Role of *E* region conductivity in the development of equatorial ionospheric plasma bubbles

Archana Bhattacharyya

Indian Institute of Geomagnetism, Navi Mumbai, India

Received 31 October 2003; revised 28 January 2004; accepted 23 February 2004; published 19 March 2004.

[1] A transmission line analogy, which has been used earlier in a linear theory for the development of equatorial plasma bubbles, is extended to study the effect of *E* region conductivity on non-linear evolution of the plasma bubbles. For this, a set of mode coupling equations are used to describe non-linear development of equatorial plasma bubbles in the presence of field-aligned currents which couple the equatorial *F* region with conjugate *E* regions. For a three mode system, these non-linear equations yield a condition for unstable fixed states. This condition shows that *E* region resistivity together with *F* region polariz-ability introduces another time scale in the non-linear evolution of equatorial bubbles, and this is the time scale for discharging the bubbles. *INDEX TERMS:* 2415 Ionosphere: Equatorial ionosphere; 2439 Ionosphere: Ionospheric irregularities; 2471 Ionosphere: Plasma waves and instabilities. **Citation:** Bhattacharyya, A. (2004), Role of *E* region conductivity in the development of equatorial ionospheric plasma bubbles, *Geophys. Res. Lett.*, *31*, L06806, doi:10.1029/2003GL018960.

1. Introduction

[2] A transmission line analogy for the development of equatorial *F* region plasma bubbles was put forward by *Bhattacharyya and Burke* [2000], where field-aligned currents (FACs) carried by oppositely propagating Alfvén waves that are launched in the equatorial *F* region as soon as a plasma bubble starts to grow there, couple the equatorial *F* region with the conjugate *E* regions. Along geomagnetic field lines connecting the equatorial *F* region with conjugate *E* regions, wherever the Pedersen conductivity is negligible, field-aligned currents close through polarization currents associated with Alfvén waves. *Keskinen et al.* [1998] were the first to introduce ion-polarization currents in simulations of the non-linear evolution of equatorial bubbles due to growth of the electrostatic Rayleigh-Taylor (R-T) instability, using a three-layer model. This model considered *E* region Pedersen conductivity effects, but FACs did not appear explicitly in the model since magnetic flux tube integration technique was used. Recently, three-dimensional non-linear evolution of equatorial spread-F (ESF) bubbles has been studied in a numerical simulation of the electrostatic R-T instability [*Keskinen et al.*, 2003], where finite parallel conductivity effects were found to slow down the evolution of the bubbles compared to the two-dimensional case, with evolution of the plasma bubble on the topside of the equatorial *F* region taking about 1 hour.

[3] The suggestion that FACs may be carried by Alfvén waves generated in the equatorial *F* region was first put forward by *Aggson et al.* [1992] who had observed simultaneous electric and magnetic field fluctuations, associated with up drafting equatorial bubbles, using DE 2 satellite data. Electromagnetic plasma waves have also been detected within and around density depletions in the topside equatorial *F* region by the electric and magnetic field sensors of the Extremely Low Frequency Wave Analyzer (ELFWA) instrument on the Combined Release and Radiation Effects Satellite (CRRES) [*Koons et al.*, 1997]. An electromagnetic version of the R-T instability could provide a possible explanation for these observations.

[4] In the transmission line analogy, parallel conductivity is assumed to be infinite, so that there is no parallel electric field. In a linear theory of development of equatorial bubbles, based on this analogy, the *E* region boundary conditions imposed stringent conditions on the growth of the plasma bubble by requiring that conjugate *E*-region conductivities in the two hemispheres should vanish simultaneously for a non-vanishing perturbation in equatorial *F*-region plasma density [*Bhattacharyya and Burke*, 2000]. This condition provided an explanation for the observation that small angles between the sunset terminator and the magnetic meridian seem to favor the development of equatorial bubbles as seen from equatorial scintillations and range spread *F* data [*Tsunoda*, 1985; *Abdu et al.*, 1992] and satellite observations of equatorial bubbles [*Huang et al.*, 2001]. In order to estimate how small the *E*-region conductivity should be for evolution of the equatorial plasma bubbles in the top-side of the *F* region, it is necessary to extend the transmission line analogy to at least a weakly non-linear situation.

