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Fuzzy clustering analysis to study geomagnetic coastal effects

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Abstract. The utility of fuzzy set theory in cluster analysis and pattern recognition has been evolving since the mid 1960s, in conjunction with the emergence and evolution of computer technology. The classification of objects into categories is the subject of cluster analysis. The aim of this paper is to employ Fuzzy-clustering technique to examine the interrelationship of geomagnetic coastal and other effects at Indian observatories. Data from the observatories used for the present studies are from Alibag on the West Coast, Visakhapatnam and Pondicherry on the East Coast, Hyderabad and Nagpur as central inland stations which are located far from either of the coasts; all the above stations are free from the influence of the daytime equatorial electrojet. It has been found that Alibag and Pondicherry Observatories form a separate cluster showing anomalous variations in the vertical (Z)-component. H- and D-components form different clusters. The results are compared with the graphical method. Analytical technique and the results of Fuzzy-clustering analysis are discussed here.

Keywords. Geomagnetism and paleomagnetism (Spatial variations attributed to sea floor spreading; Time variations, secular and long term; General or miscellaneous)

1 Introduction

Geomagnetic quiet day (Sq) variations have been widely analyzed by many scientists. There is an obvious annual increase in amplitude during the summer months and a seasonal shift of the maximum early in summer and late in winter for solar quiet day (Sq) variations at middle- and low-latitude stations (Campbell, 1997). It is only approximately true that Sq depends on local time and latitude only. There is a distinct, although small, longitude effect (Parkinson, 1983).

The quiet-day maximum-minimum range in H (Yacob and Sen, 1974) and "best" estimates of the amplitudes of diurnal and semidiurnal components of Sq(H) and their annual variations (Rangarajan, 1975), the latitudinal profile over India of Sq(H) range and of its prominent periodicities (Yacob, 1975), have been studied in detail. Quiet day mean hourly variations of the geomagnetic field have been utilized to study the solar control of the low-latitude quiet day magnetic field (Bhargava and Rangarajan, 1979) and local time and solar cycle features of the day-to-day variability in horizontal intensity (Bhargava and Yacob, 1969 and 1974); in addition, daily variations at low latitudes (Rastogi, 1992) and daily variations at low latitudes associated with stable solar wind flow (Rangarajan, 1981), etc., have been analyzed based on geomagnetic quiet day variations at Indian observatories.

Hence, it is appropriate to take the daily range of the solar quiet day hourly mean values of declination (D), horizontal (H) and vertical (Z) components of the Earth's magnetic field with the available common data for the years 1995, 1996 and 1997, namely Sq(D), Sq(H), Sq(Z) on all international quiet days. The results agree with the graphical method found by Srivastava et al. (2001) using 1995 data. The results from the analytic technique of Fuzzy-cluster analysis are highly encouraging for application of this technique to the future analysis of geomagnetic data.

The geographic and dipole coordinates of the observatories are provided in Table 1. From 1995 to 1997 there were 180 international quiet days. Daily ranges, namely the difference between daily maximum and daily minimum on individual days, are calculated, and monthly mean values of the ranges are listed in Table 2. Seasonal variations for the D-season (December solstice), comprising the months January, February, November and December; E-season (Equinoxial) comprising the months March, April, September and October; and J-season (June solstice), comprising the months May, June, July and August, are found and listed in Table 3. All the above 5 stations' westerly declination in minutes of arc have been converted to variations in nT for uniformity. Seasonal variations for the years 1995, 1996 and 1997 are plotted in Figs. 1, 2 and 3.

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Table 1. Locations of Observatories.

