Modulational instability of electron-acoustic waves in the auroral region

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Abstract. Interaction of solar wind with the Earth's magnetic field leads to the formation of the magnetosphere which has several boundary layers like magnetopause, plasmas sheet boundary layer and auroral zone etc. These boundary layers play important role in the distribution of energy extracted from solar wind to various regions of the magnetosphere. The free energy available in the boundary layers in the form of gradients and currents can drive several plasma waves at different scale lengths. Thus, the boundary layer offers an opportunity to study the physics of the micro and macro scale plasma processes. Broadband plasma waves, having wide range of frequencies from lower hybrid to electron plasma frequency and above, have been observed by many satellites on the auroral and cusp field lines. The amplitude of the electric field of these waves can be from a few tens to hundreds of mV/m. These large amplitude waves can decay into kinetic Alfvén waves (KAWs) through parametric processes. Three wave interaction process involving a high frequency pump wave (electron-acoustic wave (EAW)), a kinetic Alfvén wave and another electron acoustic wave as the daughter wave is studied in the auroral region by using multi-fluid approach. Auroral plasma is considered to consist of cold electrons, hot electrons and ions. A nonlinear dispersion relation for the three-wave interaction process is obtained. It is found that the observed amplitudes of the EAWs exceed the threshold required to excite the KAWs. This mechanism may be a plausible candidate for the generation of KAWs in the auroral region. The kinetic Alfvén waves in turn can lead to the local acceleration/heating of the auroral plasma.

Index Terms. Auroral region, electron-acoustic waves, kinetic Alfvén waves, parametric coupling**.**

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1. Introduction

Alfvén waves are the solution of the ideal magnetohydrodynamic (MHD) equations, which assumes that the plasma is frozen in the magnetic field. The pressure and magnetic field perturbations couple over characteristic perpendicular wavelengths $\lambda_\perp = 2\pi / k_\perp$ larger than the ion-acoustic gyroradius $\rho_{gs} = c_s/\omega_{ci}$; where $c_s = (T_e/m_i)^{1/2}$ is the ion-acoustic velocity and $\omega_{ci} = eB_o/m_i c$ is the ion cyclotron frequency. However, when a perturbation develops on a scale comparable to ion-acoustic gyroradius, ρ_{gs}, the system departs from the MHD equations, then, the shear or kinetic Alfvén waves can exist as normal mode of the plasma (Stefant 1970, Lashmore-Davies and May 1972, Hasegawa and Mima 1976). The kinetic regime of the KAWs can be defined by the condition $k_\perp \rho_{gs} \geq 1, 1 << \beta >> m_e / m_i$ (Goertz and Boswell, 1979). Also, MHD equations are not appropriate when perpendicular wavelength is of the order of electron inertial length $\lambda_e = c/\omega_{pe}$, where c is the velocity of light and ω_{pe} is the electron plasma frequency. These waves are characterized by $\omega_{pe}/ck_\perp \leq 1$, $\beta << m_e/m_i$ and are known as shear KAWs. The kinetic Alfvén waves have an electric field component along the ambient magnetic field and can accelerate the particles in the direction of the magnetic field (Hasegawa, 1976). KAWs have been widely observed in the Earth's magnetosphere (Gurnett et al., 1984; Wahlund et al., 1994; Wygant et al., 2002; Takada et al., 2005). These waves may arise due to the presence of intense field-aligned currents (Voitenko et al., 1990) or velocity shears (Mikhailovski and Klimenko, 1980; Lakhina, 1990; Wang et al., 1998). Satellite observations have shown the presence of kinetic Alfvén waves in the auroral region of the Earth's magnetosphere. Also, two-electron population has been observed in the auroral and other regions of the Earth's magnetosphere which supports the electron- acoustic waves that can be excited by the field-aligned currents in the various regions of the magnetosphere. Yukhimuk et al. (1998) have studied the parametric excitation of small-scale kinetic Alfvén waves in magnetized plasma through a three wave-interaction process of upper hybrid wave (pump wave), kinetic Alfvén wave and a daughter upper hybrid wave. In this paper, we study the nonlinear excitation of kinetic Alfvén waves by electron-acoustic waves in a three-component plasma through a parametric process. The plasma is considered to consist of two electrons (cold and hot) species and ions. A nonlinear dispersion relation is obtained in the next section and the results are discussed in section 3.