[5] In the present paper, the transmission line analogy is extended to a set of mode coupling equations which could describe non-linear evolution of equatorial bubbles. Basic equations which describe the model, and assumptions are described in section 2. In the next section, scaling appropriate to the problem, is used to derive equations satisfied by perturbations in plasma density, electric, and magnetic fields, both in the equatorial *F* region and outside this generator region, for a three mode system. Role played by the coupling between equatorial *F* region and conjugate *E* regions, in relating variations in field aligned currents with electric field fluctuations, is discussed in section 4. On using this information, the mode coupling equations reduce to the form of the Lorenz equations [*Lorenz*, 1963], and yield a condition for the onset of chaotic evolution of equatorial plasma bubbles. In the final section, the results

0094-8276/04/2003GL018960$05.00

obtained are discussed in the light of certain observations pertaining to equatorial ionospheric bubbles.

2. Model and Basic Equations

[6] The geomagnetic field $\vec{B}_0$ is considered to be uniform and northward. A Cartesian coordinate system is used in which the x-, y-, and z- axes point vertically upward, eastward, and northward respectively. A vertical gradient in the background plasma density n_0 (x) is assumed. For simplicity, the ambient electric field is taken to be zero, and ions are assumed to be cold (T_i = 0). Further, electron-neutral collisions are neglected, while electron-ion collisions are included in the theory [*Huba et al.*, 1985]. Basic equations of the theory are the continuity and momentum equations for ions and electrons:

$$\frac{\partial n_j}{\partial t} + \vec{\nabla}\cdot\left(n_j\vec{v}_j\right) = 0; \quad j = e, i \tag{1}$$

$$0 = -\frac{e}{m_e}\left[\vec{E} + \vec{v}_e \times \vec{B}\right] - \frac{k_B T_e}{m_e}\frac{\vec{\nabla} n_e}{n_e} - \nu_{ei}[\vec{v}_e - \vec{v}_i] \tag{2}$$

$$\frac{\partial \vec{v}_i}{\partial t} + \left(\vec{v}_i\cdot\vec{\nabla}\right)\vec{v}_i = \frac{e}{m_i}\left[\vec{E} + \vec{v}_i \times \vec{B}\right] - \nu_{ie}(\vec{v}_i - \vec{v}_e) - \nu_{in}\vec{v}_i + \vec{g} \tag{3}$$

where the terms have their usual meanings; and Maxwell's equations. In equation (3), ion inertia is retained. Perturbations in plasma density, electric and magnetic fields about their equilibrium values $n_0(x)$, 0, and $\vec{B}_0$ respectively, are n_1, $\vec{E}_1$, and $\mu\vec{H}_1$. In the transmission line picture, geomagnetic flux tubes are considered to be perfect conductors, so that $\vec{E}_1$, and $\vec{H}_1$ are two-dimensional perturbations in the x-y plane:

$$\vec{E}_1 = \vec{F}(x,y)\,V(z,t) \tag{4}$$

$$\vec{F}(x,y) = -\vec{\nabla}_\perp\,\phi(x,y) \tag{5}$$

$$\vec{H}_1 = \vec{G}(x,y)I(z,t) \tag{6}$$

$$\vec{\nabla}\cdot\vec{G} = 0 \tag{7}$$

There is a field-aligned current associated with $\vec{H}_1$:

$$j_z = \left(\vec{\nabla}\times\vec{G}\right)_z I(z,t) \tag{8}$$

Reflection of Alfvén waves from the E region loads, as well as the generator in the equatorial F region, are considered to be equivalent to closure of the FACs carried into these regions, as in the case of magnetosphere-ionosphere coupling [*Kan and Sun*, 1985]. The equatorial F region generator is assumed to be a constant-current source such that $I(0, t)$ = 0 [*Bhattacharyya and Burke*, 2000].

