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A test for the stationariness of the *Sq* current system

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Abstract. We have shown here that a single chain of geomagnetic observatories, located at a fixed longitude, can provide the longitudinal gradients in the strength of the current system. The empirical model of *Onwumechili* [1967] for low latitude *Sq* currents, which is based on the diurnal variation of the horizontal component of the geomagnetic field, has been augmented to reproduce *D* variations that are consistent with the curl-free nature of the surface magnetic field variations. It is then possible to use variations in declination to obtain the longitudinal gradients in the strength of the current system using data from a single longitude zone. The model is applied to the data from the Indian zone. A clear linear relation between the longitudinal and temporal variations of the strength of the current system is brought out. The result appears to indicate that, at least locally, in the 75°E longitude sector, the current system is stationary and the time dependence can be used to replicate the longitudinal variations.

1. Introduction

The quiet day *Sq* current system, to the first approximation, can be regarded as a stationary structure in the Sun synchronous coordinates. The diurnal variation at any surface observatory can then be regarded as a consequence of rotation of Earth under this current system, and the time variations reflect the longitude dependence of the current system. The justification of the slice method [*Campbell*, 1997], wherein the longitude coordinates and time coordinates are regarded as inter-changeable, is based on the implicit assumption of such a scenario.

The picture of Earth rotating under a fixed current system in space is conceptually elegant. It is also useful in analysis of data from a single latitudinal chain of observatories, as it allows for use of the global spherical harmonic expansion, through the use of local time as a proxy for longitude [*Campbell et al.*, 1993]. It therefore becomes necessary to determine to what extent this interchangeability of time and longitude coordinates holds. The procedure adapted here is basic in nature and follows logically from the quasi-static nature of temporal variations of the *Sq* currents and the use of the fundamental properties of electrodynamics as encapsulated in Maxwell's equations.

In the scenario determined by Earth rotating under a fixed current system in space, the horizontal component *H* at the surface tracks the overhead eastward current. A station 15° to the west of the station will see the field that the station monitored an hour earlier, and similarly, a station 15° to the east will be observing a field that the station will experience an hour later. On the other hand, there are no significant currents in the Earth ionosphere cavity, and that requires that the curl of the magnetic field has to be zero. In particular, the vertical component of the curl of the magnetic field vector has to vanish. This requires that the latitudinal variation in the declination should balance the east-west variation of the horizontal component *H* [*Patil and Rajaram*, 1997]. In other words, the actual east-west gradients in *H*, at any instant, can be estimated from the latitudinal variation of the declination, at a single longitude.

In principle, data of two low-latitude nonelectrojet stations are required for the proposed analysis. In practice, however, for a statistically reliable estimate of the *Sq* current parameters, a chain of geomagnetic observatories within an acceptable longitude slice is desirable. In this paper we use the Indian geomagnetic observatory chain (75°E geographic) and an empirical analytical model based entirely on the latitudinal variation of *H*. While any analytic empirical fit based only on *H* variations could have been adopted, we use here the model of *Onwumechili* [1967] to obtain the main *Sq* parameters and its temporal and longitudinal variations. This empirical model is known to reproduce, in a quantitative manner, the salient features of the *Sq* current system [*Onwumechili*, 1967; *Arora et al.*, 1993; *Onwumechili et al*, 1996]. The model provides for the reversal of the sign of the *H* variations across the latitude of the focus, and at the same time, the final expression for the *H* component is simple and compact. This is important, as it makes it possible to manipulate the expressions analytically and thus augment the model to include the determination of the longitudinal gradients in the *Sq* parameters from the diurnal pattern of declination seen by the observatory chain.

In section 2 an augmentation of the model of *Onwumechili* [1967] is carried out to include the determination of the longitudinal derivative of the *Sq* parameters as determined from *H* in the original model framework. In section 3 the original model and its extension are applied to the data of the Indian chain to obtain the *Sq* patterns and their longitudinal and temporal derivatives. Section 4 is dedicated to the discussions of the results in terms of the stationarity of the *Sq* current pattern. The principal conclusions are reviewed in section 5.