Station	Geographic	Dipole
1. Alibag(ALI)	Latitude 18°37′N Longitude 72°52′E	9.7°N 145.6°
2.Hyderabad(HYD)	Latitude 17°25′N Longitude 78°33′E	7.6°N 148.9°
3. Nagpur(NAG)	Latitude 21°09′N Longitude 79°05′E	11.7°N 151.9°
4. Pondicherry(PON)	Latitude 11°55′N Longitude 79°55′E	2.4°N 151.7°
5. Visakhapatnam(VIZ)	Latitude 17°41′N Longitude 83°19′E	07.8°N 155.8°

2 Concept of fuzzy clustering

The method of fuzzy clustering is based on a fuzzy equivalence relation (Klir and Yuvan, 1997). An equivalence relation is a relation defined on a set which is reflexive, symmetric and transitive, whereas a fuzzy equivalence relation is a relation defined on a set which is reflexive, symmetric, and max-min transitive. Like an ordinary equivalence relation, a fuzzy equivalence relation also induces a partition in each of its α-cuts (Zadeh, 1965). The fuzzy clustering problem can thus be viewed as the problem of identifying an appropriate fuzzy compatibility relation in terms of an appropriate distance function applied to given data (Klir and Folger, 2000). Then a meaningful fuzzy equivalence relation is defined based on the transitive closure of the fuzzy compatibility relation (Anderberg, 1973).

3 Method of fuzzy clustering

Suppose S is a set consisting of n data items. Let R be the set of all real numbers. Let p be a natural number. Suppose each data item in S is a p-tuple in R^p. Consider any two points X_i X_k in S with

$X_{i,}=(x_{i1,}x_{i2,}x_{i3,}\ldots, x_{ip})$

$X_k=(x_{k1,}x_{k2,}x_{k3}\ldots\ldots, x_{kp})$.

Let a fuzzy compatibility relation R on S be defined in terms of an appropriate distance function of the Minkowski class by the formula

$$R(x_{i,}x_k)=1-\delta\left[\sum(x_{ij}-x_{kj})^q\right]^{1/q}\ldots \quad (1)$$

For all pairs $(x_i, x_k)\in S$ where q is a positive real number and δ is a constant that ensures that $R(x_i, x_k)\in[0, 1]$. The quantity δ is the inverse value of the largest distance in S.

In general, R, defined by Eq. (1) is a fuzzy compatibility relation and it need not be a fuzzy equivalence relation. An algorithm is required to determine the transitive closure of R. Since R is a compatibility relation, one can use the following result in the formulation of an algorithm.

4 Result on max-min transitive closure

Let S be a finite universal set. Let the number of elements in S be n. Let R be a fuzzy compatibility relation on S. Then the max-min transitive closure of R is the relation $R^{(n-1)}$.

5 Transitive closure algorithm

Let R be a square matrix of order k obtained from the given data matrix by employing Eq. (1):

$$\text{Take } R^{(2)} = R\,o\,R\ldots, \quad (2)$$

where an element of R o R is max-min (X_{rj}, X_{js}) with j varying from 1 to k, where X_{rs} is an element in the r^{th} row and s^{th} column of the matrix $R^{(2)}$.

Similarly,

$$R^{(4)}=R^{(2)}\,o\,R^{(2)}\ldots \quad (3)$$

....

....

$$R^{(2k)}=R^{(2k-1)}oR^{(2k-1)}\ldots \quad (4)$$

This is continued until no new relation is produced. Thus, the max-min transitive closure of R is the relation $R^{(n-k)}$ which is denoted by $R\tau$. Finally this relation induces partitions called α-cuts in different intervals. The partitions agree with the visual perception of geometric clusters in the data.

6 Application of fuzzy clustering to geomagnetic Sq variations

The monthly mean values of the Sq range for the D, H, Z components for the 36 months from Table 2 are considered. From this table a distance matrix is obtained with the formula Distance=square root of $[(x_{11}-x_{12})^2 + (x_{21}- x_{22})^2 + \ldots\ldots + (x_{n1}-x_{n2})^2]$ etc ... for Euclidean distance. It is given as the distance matrix for each component.

The declination (D)-component will be used to explain the analysis and is applicable to horizontal (H)- and vertical (Z)-components.

