Fr can be taken care of by calibrating the crystal. But error arising from the fluctuations of the zero crossings generated by the noise component has to be treated using special techniques. This error can be minimized with different processing techniques.

In the real life situation the signal decays due to loss of coherence of the precession of the protons in the different parts of the sensor volume. The signal decay with time can be expressed through the degeneration of the signal to noise ratio R as

$$R = R_0 e^{-t/T_2}. \qquad (6)$$

Here $R_0$ is the signal to noise ratio at time zero.

We note from the second term in equation (5) that the larger the value of $T_2$, the less will be the error in B. This would imply that the larger the value of N, the larger the accuracy in the magnetic field determination provided $\Delta T$ remains constant. We note form equation (4), that $\Delta T$ is inversely proportional to R and this error will manifest itself as we increase the number of cycles N. There should be an optimum N. To get this N, we substitute R from equation (6) into equation (4) to get $\Delta T$ and use this value of $\Delta T$ in equation (5) to get the error in B. This is differentiated with respect to N and put equal to zero. We get

$$N = \frac{T_2}{T_0}$$

N gives the optimum number of cycles for which the error in the estimated field is a minimum. But in practice the number of cycles available may be less and one has to make do with a smaller value of N.

**Processing techniques**

Two techniques that are used for processing the signal are described here. Both the methods attempt to retrieve the exact period of the wave from the N measured crossing. The period has to be determined from N values tp (p=0 , N) of observed times of the zero crossings each having a variance of $\sigma_p$. As the signal amplitude decays, the variance increases, though in most practical applications it is often neglected. We retain this feature in the basic formalism although we present numerical results with the simplified scenario. If $T_c$ is the exact time period then

$$t_0 + pT_c = t_p \qquad (7)$$

Where p = 0, 1, 2...........N and $t_0$ is the uncertainty in the timing of the first crossing.
The techniques used derive $T_c$ in such a way that its variance is as small as possible.
The standard technique used attempts to minimize the mean square difference of the actual zero crossing times and those computed from the estimated fit. This is referred to as the least square method here. We shall note that the least square technique, with suitably chosen algorithms, has the advantage that it requires a relatively smaller number of computations and with suitably defined algorithms it can be used in real time. However, the method is susceptible to strong biases generated by outliers. A median fit becomes more reliable and may be a preferred method with smaller samples. We describe both the methods in some detail here.

(1) Least square method – In this method time is measured from the first zero crossing of the signal up to N number of zero crossings. $s^2$, the sum of the square of the difference between the observed time and fitted value of the zero crossings is given by

$$s^2 = \sum_0^N \frac{\left(t_0 + pT_c - tp\right)^2}{\sigma_p^2}$$

Differentiating the above expression, with respect to $T_c$ and putting the result equal to zero we get

$$t_0 \sum_0^N \frac{p}{\sigma_p^2} + T_c \sum_0^N \frac{p^2}{\sigma_p^2} - \sum_0^N \frac{ptp}{\sigma_p^2} = 0$$

A similar operation with respect to $t_0$ gives

$$t_0 \sum_0^N \frac{1}{\sigma_p^2} + T_c \sum_0^N \frac{p}{\sigma_p^2} - \sum_0^N \frac{tp}{\sigma_p^2} = 0$$

$T_c$ can be obtained from the two equations given above and can be written as

$$T_c = \frac{S_A S_E - S_B S_D}{S_A S_C - S_B^2} \quad \textbf{(8)}$$

Where

$$S_A = \sum_{p=0}^N \frac{1}{\sigma_p^2}$$

$$S_B = \sum_{p=0}^N \frac{p}{\sigma_p^2}$$

$$S_C = \sum_{p=0}^N \frac{p^2}{\sigma_p^2}$$

$$S_D = \sum_{p=0}^N \frac{tp}{\sigma_p^2}$$

$$S_E = \sum_{p=0}^N \frac{p\ tp}{\sigma_p^2}$$

The variance of $T_c$ is given by

$$\sigma\left(T_c\right) = \sqrt{\frac{S_A}{S_A S_C - S_B^2}}$$

The expressions provided by Hancke (1990) can be retrieved if it is assumed that the variance in the estimates of the time of zero crossings remains constant right through the measurement cycle. After some manipulations it is possible to express $T_c$ in the form

$$T_c = \sum_0^N \alpha_p t_p \quad \textbf{(9)}$$

Where

$$\alpha_p = \frac{p.S_A - S_B}{\sigma_p^2.(S_A.S_C - S_B^2)} \quad \textbf{(10)}$$

The form given in equation (10) is more convenient to implement in real time micro-controller computations. If we ignore the variation of the signal amplitude with time, $\sigma_p$ is independent of p and can be replaced by some typical value $\sigma_0$ and $\alpha_p$ takes a simple form

$$\alpha_p = \frac{6.(2.p - 1 - N)}{N.(N^2 - 1)} \qquad \textbf{(11)}$$

This is identical to the expression provided by Farrel et. al. (1965) using a somewhat different approach. $\alpha_p$ can be stored in a suitable array to enable quick real time computations.