2. Theoretical Background

At the equatorial and low-latitude geomagnetic observatories the *Sq*, like the equatorial electrojet, manifests itself principally in the *H* component. This prompted *Onwumechili* [1967] to propose a simple model of latitude-dependent eastward currents in the ionosphere to reproduce the observed diurnal variations in *H* on the

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ground. In its more general form the model incorporates the choice of a height distribution of the currents, but we use the simpler basic version that assumes that the ionospheric currents are flowing at a specified height h in the ionosphere. The selected model is quite adequate for fitting the surface H data. Using only Cartesian coordinates, the model adopts a functional form of the current given by

$$J(x) = J_o a^2 \frac{\left[a^2 + \alpha(x-x_0)^2\right]}{\left[a^2 + (x-x_0)^2\right]^2} , \qquad [1]$$

where x is a measure of the latitudinal distance from the dip equator; x_0 is the distance of the axis of the Sq current system from the dip equator; a is scalar constant characterizing the size of the current system; J_o is the peak current intensity at the axis of the Sq current system; and α, when negative, is the parameter that determines the latitude at which the current reverses its direction. The distance x is taken positive when the point is to the north of the dip equator. If x, x_0, and a are all measured in units of h, the height of the current layer from the ground, the surface field given by *Onwumechili* [1967] can be expressed in the form

$$H = \frac{1}{2} Ka \frac{(x-x_0)^2 + (1+2a)(1+a)^2}{\left[(x-x_0)^2 + (1+a)^2\right]^2} + \frac{1}{2} K\alpha a \frac{\left[(1+2a)(x-x_0)^2 + (1+a)^2\right]}{\left[(x-x_0)^2 + (1+a)^2\right]^2} \qquad [2]$$

In the above expression, K $(= 0.2\pi J_0)$ is the parameter representing the intensity of current system.

There are some intrinsic assumptions built into (1) and (2). They assume (1) Earth is flat and the currents can be described in Cartesian coordinates, and (2) the currents are uniform in the east-west direction.

These assumptions do not significantly affect the accurate representation of H, as it is sensitive only to the overhead east-west currents. East-west variation of the eastward current accompanied by curvature can give rise to a small declination component that is 1 order of magnitude less than the observed D variations [*Patil and Rajaram*, 1997]. These can be conveniently ignored.

The currents defined in the model represent a single local time or longitude, and the data are fitted for each local hour separately [*Onwumechili et al.*, 1996]. There is no explicit provision for north-south currents, though reversal of east-west current is adequately represented. What we do next is to augment the model so that it can be used to describe the latitudinal variations in the diurnal pattern of the declination component.

The H component varies with local time and thus with longitude at any specific instant. For such slowly varying fields the displacement current can be neglected and since the lower atmosphere cannot support steady currents, D has to vary with latitude to make the model consistent with the requirement that the curl of the magnetic field is zero. This requires that

$$\frac{\partial H}{\partial y} = \frac{\partial D}{\partial x} \qquad [3a]$$

or

$$D(x) = \int_{*}^{x} \frac{\partial H}{\partial y} dx . \qquad [3b]$$

In (3a) and (3b), y is the distance measured positive eastward in local Cartesian coordinates. The lower limit of the integral is arbitrary, and what one obtains is the departure of the declination at each hour from the value at a known fixed station, in the same latitude chain. Plugging in the expression of H as given in (1) into (3b) and integrating with respect to x, we get

$$D(x) = D_k \frac{\partial K}{\partial y} + D_\alpha \frac{\partial a}{\partial y} + D_{xo} \frac{\partial x_o}{\partial y} + D_a \frac{\partial a}{\partial y} + \text{constant} \qquad [4]$$

where

$$D_K = \frac{1}{2} a(1+a) \tan^{-1}\left(\frac{x}{1+a}\right) + \frac{1}{2} a(1-a) \frac{ax}{x^2 + (1+a)^2} , \qquad [5a]$$

$$D_\alpha = \frac{1}{2} aK \left[\tan^{-1}\left(\frac{x}{1+a}\right) - \frac{ax}{x^2 + (1+a)^2}\right] , \qquad [5b]$$

$$D_{xo} = -H(x) , \qquad [5c]$$

$$D_a = -\frac{1}{2} K(1+a) \tan^{-1}\left(\frac{x}{1+a}\right) + \frac{1}{2} K(1-3a) \frac{ax}{x^2 + (1+a)^2} - K(1-a) \frac{a^2(a+1)x}{\left[x^2(1+a)^2\right]^2} \qquad [5d]$$

We note from (5a) to (5d) that D_K, D_α, D_H and D_a are known once the fit into the latitudinal variation in H has been carried out to obtain K, a and α. It follows immediately that the unknown parameters $\partial K/\partial y$, $\partial a/\partial y$, $\partial x_o/\partial y$ and $\partial a/\partial y$ can be obtained from (4) through a least squares fitting to the latitudinal variation in D. Note that D depends on the instantaneous longitudinal *(y)* derivative of H and not on the time variation of H. The model parameters obtained at the next or previous local hours have not been used. The model parameters evaluated at different local times can be used to obtain a time derivative that will correspond to the y derivative only when we assume that the Sq current is stationary in space and Earth is moving under it.