Abbreviations:

Alibag:	ALB
Hyderabad:	HYD
Nagpur:	NAG
Pondicherry:	PON
Visakhapatnam:	VIZ

Euclidean distance matrix for declination component-D:

	ALB	HYD	NAG	PON	VIZ
ALB	0	16.97	23.98	31.03	30.397
HYD	16.97	0	17.17	29.65	18.38
NAG	23.98	17.17	0	25.14	26.32
PON	31.03	29.65	25.14	0	42.77
VIZ	30.397	18.38	26.32	42.77	0

Table 2. Monthly variations for the components D, H and Z for the years 1995, 1996 and 1997 in nT.

YEAR	ALB-D	HYD-D	NAG-D	PON-D	VIZ-D	ALB-H	HYD-D	NAG-H	PON-H	VIZ-H	ALI-Z	HYD-D	NAG-Z	PON-Z	VIZ-Z
95Jan	22	26	29	28	28	20	23	29	35	23	18	15	13	26	17
Feb	15	17	20	19	18	35	37	49	36	34	15	12	11	24	15
Mar	30	29	34	27	31	49	54	61	69	55	22	21	20	40	24
Apr	42	45	44	38	50	42	49	54	69	55	32	26	25	35	23
May	48	50	49	44	53	36	42	48	51	39	30	22	18	31	15
Jun	59	62	61	51	63	44	49	57	50	45	37	20	21	25	14
Jul	51	53	54	43	61	46	50	56	56	45	33	26	24	25	15
Aug	54	50	48	43	52	41	43	54	51	41	31	29	25	32	21
Sep	51	52	49	47	54	32	36	46	46	33	32	26	21	28	15
Oct	22	22	23	21	24	38	42	51	53	42	18	18	11	21	13
Nov	16	18	19	17	20	35	39	49	46	41	16	15	10	21	13
Dec	17	24	26	22	26	25	29	38	31	29	15	14	14	25	14
96Jan	22	18	27	24	24	31	33	40	43	33	23	17	23	26	19
Feb	23	21	27	29	21	40	44	51	50	45	16	11	13	19	13
Mar	15	16	21	21	21	38	41	50	48	43	15	11	12	22	12
Apr	42	40	41	41	51	42	52	56	65	51	30	21	23	30	22
May	45	41	43	41	51	42	50	55	53	44	29	14	20	22	17
Jun	47	39	43	38	48	42	46	55	45	42	27	14	17	21	13
Jul	33	31	33	30	39	29	31	40	39	28	26	19	18	24	14
Aug	53	41	44	44	49	25	28	42	40	25	31	21	21	30	17
Sep	51	48	51	52	57	38	46	55	59	41	35	24	22	28	19
Oct	32	29	31	28	33	40	47	52	54	46	23	13	19	23	17
Nov	15	15	16	16	15	42	46	55	48	43	15	7	8	21	13
Dec	17	15	20	19	17	28	30	40	35	29	12	9	9	18	13
97Jan	24	22	27	23	26	25	27	38	34	25	23	15	15	21	14
Feb	19	18	27	23	25	36	39	47	43	37	13	10	11	17	14
Mar	32	26	32	26	30	47	52	58	58	50	22	14	15	24	14
Apr	37	36	39	36	44	32	37	45	51	35	28	19	17	31	15
May	38	34	38	34	39	35	38	47	47	36	37	13	13	29	14
Jun	38	37	40	34	45	39	43	50	49	43	26	17	19	26	18
Jul	38	36	40	35	36	39	41	51	40	39	22	9	11	15	9
Aug	45	44	46	46	49	40	45	55	48	40	30	18	19	22	15
Sep	45	46	50	48	56	39	46	54	60	44	36	22	22	37	22
Oct	26	30	33	29	39	34	41	46	62	42	31	23	21	42	23
Nov	18	18	22	19	20	33	43	46	45	39	15	11	13	25	13
Dec	19	22	24	25	29	21	23	32	32	23	24	16	19	23	16