3. Three Mode System

[7] The basic equations given in the previous section, are cast in dimensionless forms using appropriate transformations. In a linear theory for the development of equatorial plasma bubbles using the transmission line analogy, it was shown that the transverse electric and magnetic field fluctuations were associated with a set of Alfvén waves which propagated parallel and anti-parallel to $\vec{B}_0$ with the Alfvén speed V_A (= $B_0/[\mu_0 n_0 m_i]^{1/2}$) [*Bhattacharyya and Burke*, 2000]. Hence, while x- and y- coordinates are scaled by the density gradient scale length L (= $n_0\,[\partial n_0/\partial x]^{-1}$), a different scaling is used to transform the z-coordinate:

$$\frac{1}{L}(x,y) = (x', y'); \quad \frac{z}{V_A}\sqrt{\frac{g}{L}} = z' \tag{9}$$

The other transformations that are used are:

$$\frac{n_1}{n_0} = \tilde{n}; \quad t\sqrt{\frac{g}{L}} = t'; \quad \nu_i\sqrt{\frac{L}{g}} = \tilde{\nu}_i \tag{10}$$

$$\frac{1}{B_0}\sqrt{\frac{L}{g}}\frac{\phi V}{L^2} = \tilde{\phi}\,\tilde{V}; \quad \frac{\mu_0}{B_0}\,GI = \tilde{G}\,\tilde{I}; \tag{11}$$

$$\sqrt{\frac{L}{g}}\frac{D_e}{L^2} = \tilde{D}_e \quad \frac{V_A}{L}\sqrt{\frac{L}{g}} = \tilde{V}_A \tag{12}$$

where $\nu_i = \nu_{ie} + \nu_{in}$ is the ion collision frequency with both neutrals and electrons, and $D_e = \nu_{ei}k_BT_e/m_e\Omega_e^2$ is the electron diffusion coefficient.

[8] With the above scaling, and electron and ion velocities derived from equations (2) and (3) respectively, the electron continuity equation yields

$$\frac{\partial\tilde{n}}{\partial t'} = \tilde{D}_e\,\nabla'^2\tilde{n} + \frac{\partial\tilde{\phi}}{\partial y'}\tilde{V}(0,t') + \vec{\nabla}'\tilde{n}\cdot\left(\vec{\nabla}'_\perp\tilde{\phi}\times\hat{z}\right)\tilde{V}(0,t') \tag{13}$$

and the current continuity equation in the equatorial F region reduces to:

$$\nabla'^2_\perp\tilde{\phi}\frac{\partial\tilde{V}(0,t')}{\partial t'} = -\left[\tilde{V}^2(0,t')\left(\hat{z}\times\vec{\nabla}'_\perp\tilde{\phi}\right)\cdot\vec{\nabla}'_\perp + \tilde{\nu}_i\tilde{V}(0,t')\right]\nabla'^2_\perp\tilde{\phi} + \frac{\partial\tilde{n}}{\partial y'} + \tilde{V}_A\left(\vec{\nabla}'\times\vec{\tilde{G}}\right)_z\frac{\partial\tilde{I}(z',t')}{\partial z'}\bigg|_{z'=0} \tag{14}$$

In the derivation of equation (14), for calculating the current perpendicular to the geomagnetic field, ion velocity is derived from equation (3) by an iterative procedure. In this procedure the zeroth order ion velocity is obtained by neglecting ion inertia, and this is then used to obtain the ion velocity correct to first order when ion inertia is retained as in equation (3). In equation (14), the term on the left hand side and the first term within the square brackets on the right hand side arise due to ion inertia, and the last term is the contribution of field-aligned currents to the current continuity equation. Evolution of a three-mode system is studied using the above non-linear equations, where perturbations in the normalized electric field, density, and magnetic field, are considered to have, respectively, the following forms:

$$\tilde{\phi}(x', y')\tilde{V}(z', t') = \sin(\tilde{k}\,x')\sin(\tilde{k}\,y')\tilde{V}(z', t') \tag{15}$$

$$\tilde{n}_1 = \tilde{k}\,\tilde{n} = Y_1\sin(\tilde{k}\,x')\cos(\tilde{k}\,y') + Z_1\sin(2\,\tilde{k}\,x') \tag{16}$$

$$\begin{aligned}\vec{\tilde{G}}(x', y')\tilde{I}(z', t') = &\left[-\tilde{k}\sin(\tilde{k}\,x')\cos(\tilde{k}\,y')\hat{x}\right.\\ &\left.+\ \tilde{k}\cos(\tilde{k}\,x')\sin(\tilde{k}\,y')\hat{y}\right]\ \tilde{I}(z', t')\end{aligned} \tag{17}$$

$\tilde{V}(z', t')$ and $\tilde{I}(z', t')$ are related through coupling between the equatorial F region and conjugate E regions.