The basic formalism for utilizing the H and D variations obtained from a single latitudinal chain of observatories to get the Sq parameters and their longitudinal gradients is now available. The parameters are obtained, for each hour, by fitting the H data, while their gradients are obtained, again for each local hour, through the fitting of D.

3. Data Treatment and Analysis

The model described in section 2 is used here to examine the Sq variation in the Indian (75°E geographic) zone. The stations whose data have been used are Hyderabad (HYD), Alibag (ABG), Jaipur (JAI), Sabhawala (SAB), and Gulmarg (GUL). The geographic

Table 1. Locations of the Geomagnetic Observatories

Station	Code	Geographic Latitude °N	Geographic Longitude °E	Geom. Latitude °N	Geom. Longitude °E
Hyderabad	HYD	17.41	78.56	7.6	148.9
Alibag	ABG	18.64	72.87	9.5	143.6
Jaipur	JAI	26.92	75.80	17.3	147.4
Sabhavala	SAB	30.36	77.80	20.6	149.8
Gulmafg	GUL	34.12	74.40	24.5	147.2
Shillong	SHL	25.57	91.88	14.6	162.4

and geomagnetic coordinates of the stations and consequently the latitudinal distance x from the dipole (geomagnetic) equator corresponding to the 75°E longitude are given in Table 1. In the results presented here, a is taken as 62 and x_0 as -5.0°. The position of the axis of the Sq current system, the dipole and geographic equators, and the x coordinates of the stations used are shown schematically in Figure 1.

The mean hourly patterns for H and D, at the selected stations, are obtained for each month by first correcting the hourly field values for noncyclic variations and then averaging them (for each local hour separately) over all the quiet days in a given month. This is carried out for each of the months for the entire year of 1978. The hourly monthly mean values of the departures of the hourly values from the midnight value are then computed for H and D for each month, and these are used for further analysis. The data of Hyderabad (HYD) is used as the fixed reference station for the D analysis, and deviations for each hour from this reference station are utilized for fitting the D data into the model. The fit is used to obtain the longitudinal gradients in the Sq parameters. The parameters themselves are evaluated earlier using the H data from the entire chain.

The first step in the analysis is to use (1) to obtain the Sq parameters K and α defining the daily variation in H. The parameter fitting is carried out separately for each hour from 0600 to 1800 local 75°E time. The x coordinates are expressed in units of h, the assumed height of the current system, taken here as 130 km.