Relational matrix R arising from the Euclidean distance equation for the element D is

$$R = \begin{array}{c} \\ \text{ALB} \\ \text{HYD} \\ \text{NAG} \\ \text{PON} \\ \text{VIZ} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{ALB} & \text{HYD} & \text{NAG} & \text{PON} & \text{VIZ} \end{array} \\ \begin{pmatrix} 1 & 0.60323 & 0.43933 & 0.27449 & 0.28929 \\ 0.60323 & 1 & 0.59855 & 0.30676 & 0.57026 \\ 0.43933 & 0.59855 & 1 & 0.41220 & 0.38462 \\ 0.27449 & 0.30676 & 0.41220 & 1 & 0 \\ 0.28929 & 0.57026 & 0.38462 & 0 & 1 \end{pmatrix} \end{array}$$

Next $\mathbf{R}^{(2)}$ is obtained using the transitive closure algorithm described above. The result is given below:

$$R^{(2)} = R \circ R = \begin{array}{c} \\ \text{ALB} \\ \text{HYD} \\ \text{NAG} \\ \text{PON} \\ \text{VIZ} \end{array} \begin{array}{c} \begin{array}{ccccc} \text{ALB} & \text{HYD} & \text{NAG} & \text{PON} & \text{VIZ} \end{array} \\ \begin{pmatrix} 1 & 0.60323 & 0.59855 & 0.41220 & 0.57026 \\ 0.60323 & 1 & 0.59855 & 0.41220 & 0.57026 \\ 0.59855 & 0.59855 & 1 & 0.41220 & 0.57026 \\ 0.41220 & 0.41220 & 0.41220 & 1 & 0.38462 \\ 0.57026 & 0.57026 & 0.57026 & 0.38462 & 1 \end{pmatrix} \end{array}$$

Table 3. Seasonal variations for D, H and Z nT.

YEAR	ALI-D	HYD-D	NAG-D	PON-D	VIZ-D	ALB-H	HYD-H	NGP-H	PON-H	VIZ-H	ALB-Z	HYD-Z	NAG-Z	PON-Z	VIZ-Z
95D	17.5	21.25	23.5	21.5	23	28.75	32	41.25	37	29.25	16	14	12	24	15.5
95E	36.25	37	37.5	33.25	39.75	40.25	45.25	53	59.25	46.25	26	22.75	19.25	31	18.75
95J	53	53.75	53	45.25	57.25	41.75	46	53.75	52	42.5	32.75	24.25	22	28.25	16.25
96D	19.25	19.75	22.5	22	19.25	35.25	38.25	46.5	44	37.5	16.5	11	13.25	21	14.5
96E	35	43.75	36	35.5	40.5	39.5	46.5	53.25	56.5	45.25	25.75	17.25	19	25.75	17.5
96J	44.5	43.75	40.75	38.25	46.75	34.5	38.75	48	44.25	34.75	28.25	17	19	24.25	15.25
97D	21	23	25	22.5	25	28.75	33	40.75	38.5	31	18.75	13	14.5	21.5	14.25
97E	38	39.5	38.5	34.75	42.25	38	44	50.75	57.75	42.75	29.25	19.5	18.75	33.5	18.5
97J	44	43	41	37.25	43	38.25	41.75	50.75	46	39.5	28.75	14.25	15.5	23	14