[9] Outside the generator region gravity and Pedersen conductivity may be neglected. Therefore the current continuity equation in this region is obtained by dropping the terms containing $\tilde{\nu}_i$ and $\partial\tilde{n}/\partial y'$ in equation (14), and for the three mode system with $\tilde{\phi}(x', y')$ and $\vec{\tilde{G}}(x', y')$ given by equations (15) and (17), it reduces to

$$\frac{\partial\tilde{V}(z', t')}{\partial t'} = \tilde{V}_A\frac{\partial\tilde{I}(z', t')}{\partial z'} \tag{18}$$

Also $\vec{\nabla} \times \vec{E} = -\ \partial\vec{B}/\partial t$ yields

$$\frac{\partial\tilde{V}(z', t')}{\partial z'} = \tilde{V}_A\frac{\partial\tilde{I}(z', t')}{\partial t'} \tag{19}$$

General solutions of these equations satisfied by $\tilde{V}(z', t')$ and $\tilde{I}(z', t')$ in the region between the equatorial F region generator and conjugate E region loads may now be written as

$$\tilde{V}(z', t') = V_+ f(t' - z') + V_- f\ (t' + z') \tag{20}$$

$$\tilde{I}(z', t') = I_+ f(t' - z') + I_- f\ (t' + z') \tag{21}$$

where, in the non-linear regime, $f(t' \pm z')$ may no longer be considered to be plane waves propagating anti-parallel and parallel to $\vec{B}_0$, as was the situation for the linear case [*Bhattacharyya and Burke*, 2000]. Rather, they represent finite amplitude fluctuations in electric and magnetic fields, which propagate between the equatorial F region and conjugate E regions with the Alfvén speed.

4. *E* Region Boundary Conditions

[10] In the E region, polarization current is negligible in comparison to the Pedersen current that flows there. Hall current, although not negligible, is not considered here. Hence, field-aligned currents which flow into or out of the E regions are closed by the Pedersen currents that flow in the E regions. Current continuity in the northern and southern hemispheric E regions, located at $z' = \pm l'$ respectively, yield the following conditions [*Bhattacharyya and Burke*, 2000]:

$$\begin{aligned}\tilde{\Sigma}^E_{P(N,S)}[V_+ f(t'\ \mp\ l') + V_- f(t'\ \pm\ l')] = \mp\,[&I_+ f(t'\ \mp\ l')\\ &+ I_- f(t'\ \pm\ l')]\end{aligned} \tag{22}$$

where $\tilde{\Sigma}^E_{P(N,S)} = \mu_0\sqrt{gL}\ \Sigma^E_{P(N,S)}$ are the dimensionless field-line integrated, northern and southern E region Pedersen conductivities, respectively. For typical nighttime equatorial ionospheric conditions, $n_0 \approx 4 \times 10^{11}$ m^{-3}, $V_A \approx 300$ km/s, $L \approx 10$ km, and the length of the geomagnetic field line from the equatorial F region, which is assumed to be at an altitude of about 500 km, to conjugate E regions may be taken as $l \approx 1600$ km, such that l' (= $[l/V_A]$ $[g/L]^{1/2}$) ≈ 0.2. Since $l' \ll 1$, both sides of equation (22) may be expanded in a power series in l' to obtain:

$$\tilde{\Sigma}^E_{P(N,S)}[V_+ + V_-]\,f(t') = \mp[I_+ + I_-]\,f(t') \mp \left(\frac{\partial\tilde{I}(z', t')}{\partial z'}\right)\bigg|_{z'=0}(\pm l') \tag{23}$$

Only the lowest order term is retained on the left hand side while the next order term is also retained on the right hand side, because the assumption of a constant current source in the equatorial F region implies that $I_+ + I_- = 0$. Therefore, to lowest order, a non-zero electric field perturbation, and hence, a non-vanishing density perturbation, requires that $\Sigma^E_{P\ N} = \Sigma^E_{P\ S} = 0$ [*Bhattacharyya and Burke*, 2000]. Retention of the next order term on the R.H.S. in equation (23) allows the existence of non-vanishing perturbations in the presence of non-zero E region conductivity. In this situation, equation (23) requires that $\tilde{\Sigma}^E_{P\ N} = \tilde{\Sigma}^E_{P\ S}$ (= $\tilde{\Sigma}^E_P$) and further yields

$$\frac{\partial\tilde{I}(z', t')}{\partial z'}\bigg|_{z'=0} = -\frac{1}{l'}\tilde{\Sigma}^E_P\ \tilde{V}(0, t'). \tag{24}$$