2. Nonlinear dispersion relation

Let us consider an electron-acoustic (ω_o, $\mathbf{k_o}$) wave propagating in a magnetized plasma, having the ambient magnetic field B_o directed along the z-axis, with electric field $\mathbf{E_o}$ given by

$$\mathbf{E}_0 = \left(E_{ox}\,\mathbf{X} + E_{oz}\,\mathbf{Z}\right)e^{-i(\omega t - k_{ox}x - k_{oz}Z)} + c.c. \quad (1)$$

decays into a low frequency (ω, $\mathbf{k}$) kinetic Alfvén wave (KAW) and a sideband electron acoustic (ω_1, $\mathbf{k_1}$). For the efficient three wave interaction, the following phase matching conditions have to be satisfied:

$$\omega_o = \omega + \omega_1;\ \mathbf{k_o} = \mathbf{k} + \mathbf{k_1} \quad (2)$$

The physics of the problem is as follows: The electron-acoustic pump wave interacts with the low frequency kinetic Alfvén waves and give rise to sideband electron-acoustic wave. The sideband wave again interacts with the pump wave and enhances the low-frequency kinetic Alfvén wave. The multi-fluid MHD equations for the model are given by

$$\frac{\partial \mathbf{v}_\alpha}{\partial \boldsymbol{t}} = \frac{1}{m_\alpha}(q_\alpha \mathbf{E} + \mathbf{F}_\alpha) + \mathbf{v}_\alpha \times \boldsymbol{\omega}_{c\alpha} - v_{t\alpha}^2 \frac{\nabla n_\alpha}{\boldsymbol{n}_\alpha} \quad (3)$$

$$\frac{\partial n_\alpha}{\partial t} + \nabla.(n_\alpha \mathbf{v}_\alpha) = 0, \quad (4)$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \quad (5)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \quad (6)$$

$$\nabla.\mathbf{E} = 4\pi\rho, \quad (7)$$

where the ponderomotive force $\mathbf{F}_\alpha$ is given by

$$\mathbf{F}_\alpha = \frac{q_\alpha}{\boldsymbol{c}}(\mathbf{v}_\alpha \times \mathbf{B}) - m_\alpha(\mathbf{v}_\alpha.\nabla)\mathbf{v}_\alpha, \quad (8)$$

where $\mathbf{J}=e(n_i\mathbf{v}_i - n_e\mathbf{v}_e)$, $\rho = e(n_i - n_e)$.

The index α=c,h,i stands for cold and hot electrons and ions respectively and $v_{t\alpha}=(T_\alpha/m_\alpha)^{1/2}$ and $\omega_{c\alpha}=q_\alpha B_o/cm_\alpha$ the thermal velocity and cyclotron frequency.

The electric field for the kinetic Alfvén waves is given by

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\frac{\partial A_z}{\partial t}\mathbf{z} \quad (9)$$

where A_z is the vector potential. From equations (3)-(8), we find the following expressions for the perturbed electron and ion densities

$$\frac{n_\alpha^{/}}{n_o} = \frac{e}{T_\alpha}\frac{1}{(1-v_{ph}^2/v_{t\alpha}^2)}[\phi - A + \frac{k_x}{ek_z^2}\frac{\omega}{\omega_{ce}}(\frac{i\omega}{\omega_{ce}}F_{\alpha x} + F_{\alpha y}) - \frac{iF_{\alpha z}}{ek_z}], \quad (10)$$

$$\frac{n_i^{/}}{n_o} = -\frac{e}{T_i}\frac{k_x^2\rho_i^2}{1+k_x^2\rho_i^2}[\phi + \frac{k_z^2\omega_{ci}^2}{k_x^2\omega^2}A], \quad (11)$$

where $\rho_i = v_{ti}/\omega_{ci}$, $A=\omega A_z/ck_z$ and $v_{ph}=\omega/k_z$.