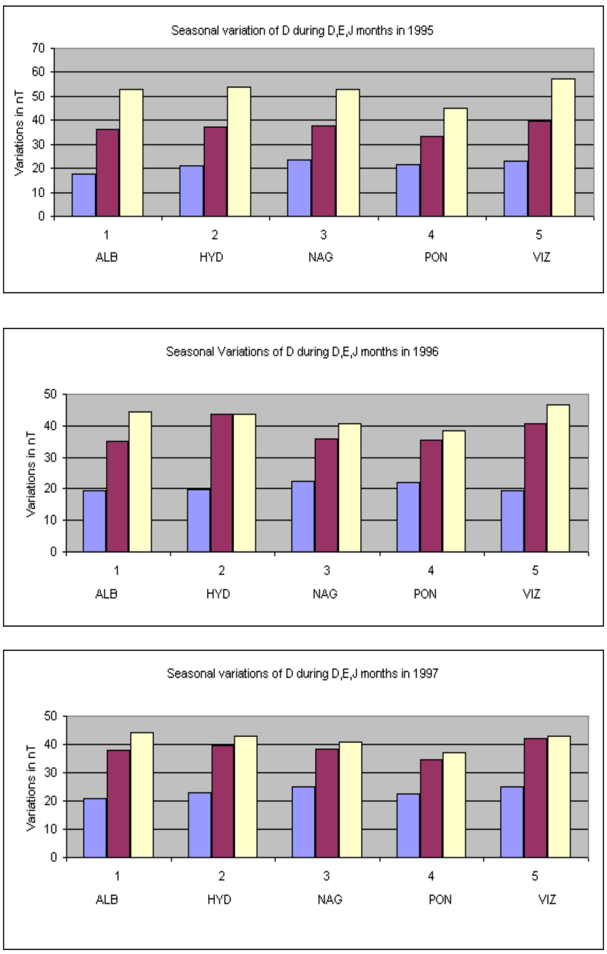

Fig. 1. Seasonal variations of the declination D-component during D, E, J months.

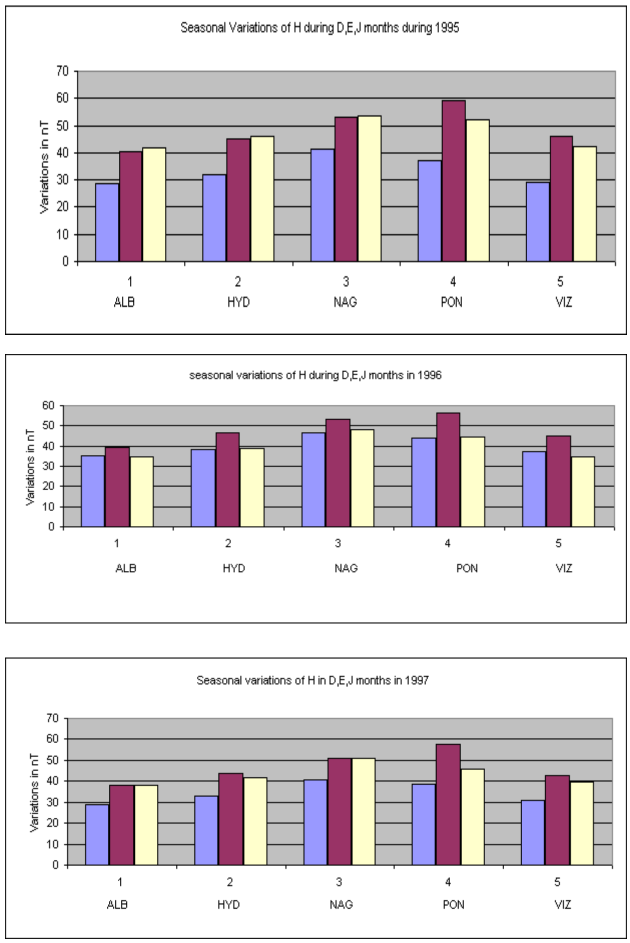

Fig. 2. Seasonal variations of the horizontal H-component during D, E, J months.

Using the transitive algorithm, $R^{(4)}$ is obtained as follows:

$R^{(4)} = R^{(2)} \text{ o } R^{(2)} =$

	ALB	HYD	NAG	PON	VIZ
ALB	1	0.60323	0.59885	0.41220	0.57026
HYD	0.60323	1	0.59855	0.41220	0.57026
NAG	0.59855	0.59855	1	0.41220	0.57026
PON	0.41220	0.41220	0.41220	1	0.41220
VIZ	0.57026	0.57026	0.57026	0.41220	1

Again using the transitive algorithm, $R^{(8)}$ is obtained as follows:

$R^{(8)} = R^{(4)} \text{ o } R^{(4)} =$

	ALB	HYD	NAG	PON	VIZ
ALB	1	0.60323	0.59885	0.41220	0.57026
HYD	0.60323	1	0.59855	0.41220	0.57026
NAG	0.59855	0.59855	1	0.41220	0.57026
PON	0.41220	0.41220	0.41220	1	0.41220
VIZ	0.57026	0.57026	0.57026	0.41220	1

Next it is observed that

$R^{(8)} = R^{(4)} \text{ o } R^{(4)}$

$= R^{(4)}$

and $R^{(16)} = R^{(8)} \text{ o } R^{(8)}$

$= R^{(4)} \text{ o } R^{(4)}$

$= R^{(4)}$, etc. . . .