[11] This relation is now used in equation (14), such that for the three mode system, the following coupled, non-linear differential equations are obtained from equations (13) and (14):

$$\dot{X} = -\sigma X + \sigma Y \tag{25}$$

$$\dot{Y} = rX - Y - XZ \tag{26}$$

$$\dot{Z} = -2Z + XY \tag{27}$$

where $\sigma = \hat{\nu}_i/2\ \hat{D}_e$ with $\hat{\nu}_i = \tilde{\nu}_i + \tilde{\sum}^E_P\tilde{V}_A/l'$; $r = [4\ \hat{D}_e\hat{\nu}_i]^{-1}$ with $\hat{D}_e = \tilde{k}^2\tilde{D}_e$; $X = X_1\sqrt{r\sigma}$ with $X_1 = \tilde{k}^2\tilde{V}\ (0,\ t')$; $Y = Y_1\sqrt{r\sigma}/2\ \hat{\nu}_i$; $Z = r\,Z_1$; and the dots represent derivatives with respect to a time variable $\tau = 2\ \hat{D}_e t'$. These equations are identical to those obtained by *Huba et al.* [1985] for an electrostatic interchange instability, except that $\hat{\nu}_i$ has an additional term which depends on Σ_P^E. Since these equations correspond exactly to the Lorenz equations [*Lorenz*, 1963], the analysis that follows is well known. For $r < 1$, $X = Y = Z = 0$ is the only stable steady state, whereas for $r > 1$, two additional fixed states are allowed:

$$X = Y = \pm[2(r-1)]^{1/2}; \quad Z = r - 1 \tag{28}$$

Analysis of the linear stability of these fixed states yields a critical value r_c of r for instability: $r_c = \sigma\,(\sigma + 5)/(\sigma - 3)$, so that for $1 < r < r_c$ or $\sigma < 3$ these fixed states are stable and no chaotic evolution of the bubbles would take place. For the equatorial F region, spatial scale of the plasma density variations may be taken to be $\sim$1 km, so that σ takes on a value exceeding 10^3. Hence, the condition for unstable fixed states becomes $r > r_c \approx \sigma$, which translates to

$$\nu_i + \frac{\mu_0 V_A^2 \Sigma_P^E}{l} < \sqrt{\frac{g}{2L}} \tag{29}$$

Thus the E region conductivity together with the equatorial F region polarizability introduces a new time scale in the phenomenon.

5. Conclusion

[12] The additional time scale introduced in equation (29) by a non-zero E region Pedersen conductivity represents the time required to discharge the bubble. With $\Sigma_P^E \approx 1$ mho, $l \approx 1600$ km, and $V_A \approx 300$ km/s, it is seen that $\mu_0 V_A^2\ \Sigma_P^E/l \approx 0.01\ \text{s}^{-1}$. This is comparable to the value of $(g/2L)^{1/2} \approx 0.02\ \text{s}^{-1}$ for $L = 10$ km. For the electrostatic process considered by *Huba et al.* [1985], the condition for chaotic evolution of the equatorial bubbles was simply $\nu_i < (g/2\ L)^{1/2}$, which required that the altitude of the equatorial F region peak be greater than 400–500 km. The effect of non-vanishing E region conductivity is to put a more stringent condition on the altitude of the equatorial F region, which would need to be greater for higher E region conductivity, in order that the equatorial bubbles evolve chaotically. Using parameter values relevent to the post-sunset equatorial ionosphere, perturbations associated with the fixed states are estimated to be $n_1/n_0 \approx 0.01$, $E_1 \approx 5 \times 10^{-3}$ mV/m and $B_1 \approx 0.01$ nT. The results presented here are based on a number of simplifying assumptions. A three-dimensional simulation of this electromagnetic version of the R-T instability needs to be carried out to obtain more realistic results.