Using the Ampere's law and equation of motion we obtain electron density perturbation as

$$n_c^{'}T_c + n_h^{'}T_h = n_o\, e\left[\phi - (1+K_e)A + \frac{n_{oc}F_{cz} + n_{oh}F_{hz}}{i\,n_o e\,k_z}\right], \quad (12)$$

where, $K_e = c^2 k_x^2/\omega_{pe}^2$. The dispersion equation for the KAWs can be obtained by assuming

$$n_e^{'} = n_i^{'}, \quad (13)$$

where $n_e^{'} = n_c^{'} + n_h^{'}$ and $n_i^{'}$ are the perturbed electron and ion densities associated with the slow motions, respectively. Using equations (10)-(12) in (13), we obtain the following dispersion equation for the kinetic Alfvén wave

$$\varepsilon\phi = -\frac{ek_z^2\, v_A^2(1+\mu_i)k_{0z}k_{1z}\ \phi_0\phi_1}{m_e(1+K_e)\ \omega_0\omega_1}\left[1 + \frac{(1+K_e)}{K_e}\frac{\omega\omega_0 k_x(k_{0x}+k_{1x})}{\omega_{ce}^2 k_z^2}\right], \quad (14)$$

where,

$$\varepsilon = \omega^2 - k_z^2\, v_A^2\left[\frac{\mu_i t + (1+\mu_i)\left(\frac{T_c}{T_{eff}}\right)}{1+K_e}\right] = 0 \quad (15)$$

gives the linear dispersion relation for the KAWs, $\mu_i = k_x^2\rho_i^2$, $t=T_c/T_i$, and v_A is the Alfvén speed.

A dispersion equation for the sideband electron-acoustic wave at frequency ω_1 can be obtained by solving a set of equations (3)-(8) and is given by

$$\varepsilon_1\phi_1 = \frac{e\,\omega_{pe}^2\ K_e k_{0z}k_{1z}\ \phi_0\phi}{m_e\ k_1^2 v_A^2\ (1+\mu_i)\ \omega_0\omega_1}, \quad (16)$$

where

$$\varepsilon_1 = 1 + \frac{\omega_{ph}^2}{k_1^2\ v_{th}^2} + \frac{\omega_{pc}^2}{\omega_{ce}^2}\sin^2\theta_1 - \frac{\omega_{pc}^2}{\omega_1^2}\cos^2\theta_1 = 0 \quad (17)$$

will give the linear dispersion relation for the sideband electron-acoustic waves in a magnetized plasma and θ_1 is the angle made by the wave vector $\mathbf{k_1}$ with the ambient magnetic field.

From equations (14) and (16), we obtain the following nonlinear dispersion relation for the kinetic Alfvén waves by eliminating ϕ and ϕ_1

$$\varepsilon\varepsilon_1 = \mu,$$

where

$$\mu = -\frac{k_x^2\left|V_{0z}\right|^2}{\omega_1^2}\cos^2\theta_1\left[\frac{c^2k_z^2}{(1+K_e)} + \omega\omega_0\frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{(k_{0x}+k_{1x})}{k_x}\right] \quad (18)$$

is the coupling coefficient and $\left|V_{0z}\right| = \dfrac{eE_{0z}}{m_e \omega_0}$ is the z-component of the oscillatory velocity due to the pump electron-acoustic wave.

It may be pointed out here that both kinetic Alfvén waves and daughter electron-acoustic waves can grow at the same time. However, our purpose it to study the growth of kinetic Alfvén waves. Thus, the growth rate (γ) for the kinetic Alfvén waves can be obtained by using the following expression (Liu and Tripathi 1986).

$$\gamma^2 = - \frac{\mu}{\dfrac{\partial \varepsilon}{\partial \omega}\dfrac{\partial \varepsilon_1}{\partial \omega_1}}, \qquad (19)$$

Thus, from equations (15), (17)-(19), the growth rate can be given by

$$\gamma = \frac{k_x \left|V_{oz}\right|}{2\,\omega_{ce}} \left(\frac{n_0}{n_{0c}}\right)^{\frac{1}{2}} \left[\frac{(m_i/m_e)\,\omega\omega_1}{\mu_i t + (1+\mu_i)\left(\dfrac{T_c}{T_{eff}}\right)} + \frac{\omega_0 \omega_1 (k_{ox} + k_{1x})}{k_x} \right]^{\frac{1}{2}} \qquad (20)$$

3. Discussion

Large amplitude kinetic Alfvén waves propagating along the geomagnetic field lines have been observed by the satellites in the auroral zones. FREJA and FAST satellite observations have shown the existence of the nonlinear coherent structures associated with kinetic Alfvén waves. These waves are usually associated with broadband electrostatic activities at higher frequencies and shorter scale lengths. We have examined the parametric excitation of the kinetic Alfvén waves by high frequency electron-acoustic wave. Electron-acoustic wave decays into a kinetic Alfvén wave and a sideband electron-acoustic wave. A nonlinear dispersion relation has been obtained for the kinetic Alfvén waves through a three wave interaction parametric process. An analytical expression for the growth rate of the KAW has been obtained by solving the three-fluid equations.