Consequently, no new matrix is obtained after $R^{(8)}$. This indicates that the stopping condition for the algorithm has been reached. As a result, the final matrix $R\tau$ is taken as $R^{(8)}$. The transitive closure of R is taken as $R^{(8)}$ (Friedman and Kandel, 1999).

7 Dendogram

The result of single linkage clustering (Maskay, 1998) is displayed graphically in the form of a diagram called dendogram (Everstt, 1985). The term "dendogram" is used in numerical taxonomy for any graphical drawing or diagram giving a tree-like description of a taxonomic system. More generally, Calinski (1988) describes a dendogram as a 2-D diagram representing a tree of relationships, whatever their nature. The representation of a taxonomic system by a dendogram is particularly suitable in connection with a cluster analysis, to investigate the structure of the corresponding operational taxonomic units, that is, entities or individuals considered as the lowest-ranking taxa within the system.

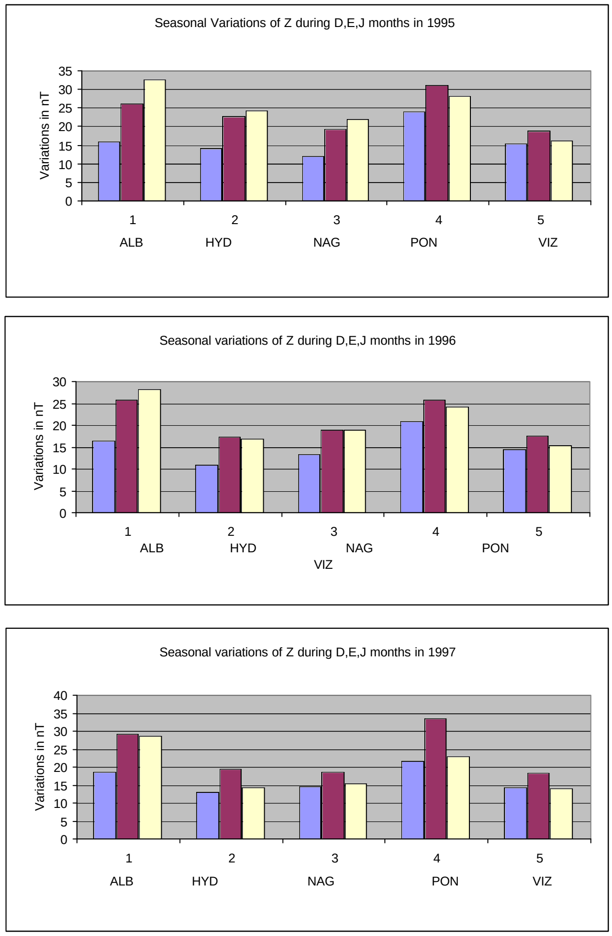

Fig. 3. Seasonal variations of the vertical Z-component during D, E, J months.

From the above table $R^{(8)}$, it is observed that the transitive closure leads to the cuts in the interval [0,1] at 0.60323, 0.59855, 0.57026, 0.41220, as shown below.
α-cuts:

0−− ...0−−−...−−−0−−−0−−−0−−−0−−− ... −−−0−−−...−−−0−−−

0 0.60323 0.59855 0.57026 0.41220 1

Thus, the following α-cuts are formed by $R\tau$

$\alpha \in$ (0.60323, 1] : {(ALB), (HYD), (NAG), (PON), (VIZ)}
$\alpha \in$ (0.59855, 0.60323] : {(HYD, ALB), NAG, PON, VIZ}
$\alpha \in$ (0.57026, 0.59855] : {(HYD, NAG), (NAG, ALB), (ALB, NAG)}
: {(HYD, ALB, NAG), (PON), (VIZ)}
$\alpha \in$ (0.41220, 0.57026) : {(ALB, VIZ), (HYD, VIZ), (NAG, VIZ), (VIZ, ALB)}
: [(HYD, ALB, NAG, VIZ), PON]

The α-cuts are considered one-by-one, starting from the right-hand side and moving in the left direction. The first α-cut to be considered is (0.60323, 1].