(2) Robust estimation using the median fit: It can be shown (Press et al, 1992) that the median value of a given set of numbers is a truly representative value of the sample as the sum of its absolute difference from the set of points is the minimum. This property can be used to obtain a robust estimate of the linear fit (cf. Press et. al., 1992). We use this technique estimate period. Following the reference cited above, we use the least square estimate as a starting value to get the median of the distribution. This method minimizes the effect of outliers in a distribution. As we shall note later the use of robust technique is not recommended when large number of cycles are available for the estimation of the time period, as the computational overheads are not justified by the corresponding accuracies in the estimation. For smaller samples however, the robust estimate provides more reliable values.

## Results

The least square algorithms have been implemented both in a PC based system as well as in a micro-controller based instrument. The micro-controller-based PPM was taken to the Xth IAGA workshop on instrumentation at Hermanus for inter-comparison and calibration and shown to generate field values accurate to 0.1 nT. We present here some results obtained using the PC-based system to demonstrate its sensitivity. The instrument was operated at the Alibag observatory and the results were compared with a standard fluxgate magnetometer developed by the Danish Meteorological Observatory with 0.1 nT accuracy. Figures 1 and 2 provide a good account of the sensitivity of the PPM.

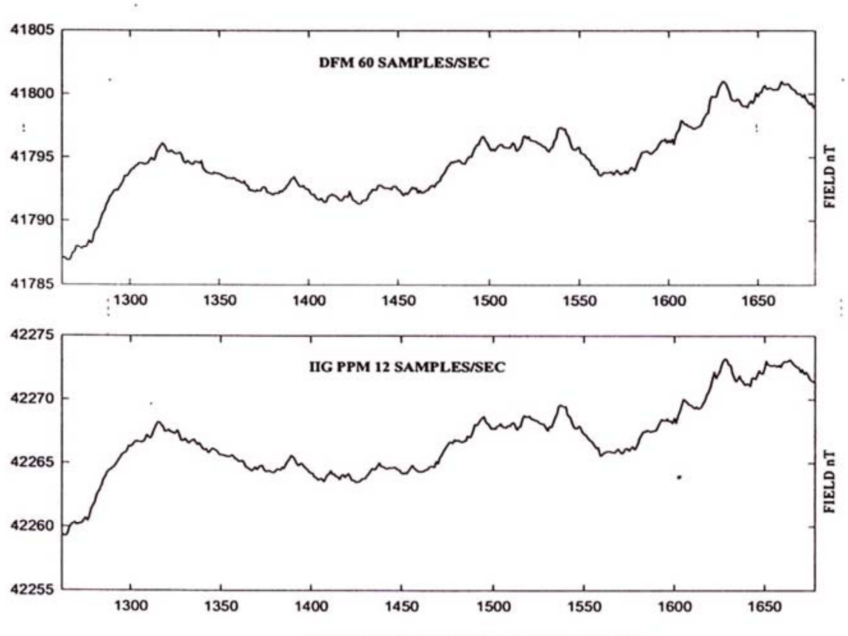

**Fig 1.** Plot of F, the total magnetic field variation at Alibag from the PPM and the corresponding value derived from fluxgate magnetometer developed by the Danish Meteorological Observatory with 0.1 nT . The offset in the figures is because of different location of the instruments.