Equation (1) is nonlinear in the parameters a and x_0. As pointed

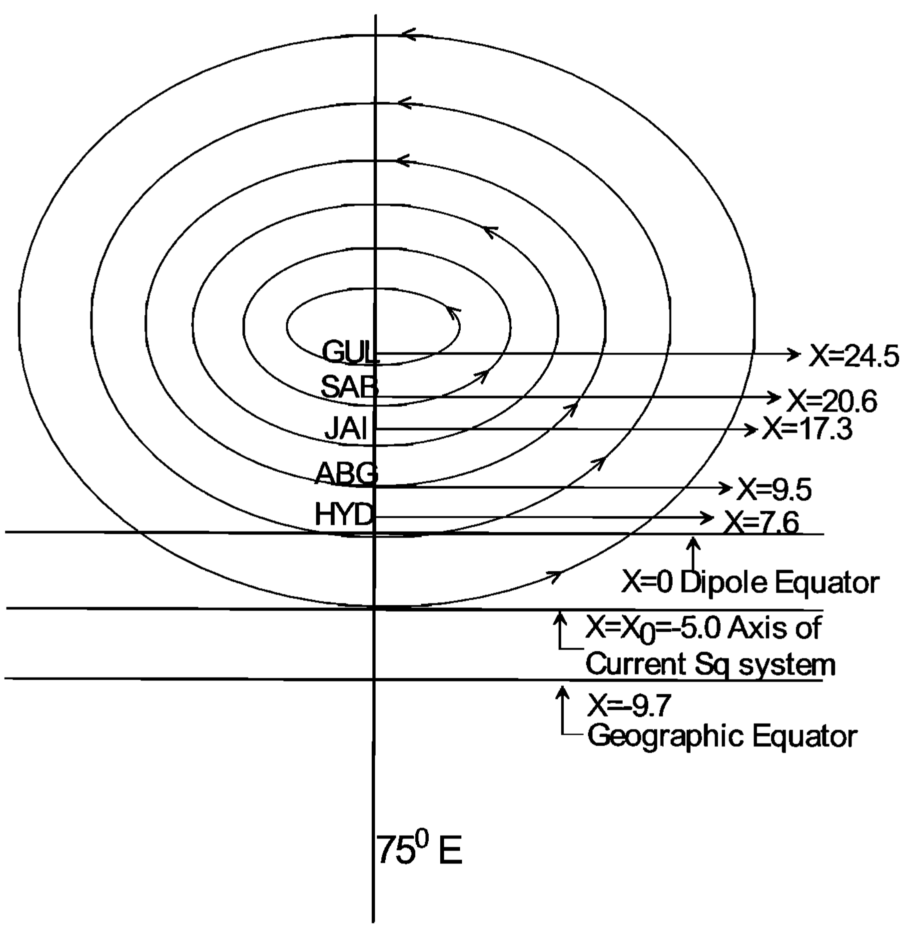

Figure 1. A schematic diagram showing the Sq current system along with the coordinates of the stations used in the 75°E Indian chain of geomagnetic observatories (see Table 1). Shillong is ~ 15° east of this chain and is not shown here.