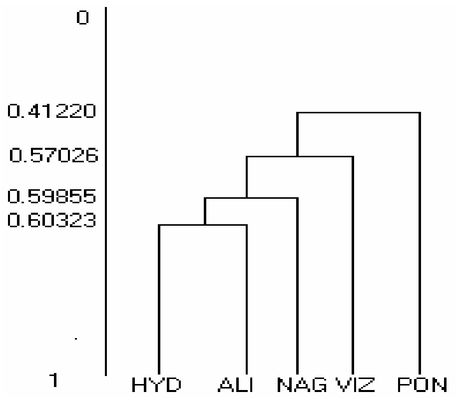

Fig. 4. Dendogram for declination D-component.

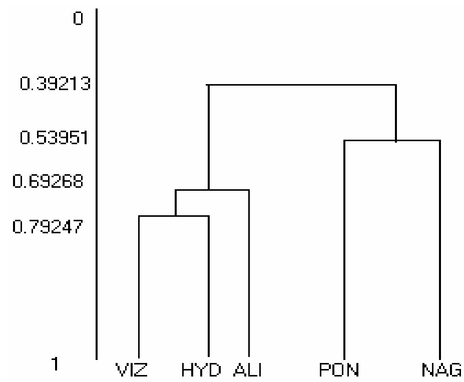

Fig. 5. Dendogram for horizontal H-component.

The entries in the $R^{(8)}$ which are greater than 0.60323 and less than or equal to 1 are 1,1,1,1,1. These values correspond to the pairs ALB–ALB, HYD–HYD, NAG–NAG, PON–PON, VIZ–VIZ. Thus, corresponding to the first α-cut (0.60323, 1), each one of the five stations, namely ALB, HYD, NAG, PON and VIZ, form a distinct partition.

The next α-cut has to be considered by moving from the right end of the interval to the left. Thus, one has to consider the α-cut (0.59855, 0.60323). The entries which are greater than 0.59855 and less than or equal to 0.60323 correspond to the pairs HYD-ALB. Consequently, it is concluded that the pair HYD-ALB forms a cluster corresponding to the α-cut (0.59855, 0.60323).

Next, the α-cut (0.57026, 0.59855) is taken up for consideration. The pairs associated with this α-cut are HYD-NAG, NAG-ALB, and ALB-NAG. Therefore, HYD, ALB and NAG form a cluster and PON and VIZ are isolated.

Next, the α-cut (0.41220, 0.57026) is taken up for consideration. Here VIZ joins with the last cluster and PON is isolated.

The first cluster in the dendogram for the declination component D (Fig. 4) indicates that HYD and ALB have similar characteristics in terms of its variations in the declination component. It is also seen from this figure that the variations in the declination component for PON stand apart from the cluster.

By applying the same procedures, the values of $R\tau$, which induces the following partition for its α-cuts for horizontal component H and vertical component Z, are found. The details are shown for component D and are similar for the H- and Z-components.

For the horizontal H-component $R\tau$ induces the following four partitions for its α-cuts:

$\alpha \in (0.79247, 1]$: {(ALB), (HYD), (PON), (NAG), (VIZ)]
$\alpha \in (0.69268, 0.79247]$: {(VIZ, HYD), ALB, PON, NAG, VIZ]
$\alpha \in (0.53951, 0.69268]$: {(ALB, HYD), (ALB, VIZ)}
: {(ALB, HYD, VIZ), PON, NAG}
$\alpha \in (0.39213, 0.53951]$: {(NAG, PON)]
: [(ALB, HYD, VIZ), {PON, NAG}] .

