Generation mechanism for electron acoustic solitary waves

A. P. Kakad,[a)] S. V. Singh, R. V. Reddy, and G. S. Lakhina
Indian Institute of Geomagnetism, New Panvel (West), Navi Mumbai-410 218, India

S. G. Tagare
Department of Mathematics, Addis Ababa University, Addis Ababa, Ethiopia

F. Verheest
Sterrenkundig Observatorium, Universiteit Gent, Krijglaan 281, B-9000 Gent, Belgium and School of Physics, University of KwaZulu-Natal, Private Bag X54001, Durban 4000, South Africa

(Received 6 February 2007; accepted 27 March 2007; published online 14 May 2007)

Nonlinear electron acoustic solitary waves (EASWs) are studied in a collisionless and unmagnetized plasma consisting of cold background electrons, cold beam electrons, and two different temperature ion species. Using pseudopotential analysis, the properties of arbitrary amplitude EASWs are investigated. The present model supports compressive as well as rarefactive electron acoustic solitary structures. Furthermore, there is an interesting possibility of the coexistence of compressive and rarefactive solitary structures in a specific plasma parameter range. The application of our results in interpreting the salient features of the broadband electrostatic noise in the plasma sheet boundary layer is discussed. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2732176]

I. INTRODUCTION

Solitary waves and bipolar pulses in the electric field parallel to the background magnetic field have been identified throughout the Earth's magnetosphere at narrow boundaries, such as the plasma sheet boundary layer,[1,2] the magnetosheath,[3] the bow shock,[4] in strong currents, such as those associated with the auroral acceleration region[5] and also in solar wind plasma[6] near the Lagrangian point L1. Recently, several observations report the evidence of tripolar waveforms (two positive peaks and one negative peak in the electric field amplitude or vice versa) in addition to the standard bipolar pulses in the electric field.[7,8] Except in the ion beam regions of the auroral zone, the solitary waves are positive potential structures moving at velocities comparable to the electron thermal velocity ($\sim$1000s of km/s) and are commonly interpreted to be either electron holes, such as Bernstein-Greene-Kruskal modes[9–12] arising from the evolution of a bump on tail instability and/or electron two-stream instability,[13,14] or in terms of electron acoustic solitons. The existence of these positive potential electrostatic structures is associated with the high frequency disturbances.[1]

An electron-acoustic wave can either exist in a two-temperature (cold and hot) electron plasma[15] or in an electron-ion plasma with ions hotter than electrons.[16] The earlier models based on electron acoustic solitons could explain the spacecraft observations (e.g., Viking) of solitary waves that had negative potentials in two- and three-temperature electron plasmas.[17–22] To explain the positive structures, recently attempts have been made to study electron acoustic solitons in three electron (cold, hot, beam) component plasmas.[23–25] These models show that depending on the beam density and temperature and below a critical velocity of the electron beam, nonlinear structures can have a positive potential signature.[23] More recently, Verheest *et al.*[26] have pointed out the possibility to obtain compressive electron acoustic solitons, i.e., those having positive potentials, even without the electron beam component, provided the hot electron inertia is retained in the analysis.

Observational evidence gives a clear understanding about the role of ion and electron beams to drive the broadband electrostatic waves.[27–30] These solitary waves have amplitudes typically a few mV/m in the plasma sheet boundary layer, but they can be as large as 200 mV/m at polar altitudes.[31] Such nonlinear solitary structures observed in the plasma sheet boundary layer regions may play a key role in supporting parallel electric fields in the magnetotail.