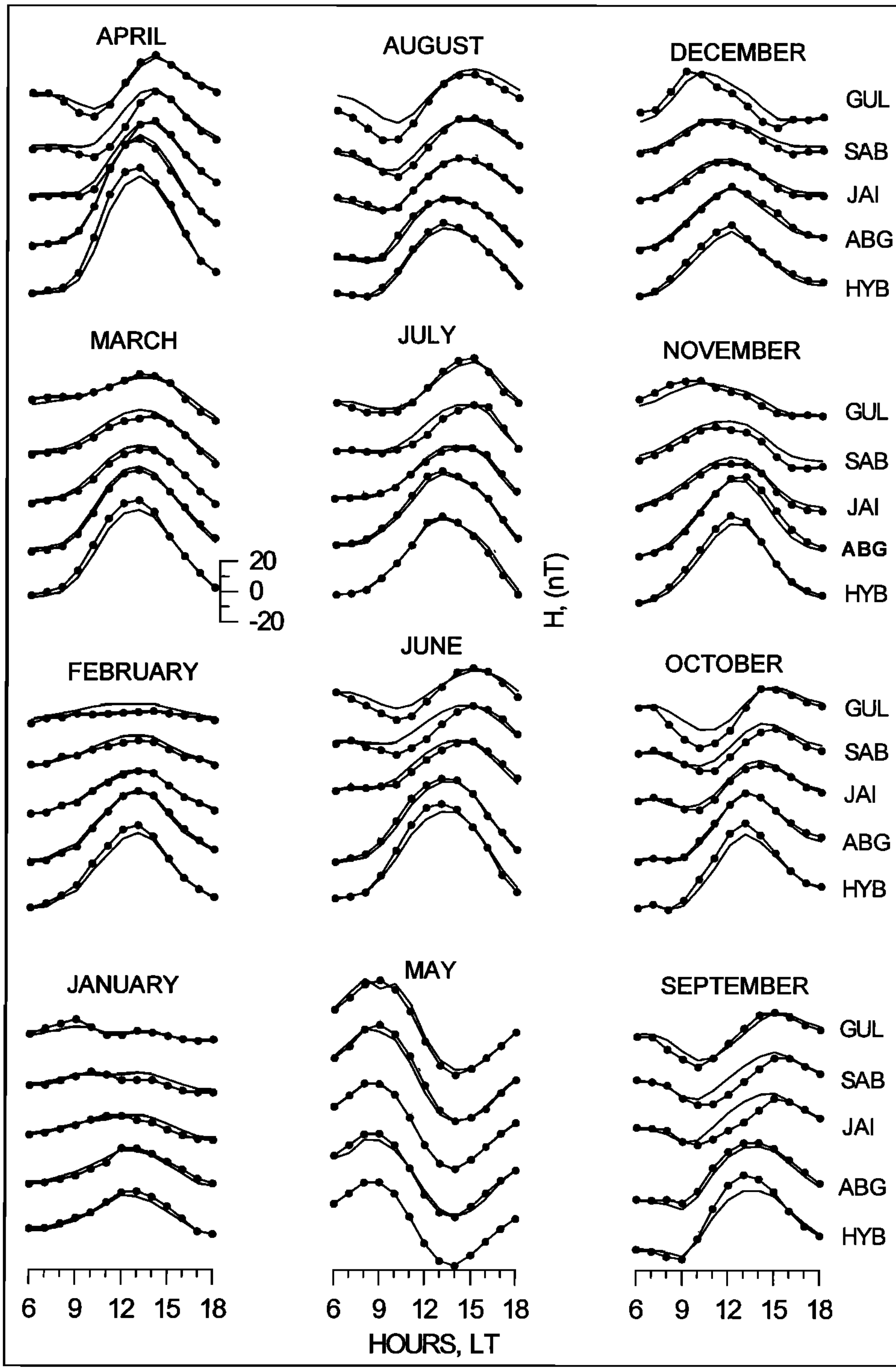

Figure 2. Daily variation of H at all the observatories for each of the calendar months of 1978 along with the values calculated using the fit. The lines with circles show the observed daily variation, and the solid lines show the calculated values.

out, a is taken as 62 and x_0 as -5.0°. The values of a and x_o selected are consistent with those determined for the Indian zone by *Onwumechili et al.* [1996]. The center of the Sq current system lies between the dipole and geographic equators but closer to the geographic axis. Once a and x_0 are fixed, (1) becomes linear in K and $K\alpha$, and these can be obtained for each local time by the least squares method using the data from the five observatories listed in Table 1. However, before finally accepting the values, we confirmed that the quality of fitting is not very sensitive to the choice of the values x_0 and a. We, however, do not rule out the possibility that the choice of a and x_0 could still bias the estimates of K and α, eventhough the quality of fit is insensitive to it. The bias, even if present, will not, however, in any way change the conclusions reached here.

In Figure 2 we reproduce the daily variation of H at all the observatories (lines with circles) for each of the calendar months