We have solved equations (15) and (20), numerically for the real frequency and the growth rate of the kinetic Alfvén waves. Fig. 1 shows the variation of normalized growth rate (γ/ω_{ci}) versus normalized perpendicular wave number ($k_x c/\omega_{pe}$) for different values of the fractional cold electron density (n_c/n_o). As the fractional cold electron density increases the growth rate decreases. Fig. 2 describes the variation of normalized growth rate with respect to the normalized perpendicular wave number for various values of the k_z/k_x. The numerical results show that these waves can be excited for $k_z/k_x \leq 0.1$ for which our approximation ($\omega < \omega_{ci}$) for kinetic Alfvén waves is satisfied.

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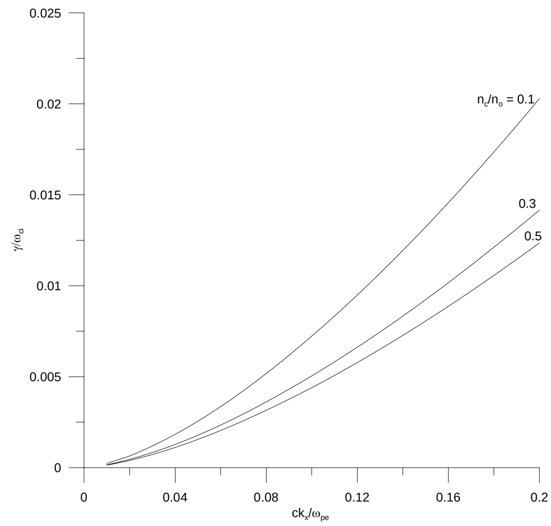

Fig. 1. Variation of normalized growth rate (γ/ω_{ci}versus normalized perpendicular wave number ($k_x c/\omega_{pe}$) for different values of the fractional cold electron density (n_c/n_o) for the parameters, k_z/k_x= 0.01, T_c/T_h=0.01, T_c=T_i , V_{oz}/c=0.01 and ω_{pe}/ω_{ce}=1.

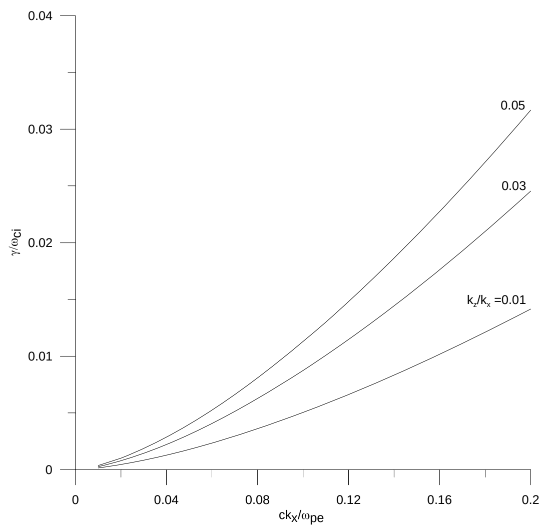

Fig. 2. Variation of normalized growth rate (γ/ω_{ci}) versus normalized perpendicular wave number ($k_x c/\omega_{pe}$) for different values of k_z/k_x for n_c/n_o =0.3. Other parameters are same as in Fig. 1.

The maximum real frequency and growth rate of the kinetic Alfvén waves comes out to be 1.3 s^{-1} and 0.2s^{-1}, respectively for n_c/n_o = 0.3 for the ion cyclotron frequency of about 15Hz and parallel electric field of pump electron-acoustic wave, E_{oz}=20 mV/m. and the corresponding pump electron-acoustic wave frequency of 2 kHz. The parameters used in our calculations of the growth rate and real frequency are representative of the auroral region of the Earth's magnetosphere (Yukhimuk et al., 1998, Voitenko, 1998). The corresponding growth time of these waves is about 6 seconds. Thus, the growth of the kinetic Alfvén waves is quite fast and hence, they can be excited through this mechanism in the auroral region of the magnetosphere.

Acknowledgments. GSL thanks the Council of Scientific andIndustrial Research (CSIR), Government of India for the support under the Emeritus Scientist Scheme. This work was carried out under the CAWSES-India project. The authors would like to thank ISRO Bangalore for providing the funding.

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