The spacecraft observations[27–30] in the Earth's plasma sheet boundary layer show the existence of background electrons, cold electron beams having energies of the order of a few eV to a few hundreds of eV, and background cold ions and warm ions/ion beams with energies from a few keV to tens of keV. In view of this, we study here the generation of nonlinear electron acoustic solitary waves in a four-component plasma system consisting of cold electrons, cold beam electrons, and two different temperature ion species (i.e., cold and hot ions). For simplicity, we have not considered the relative speed between the cold and hot ions. Further, the plasma is considered as collisionless and unmagnetized. It is shown that for certain plasma conditions there is the possibility of coexistence of rarefactive and compressive modes in the plasma sheet boundary layer. The paper is organized as follows. The basic equations governing the plasma model are given in Sec. II. The formulation and the numerical study of arbitrary amplitude electron acoustic solitary waves (EASWs) in three- and four-component plasmas are given in Sec. III. A brief discussion with application to understand broadband electrostatic noise in the Earth's plasma sheet boundary layer is given in Sec. IV.

[a)]Author to whom correspondence should be addressed. Electronic mail: amar@iigs.iigm.res.in

 © 2007 American Institute of Physics

Downloaded 18 Sep 2008 to 203.193.153.35. Redistribution subject to AIP license or copyright; see http://pop.aip.org/pop/copyright.jsp

II. GOVERNING EQUATIONS

In order to construct the model, we consider an infinite, collisionless, and unmagnetized plasma with four components; namely, cold plasma electrons, cold beam electrons, and isothermal ions with two different temperatures. For simplicity, we have not considered the ion dynamics here. We assume that these ions obey Boltzmann type distributions as they are much hotter than the electrons. The normalized fluid equations for the cold plasma electrons and cold beam electrons are given by

$$\frac{\partial n_s}{\partial t} + \frac{\partial (n_s u_s)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u_s}{\partial t} + u_s \frac{\partial u_s}{\partial x} = \frac{\partial \phi}{\partial x}. \tag{2}$$

Here, the subscript $s=c$ (cold electrons) and b (cold beam electrons). In addition, n_s and u_s are plasma density and velocity, respectively, for the species s, and ϕ is the electrostatic potential. Furthermore, the system is closed by the Poisson equation

$$\frac{\partial^2 \phi}{\partial x^2} = n_c + n_b - n_{il} - n_{ih}, \tag{3}$$

where

$$n_{il} = \mu e^{-\phi/(\mu+\nu\beta)}, \quad n_{ih} = \nu e^{-\beta\phi/(\mu+\nu\beta)} \tag{4}$$

are the densities of the isothermal ions with two temperatures, one species with low-temperature T_l and initial normalized equilibrium density μ, and the second ion species with high temperature T_h and initial normalized equilibrium density ν. Here, $\mu+\nu=1$, $\beta=T_l/T_h$, and $T_{\mathrm{eff}}=T_l T_h/(\mu T_h+\nu T_l)$. The subscripts c, b, l, and h refer to cold plasma electrons, cold beam electrons, low-temperature ions, and high-temperature ions, respectively. In the above set of Eqs. (1)–(4), the plasma densities, plasma velocities, electrostatic potential, distance x, and time t are normalized to the unperturbed plasma density n_0, effective electron acoustic speed $C_{\mathrm{eff}}=(T_{\mathrm{eff}}/m_e)^{1/2}$, effective electrostatic potential T_{eff}/e, effective Debye length $\lambda_{\mathrm{Deff}}=(T_{\mathrm{eff}}/4\pi n_0 e^2)^{1/2}$, and inverse plasma frequency ω_{pe}^{-1}, ($\omega_{pe}=\sqrt{4\pi n_0 e^2/m_e}$), respectively.

A linear dispersion relation for electron-acoustic waves can be obtained by linearizing and solving a set of Eqs. (1)–(4) and is given by

$$\frac{\omega_{pc}^2}{\omega^2} + \frac{\omega_{pb}^2}{(\omega - k u_0)^2} = \frac{1 + k^2\lambda_{\mathrm{Deff}}^2}{k^2\lambda_{\mathrm{Deff}}^2}, \tag{5}$$

where $\omega_{pc,b}$ are the cold and beam electron plasma frequencies, respectively. For the case when the cold electron beam is absent, i.e., $n_b=0$, and limit $k\lambda_{\mathrm{Deff}} \ll 1$ is considered, Eq. (5) reduces to $\omega=kC_{\mathrm{eff}}$, which is similar to the dispersion relation for the electron-acoustic modes derived by Fried and Gould[16] for the case of electron-ion plasma with hot ions; i.e., $T_i \gg T_e$. However, when the cold electron beam is present, the dispersion relation [Eq. (5)] is more complicated as the beam mode and the electron acoustic modes are coupled.