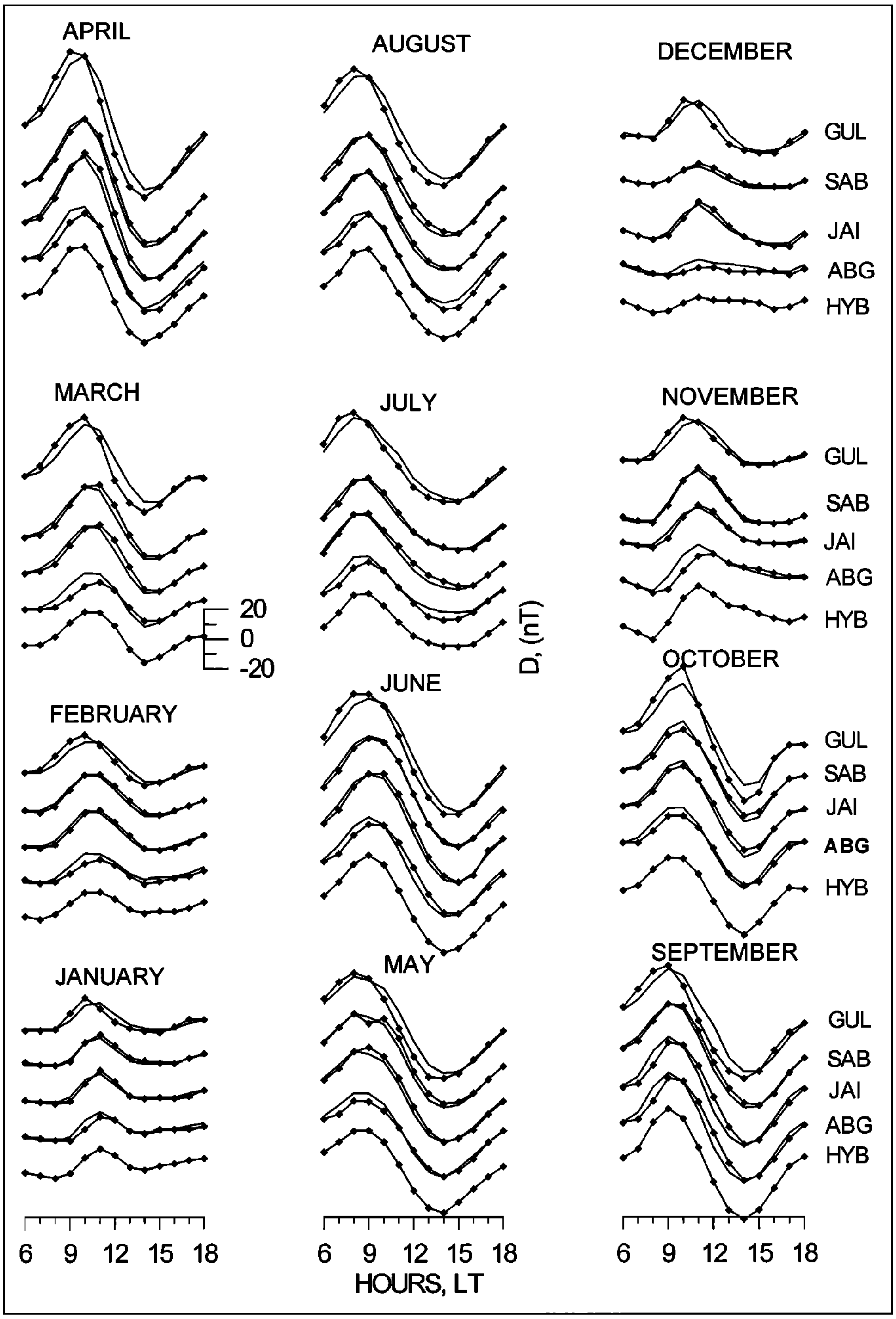

Figure 3. Daily variation of D at all the observatories for each of the calendar months of 1978 along with the values calculated using the fit. The lines with circles show the observed daily variation, and the solid lines show the calculated values using the best fit for $\partial K/\partial y$.

along with the values calculated using the fit (solid line). The fit is excellent for most of the stations in all the months. There are some discrepancies at Gulmarg during a few hours in August and October. The computed H values also differ from the actual values at Jaipur and Sabhawala in the afternoon hours in September. These minor problems may be overlooked, considering the overall quality of the fit.

Having obtained the values of K and α for chosen a and x_0, we next estimate the y derivatives of K and α using the D data. The spatial derivatives of a and x_0 are zero as a and x_0 are fixed at 62 and -5.0°, respectively. Thus only $\partial K/\partial y$ and $\partial \alpha/\partial y$ have to be evaluated from (4). The constant is automatically eliminated, as the reference field D at Hyderabad has been subtracted out. Here $\partial K/\partial y$ and $\partial \alpha/\partial y$ are obtained through a least squares fitting procedure from (4) using D_K and D_α defined in (5a) and (5b), respectively. D_K and D_α have to be evaluated for each hour separately using K and α computed from the fit to the H data for that hour.

The basic input is the difference between D at a given station and the corresponding D at Hyderabad at the same hour. It should be noted that there are effectively only four values available for each hour from which the two derivatives have to be obtained by the least squares fitting procedure. These derivatives are then used to recompute the D variations at all latitudes by adding the hourly D at Hyderabad at each hour as the constant in (4).

The daily variations of D, observed (line with circles) and computed (solid line), are presented in Figure 3 for each of the months. The observed and computed variations match perfectly at Hyderabad for obvious reasons. On the whole, the observed and computed values match quite well. Gulmarg is the station where the two deviate most from each other. This could be due to basic limitations of the model in describing the fields at these latitudes or to limited data being available for the fitting procedure.

4. Is the *Sq* Current System Stationary?

In section 3 we showed that the longitudinal gradients of K and α could be obtained for each hour using D variations from the Indian chain. Since we have also obtained from the H variations the K and α at each hour, we can get the time variations of K and α centered at each hour. If the two are equal, we can conclude that the current system is stationary. If they differ widely, we can doubt the validity of a stationary model of the *Sq* current system. In the next paragraph we explain the procedure with a simple example.

For 1000 LT in the month of January, least squares fit for (1) yielded $K = 19.5$ and $\alpha = -1.5$. Using these values in (4) and (5a)-(5d), the least squares fit yielded a value of 8.9 for $\partial K/\partial y$. Also using the value of $K = 12.2$ for hour 9 and $K = 28.9$ for hour 11, the central difference formula yielded $\partial K/\partial t = 8.4$ at 1000 LT. A similar methodology was used for α. The derivatives represent change per hour (or, equivalently, change per 15° longitude).