[13] Although the three-mode system considered here is an over-simplification, the stable fixed states obtained when the condition for instability is not met may correspond to the bottom-side equatorial F region irregularities which have been observed to be confined to a limited altitude extent of thickness 50–100 km [*Valladares et al.*, 1983; *Hysell*, 1998]. Another result obtained from long term radar observations at Jicamarca, which is of relevance here, is that the threshold vertical drift velocity for the generation of strong early night ESF irregularities increases with solar flux [*Fejer et al.*, 1999]. As the ion-neutral collision frequency increases with solar flux, the condition given in equation (29) would require a higher altitude of the equatorial F region with increasing solar flux. Earlier observations of magnetic field fluctuations associated with equatorial plasma bubbles [*Aggson et al.*, 1992; *Koons et al.*, 1997], the more recent observations of this nature made using the total magnetic field data from CHAMP satellite [*Lühr et al.*, 2002], and the present work, call for a detailed study of these magnetic field fluctuations using the vector magnetic field data from CHAMP.

References

Abdu, M. A., I. S. Batista, and J. H. A. Sobral (1992), A new aspect of magnetic declination control of equatorial spread *F* and *F* region dynamo, *J. Geophys. Res.*, *97*, 14,897–14,904.

Aggson, T. L., et al. (1992), Equatorial bubbles updrafting at supersonic speeds, *J. Geophys. Res.*, *97*, 8581–8590.

Bhattacharyya, A., and W. J. Burke (2000), A transmission line analogy for the development of equatorial ionospheric bubbles, *J. Geophys. Res.*, *105*, 24,941–24,950.

Fejer, B. G., E. R. de Paula, and L. Scherliess (1999), Effects of the vertical plasma drift velocity on the generation and evolution of equatorial spread *F*, *J. Geophys. Res.*, *104*, 19,859–19,869.

Huang, C. Y., et al. (2001), DMSP observations of equatorial plasma bubbles in the topside ionosphere near solar maximum, *J. Geophys. Res.*, *106*, 8131–8142.

Huba, J. D., et al. (1985), Ionospheric turbulence: Interchange instabilities and chaotic fluid behavior, *Geophys. Res. Lett.*, *12*, 65–68.

Hysell, D. L. (1998), Imaging coherent sctter radar studies of bottomside equatorial spread *F*, *J. Atmos. Sol. Terr. Phys.*, *60*, 1109–1122.

Kan, J. R., and W. Sun (1985), Simulation of the westward traveling surge and Pi 2 pulsations during substorms, *J. Geophys. Res.*, *90*, 10,911–10,922.

Keskinen, M. J., S. L. Ossakow, S. Basu, and P. J. Sultan (1998), Magnetic-flux-tube-integrated evolution of equatorial ionospheric plasma bubbles, *J. Geophys. Res.*, *103*, 3957–3967.

Keskinen, M. J., S. L. Ossakow, and B. G. Fejer (2003), Three-dimensional nonlinear evolution of equatorial ionospheric spread-*F* bubbles, *Geophys. Res. Lett.*, *30*(16), 1855, doi:10.1029/2003GL017418.

Koons, H. C., J. L. Roeder, and P. Rodriguez (1997), Plasma waves observed inside plasma bubbles in the equatorial *F* region, *J. Geophys. Res.*, *102*, 4577–4583.

Lorenz, E. N. (1963), Deterministic nonperiodic flow, *J. Atmos. Sci.*, *20*, 130–141.

Lühr, H., et al. (2002), First in-situ observation of night-time *F* region currents with the CHAMP satellite, *Geophys. Res. Lett.*, *29*(10), 1489, doi:10.1029/2001GL013845.

Tsunoda, R. T. (1985), Control of the seasonal and longitudinal occurrence of equatorial scintillations by the longitudinal gradient in integrated *E* region Pedersen conductivity, *J. Geophys. Res.*, *90*, 447–456.

Valladares, C. E., et al. (1983), Bottomside sinusoidal irregularities in the equatorial *F* region, *J. Geophys. Res.*, *88*, 8025–8042.

A. Bhattacharyya, Indian Institute of Geomagnetism, Kalamboli, New Panvel, Navi Mumbai 410218, India. (archana@iigs.iigm.res.in)