III. THEORY FOR ARBITRARY AMPLITUDE EASW

We seek solutions to Eqs. (1)–(4) that are stationary in a frame moving with velocity V. To study the properties of stationary arbitrary amplitude EASWs, we assume that all variables in Eqs. (1)–(4) depend only on a single variable, i.e., $\xi=x-\mathrm{Ma}\,t$ (here, $\mathrm{Ma}=V/C_{\mathrm{eff}}$ is the Mach number). We assume the following appropriate boundary conditions for the localized disturbances

$$n_c \to 1-\alpha, \quad u_c \to 0, \quad n_b \to \alpha, \quad u_b \to u_0, \quad n_{il} \to \mu,$$
$$n_{ih} \to \nu = 1-\mu, \quad \phi \to 0, \quad \text{as } |x| \to \infty. \tag{6}$$

Using the transformation to a single independent variable ξ in Eqs. (1)–(4) and applying the boundary conditions from Eq. (6), along with $\phi=0$ and $d\phi/d\xi=0$ at $\xi\to\pm\infty$, the Poisson equation (3) can be integrated to yield the following form of an energy integral:[21]

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi,\mathrm{Ma}) = 0, \tag{7}$$

where

$$\begin{aligned} V(\phi,\mathrm{Ma}) = {} & \mathrm{Ma}^2(1-\alpha)\left[1-\left(1+\frac{2\phi}{\mathrm{Ma}^2}\right)^{1/2}\right] \\ & + (\mathrm{Ma}-u_0)^2\alpha\left[1-\left(1+\frac{2\phi}{(\mathrm{Ma}-u_0)^2}\right)^{1/2}\right] \\ & + \mu(\mu+\nu\beta)[1-e^{-\phi/\mu+\nu\beta}] + \frac{\nu(\mu+\nu\beta)}{\beta} \\ & \times[1-e^{-\phi\beta/\mu+\nu\beta}]. \end{aligned} \tag{8}$$

In order to have soliton solutions, the pseudopotential $V(\phi,\mathrm{Ma})$ must satisfy the following conditions: $V(\phi)=0$, $dV(\phi)/d\phi=0$, and $d^2V(\phi)/d\phi^2<0$ at $\phi=0$, $V(\phi)=0$ at $\phi=\phi_0$, and $V(\phi)<0$ for $0<|\phi|<|\phi_0|$.

It is seen that Eq. (8) satisfies two of the soliton conditions at $\phi=0$. The third condition, i.e., $d^2V(\phi)/d\phi^2<0$ at $\phi=0$, is satisfied provided $\mathrm{Ma}>\mathrm{Ma}_0$. Here, Ma_0 is the solution of $f(\mathrm{Ma})=0$, where

$$f(\mathrm{Ma}) = \frac{(1-\alpha)}{\mathrm{Ma}^2} + \frac{\alpha}{(\mathrm{Ma}-u_0)^2} - 1, \tag{9}$$

which is a fourth-degree equation in Ma. Without the electron beam, Eq. (9) yields $\mathrm{Ma}_0=1$. For the soliton solution to exist, the pseudoparticle must be reflected at $\phi=\phi_0$ and return to its rest point ($\phi=0$), which requires[32] that

$$\begin{aligned} \frac{dV(\phi_0,\mathrm{Ma})}{d\phi} & > 0 \quad \text{for } \phi_0 > 0, \\ & < 0 \quad \text{for } \phi_0 < 0, \end{aligned} \tag{10}$$

together with $V(\phi_0,\mathrm{Ma})=0$. For compressive solitons, those having $\phi_0>0$, these two conditions need to be solved simultaneously to get the value of ϕ_0. However, for a rarefactive soliton (i.e., $\phi_0<0$), the condition (10) demands that $\phi_0 < \min\{-\mathrm{Ma}^2/2, -(\mathrm{Ma}-u_0)^2/2\}$. This shows that the maxi-