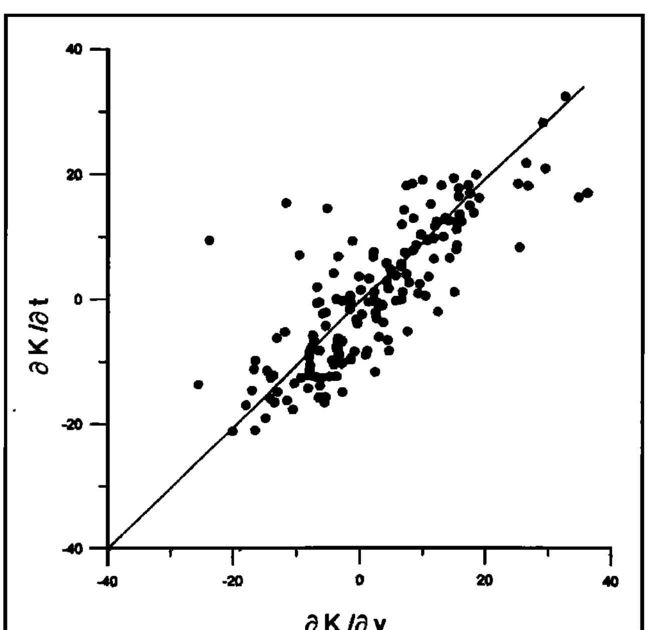

Figure 4. A scatterplot of $\partial K/\partial y$ and $\partial K/\partial t$, where both are expressed in units of change of K per 15° longitude.

Table 2. Correlation Between Temporal and Longitudinal Gradients

Month	Correlation
January	0.83
February	0.87
March	0.90
April	0.98
May	0.92
June	0.91
July	0.60
August	0.67
September	0.56
October	0.49
November	0.97
December	0.95
Annual	0.80

The time derivatives were evaluated for each local time for each month. At the end points (0600 and 1800 LT) the forward and backward difference formula was used instead of the central difference formula mentioned in the example above.

In Figure 4 we depict a scatterplot of $\partial K/\partial y$ versus $\partial K/\partial t$, where both coordinates are expressed in the units of change per 15° longitude. The scatterplot clearly brings out the linear relationship between the parameters, the correlation coefficient being as high as 0.8, which, with 155 points used, is significant at the 99.9% confidence level. D variations were also evaluated using the time derivative in place of the longitudinal derivative in Figure 3, but results are not shown because the values are very close to the former and the diagram would get cluttered.

To demonstrate the persistence of this relationship between the y and t derivatives, the correlation coefficient for each month and for the year are presented in Table 2. Only in the month of October does the significance level of the correlation between $\partial K/\partial y$ and $\partial K/\partial t$ fall below the 95% mark. It may be noted that the fits were also less satisfactory for the months of September and October. The correlation coefficient is significant at better than 99% in 9 out of the 12 months in the year under study. In 7 of these months the confidence level is better than 99.9%. All of these facts demonstrate that the time variations and longitude variations vary in a similar fashion.

Table 2 shows a decidedly poorer correlation in the months from July to October. Figure 3 also shows that the fits are less satisfactory in these months. While it is difficult to provide a definitive explanation for this, it is possible to speculate that the position of the axis was farther north because of summer conditions in the Northern Hemisphere persisting until late October. A wrong choice of x_0 or a can be expected to provide a less reliable determination of the *Sq* parameters. On the other hand, if x_0 and a were also varied, by only 5 points, the statistical reliability and objectivity of the analysis would have to be sacrificed.

Finally, a closer look at the scatterplot in Figure 4 does suggest that the distribution of points is not symmetrical about the 45° line. This may be because a small bias could be introduced by the initial choice of a and x_0, which could be a subject for future investigation.