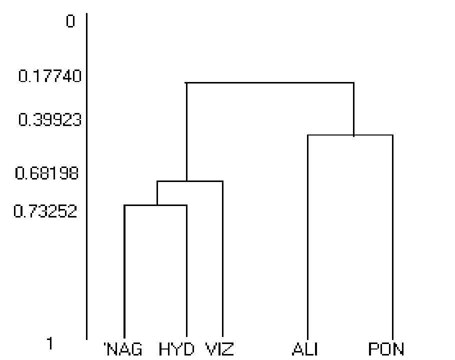

Fig. 6. Dendogram for vertical Z Component.

The dendogram for the horizontal component H (Fig. 5) indicates that the variations in H for the stations VIZ and HYD form a cluster, and ALB joins VIZ in the cluster, and PON and NAG stand apart from the first cluster.

For the vertical Z-component $R\tau$ induces the following four partitions for its α-cuts:

$\alpha \in (0.73252, 1]$: {(ALB), (HYD), (NAG), (VIZ)]
$\alpha \in (0.68198, 0.73252]$: {(NAG, HYD), ALB, PON, VIZ]
$\alpha \in (0.39923, 0.68198]$: {(HYD, NAG, VIZ), ALB, PON]
$\alpha \in (0.17740, 0.39923]$: [(HYD, NAG, VIZ), (ALB, PON)] .

The dendogram for the vertical component Z (Fig. 6) indicates that the variations in Z for the stations NAG and HYD, central inland stations, form a cluster, and VIZ joins the first cluster, and ALB and PON stand apart from the first cluster and form a different cluster.

8 Findings of the study

Seasonal variations of D, H and Z in Table 3 are plotted (Figs. 1, 2, 3) for all the stations for the years 1995 to 1997 and they are compared.

Fuzzy clustering has been achieved with continuous data from the 36 months listed in Table 2, for each component separately, to study the overall pattern and proximity of one observatory to the other. Seasonal variation of the individual observatories forms the same pattern for all 3 years.

An examination of Figs. 1, 2, and 3 – in comparison with the respective dendograms (Figs. 4, 5 and 6) show the following:

- Of the Sq(D) ranges observed among the 5 stations, ALB and HYD show the same range of variations, NAG and VIZ show marginally enhanced variations and PON stands apart. The dendogram of D agrees with this result.

- The Sq(H) range of values at ALB is less compared to HYD but no definite trend emerges at VIZ. The Sq(H) range for VIZ is comparable to HYD. The results of Srivastava et al. (2001) agree with this present study. NAG and PON values are high compared to the other three places in all seasons. This result has also been established with the Fuzzy clustering technique. NAG and PON form a separate cluster VIZ, HYD and ALB form a separate cluster which results in the dendogram for H variations.

- Sq(Z) range values at ALB and PON are high compared to HYD for all three seasons. Sq(Z) values at VIZ are less compared to HYD, except in summer months. NAG and HYD, which are far away from either of the coasts, are nearer to each other in variations of the vertical component. The dendogram for Z-variations confirms the result.

It is necessary to recollect the findings of Srivastava et al. (2001) that vast deposits of limestone have been discovered in the Bay of Bengal off Visakhapatnam and that a relatively deep resistive body may not allow the (Sq)-induced currents in the seawater to concentrate near the Visakhapatnam coast. In the case of Alibag on the west coast, however, such induced (Sq) currents in the Arabian Sea do concentrate along the coast line and the continental margins give rise to an enhanced daily range in Z and reduced daily ranges in H and D, as compared to those at the Hyderabad station.

Although PON and VIZ are in the same coastal region only ALB and PON form the same pattern. An examination of the graphical plots and dendogram for Z reveal this fact.

9 Conclusion

The application of fuzzy concepts for pattern recognition and classification have been used for numerous applications in astronomy, meteorology, geology for planetary exploration, terrestrial geologic feature analysis, cartography and geodesy, surface model fittings, satellite data analysis, artificial intelligence, etc. This technique is used here to study the identical pattern of geomagnetic variations at Indian observatories. Generally, for a huge volume of data in a complicated analysis this technique yields accurate results. As a result of this study, it is expected that future usage of this technique may be appropriate for exploring some new results in geomagnetism.

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