Downloaded 18 Sep 2008 to 203.193.153.35. Redistribution subject to AIP license or copyright; see http://pop.aip.org/pop/copyright.jsp

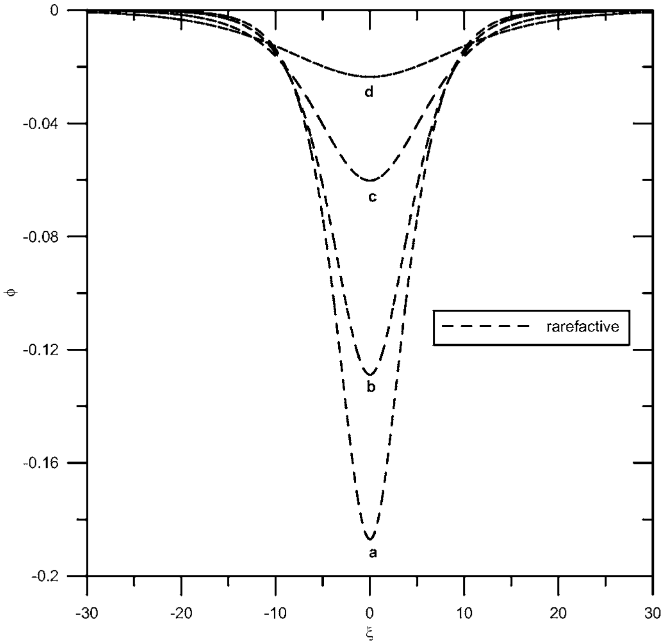

FIG. 1. Variation of potential ϕ with ξ for different electron beam velocities: (a) u_0=0.3, (b) u_0=0.4, (c) u_0=0.5, and (d) u_0=0.55. Other parameters are: Ma=1.1, μ=0.0, α=0.06, β=1.

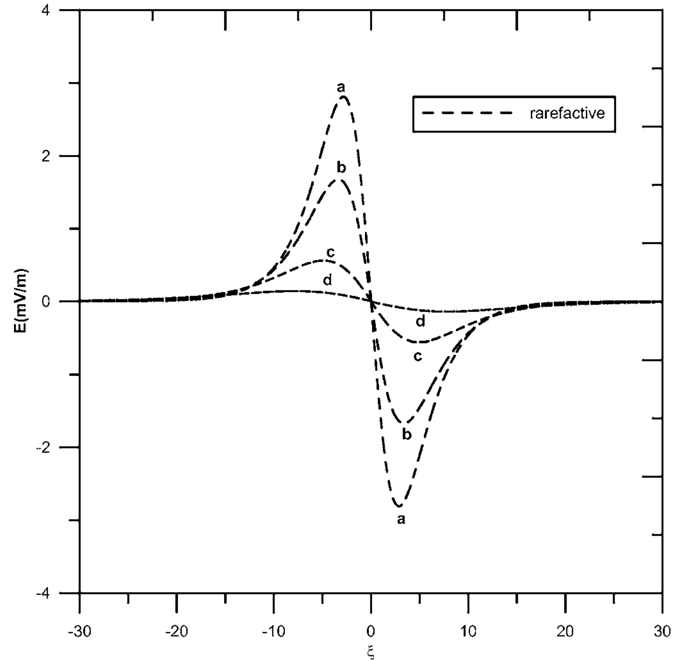

FIG. 2. Variation of electric field E with ξ for different electron beam velocities: (a) u_0=0.3, (b) u_0=0.4, (c) u_0=0.5, and (d) u_0=0.55. Here, T_l=5 eV, n_0=0.1 cm^{-3}, and other parameters are same as used in Fig. 1.

mum amplitude of the rarefactive solitons should decrease with an increase of the electron beam speed.