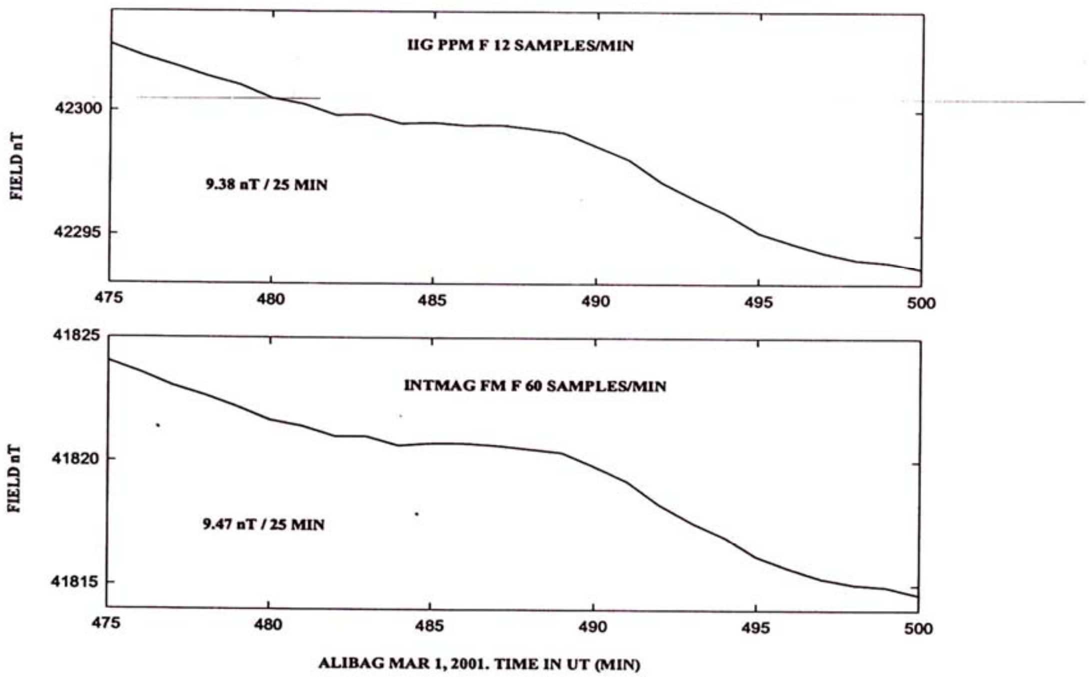


**Fig 2.** Same as Figure 1 but confined to a shorter period to demonstrate the authenticity of the short period variations of the PPM.

These plots show field variations at Alibag (18.64° N, 72.87° E geographic co-ordinates). Here the plotted values are one minute values that are averages of 60 samples in case of the Digital Fluxgate Magnetometer (DFM) and 12 samples in case of the PPM. The number of signal cycles of the proton precession utilized was 1000. The signal to noise ratio was 10 in the beginning and the signal decay time $T_2$ was around 1.5 seconds ensuring that over the entire measurement period the signal to noise ratio was above 6. The remarkable similarity in the long period variations and trends (Figure 1) and in the short data segment (Figure 2) brings out very effectively the authentic response of the PPM to changes even of the order of 0.1 nT. A DC offset between the two instruments is due to the different locations.

Very often a PPM output is required with a sampling rate of one per second or higher. In such cases the number of cycles available could be as low as 300. The signal to noise ratio also may not be as high. We therefore examine how the two methods described in the last section perform when only short periods of a signal output is available and the signal to noise ratio is also weaker. A standard signal source of very high stability with a frequency corresponding to 46974.3 nT was used to simulate the proton output. The signal to noise ratio was maintained at around 4 and the utilized measuring time for the data was varied to examine how the number of cycles used, controls the reliability of the measurement. Sixty independent samples were taken and using 300, 600 and 1000 cycles, the 'field' was computed for each of the sixty samples. The standard deviation and the probabilities of the computed field deviating (a) by more than 1 nT and (b) by more than 0.1 nT was estimated. The results of the computations are presented in. Table 1

| Number of samples | Least Square estimates % Points outside | | | Robust Estimates % Points outside | | |
|---|---|---|---|---|---|---|
| | σ(Tc) | 0.1nT | 1 nT | $\sigma(T_C)$ | 0.1 nT | 1 nT |
| 300 | 1.286 | 98 | 13 | 0.357 | 58 | 1 |
| 600 | 1.116 | 96 | 18 | 0.206 | 41 | 0 |
| 1000 | 0.021 | 0 | 0 | 0.027 | 0 | 0 |

**Table 1.** Performance summary for the least square and robust techniques.

We note that when 1000 cycles are available, the least square estimates lie within 0.1nT of the expected values and no advantage is gained by using the more time consuming but robust median fit. The variance is also not significantly different between the two methods. But the situation is dramatically different when a smaller number of cycles are used. The variance in the estimated periods is drastically less when a robust estimation is used.

When only 600 cycles are used, only 4% of the observations lies within a 0.1 nT window for the least square estimate and as many as 18% of the estimates lies outside 1 nT. On the other hand 59% of the points lie within the 0.1 nT window, and all the estimates are accurate to better than 1 nT.

When 300 points are used, only 2% of the least square estimates lies within 0.1 nT while 42% of the robust estimates lies within this window. 87 % of the least square estimates lies within 1 nT while in the case of the robust estimates 99% of the observations lies within 1 nT.

## Conclusion

Using sophisticated numerical techniques; the accuracy and the sensitivity of the PPM measurements can be enhanced. For 5 second sampling, the PPM estimates based on least square algorithms can generate values reliable up to less than 0.1 nT. However, when a higher sampling rate is desired or when the signal to noise ratio is less due to the environmental conditions, the robust median fit is the right choice and can provide more reliable and accurate values.

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