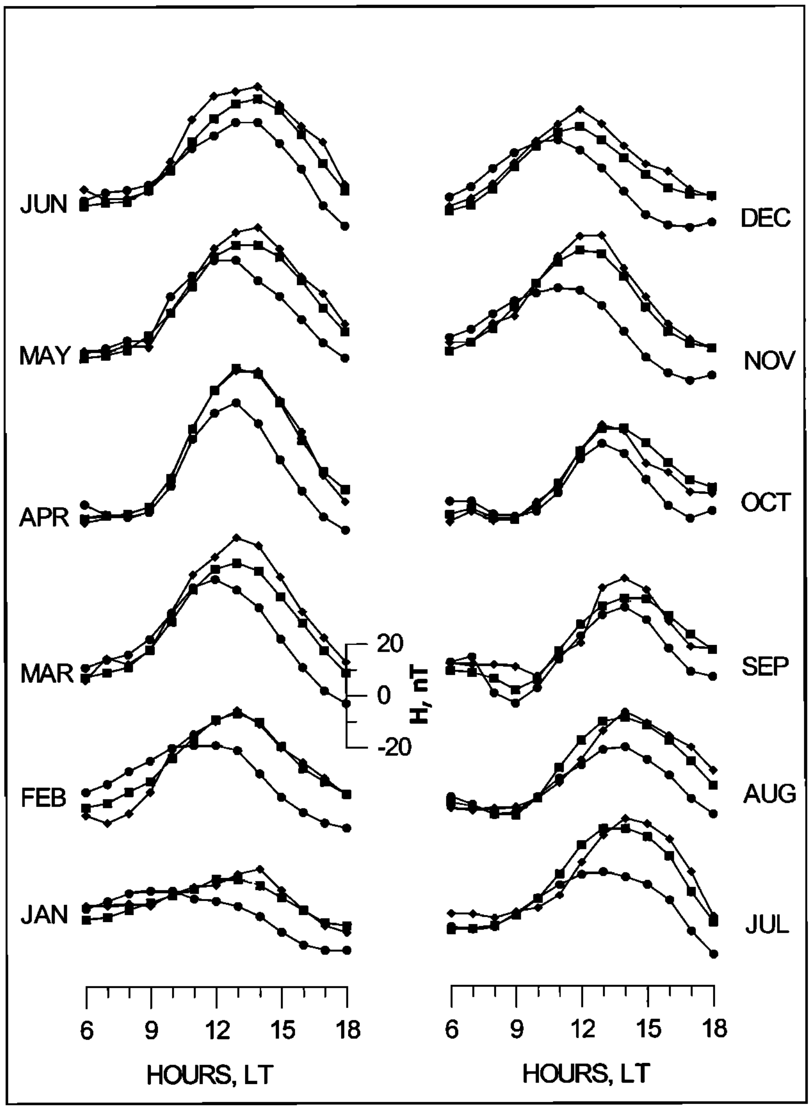

Figure 5. Daily variations of H at Shillong for each of the calendar months of 1978. The lines with circles show the observed daily variation, and lines with diamonds show the calculated values at the position of Shillong. The diurnal variations calculated for a station at the latitude of Shillong but located in the 75°E longitude sector are shown by lines with squares.

For the sake of completeness a scatterplot of $\partial\alpha/\partial y$ versus $\partial\alpha/\partial t$ was also generated. It was found that the changes in α are less significant and there was a cluster of points around the origin. A linear relationship was suggested, though not as clearly as in K.

Finally, the result should not be overinterpreted, leading to a conclusion that longitudinal variations are absent in the Sq current profiles. Our results only provide local longitudinal gradients in the current intensity, and a highly anomalous region to the east or west of the chain is not ruled out. This is effectively brought out when we use the longitudinal gradients in K and α calculated for each month to compute the value of K and α at the longitude of Shillong (SHL) and determine the diurnal pattern in H at that station. The results are presented in Figure 5. The actual observations at Shillong (line with circles) show that the diurnal pattern is highly suppressed at the station. The other two curves representing the computed field for the actual position of Shillong (line with diamonds) and the field computed for the latitude of Shillong at the 75°E meridian (line with squares) do not reproduce this suppression. At the latitude of Shillong the H variations pick up only in the afternoon hours, and it is during these times that the inadequacies of the fit are best brought out. This may suggest that Shillong is in a zone of highly anomalous subsurface conductivity.

5. Summary and Conclusions

In this paper we have augmented a model developed by *Onwumechilli* [1967] to obtain a prescription for determining the longitudinal gradients from the H and D data collected in a single longitude belt. The model is used to determine the parameters and their gradients using the data from the Indian geomagnetic observatory chain. Since the model is applied to determine the model parameters for each hour, both the temporal variations, as seen by the observatory chain, and the longitudinal gradients are estimated for each hour. The comparison shows that the time and longitudinal variations of the current strength have a strong linear relationship. This suggests that the *Sq* variations in the Indian region are at least locally stationary and the use of local time as proxy for longitude by earlier workers [*Campbell et al.*, 1993] is justified.

We believe that the methodology developed here must be tested with larger data samples collected from different regions of Earth to confirm how robust the technique works in practice. Though a particular model has been used here for practical convenience, the basic methodology can and should be used with a wider range of models. It will also be interesting to include the internal image current in future calculations. The scheme described here may be most effectively used to study the equatorial electrojet, provided a reasonable latitudinal chain of geomagnetic observatories is available within the elctrojet belt.

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