We have numerically solved Eq. (7) for a three-component plasma; i.e., cold electrons, cold beam electrons, and hot temperature ions. In this case it is found that this type of plasma composition supports only EA rarefactive solitary structures. Figure 1 shows the variation of the electrostatic potential ϕ, associated with rarefactive EA solitons for different electron beam speeds u_0. It is seen that the maximum electrostatic potential ϕ decreases with the increase of the electron beam speed u_0. The width of the EA solitons is found to be increase with the increase of electron beam speed. The bipolar electric field structures associated with these EA solitons are shown in Fig. 2.

To infer the properties of the EASW structures in the case of a four-component plasma, we have numerically solved Eq. (7) for the potential ϕ, for the typical parameters, which are shown in Fig. 3. It is found that for the beam velocities $u_0<0.45$, there exist only compressive electron acoustic solitary structures. It is also interesting to note that, for the beam velocities in a narrow range of $0.45 \leq u_0 < 0.6$, this model supports the coexistence of both rarefactive as well as compressive solitary structures. It is seen that the width and amplitude of both rarefactive and compressive modes, respectively, increase and decrease with the electron beam velocity. The corresponding electric field variation with ξ is shown in Fig. 4 for different electron beam velocities.

IV. DISCUSSION AND APPLICATION

In this paper we have theoretically investigated the nonlinear propagation of electron acoustic waves in three- (cold plasma electrons, cold beam electrons, and hot Boltzmann ions) and four-component (cold plasma electrons, cold beam electrons, and two species of cooler and hot Boltzmann ions) unmagnetized plasma. The properties of these arbitrary amplitude electron acoustic waves for both cases are studied by employing the pseudopotential technique. We have found that in the case of a three-component plasma this model supports only rarefactive EA structures, whereas a four-component plasma model supports rarefactive as well as

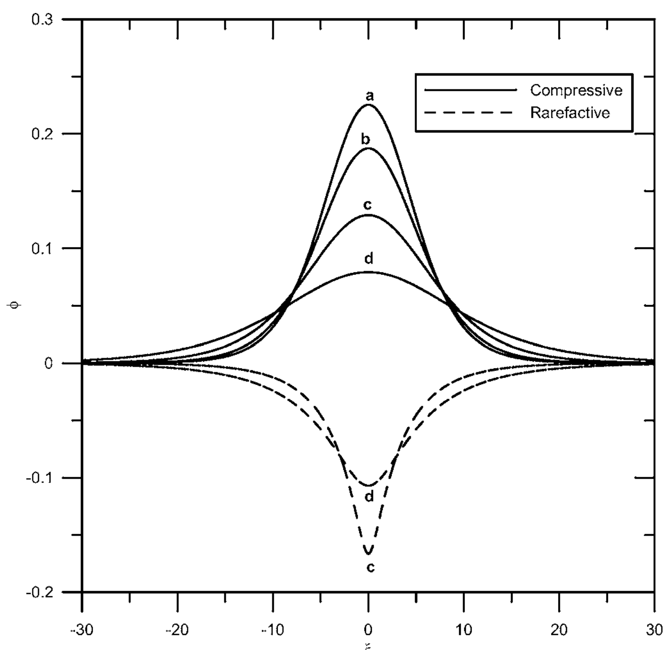

FIG. 3. Variation of potential ϕ with ξ for different electron beam velocities: (a) u_0=0.3, (b) u_0=0.4, (c) u_0=0.5, and (d) u_0=0.55. Other parameters are: Ma=1.1, μ=0.2, α=0.06, β=0.01.

Downloaded 18 Sep 2008 to 203.193.153.35. Redistribution subject to AIP license or copyright; see http://pop.aip.org/pop/copyright.jsp

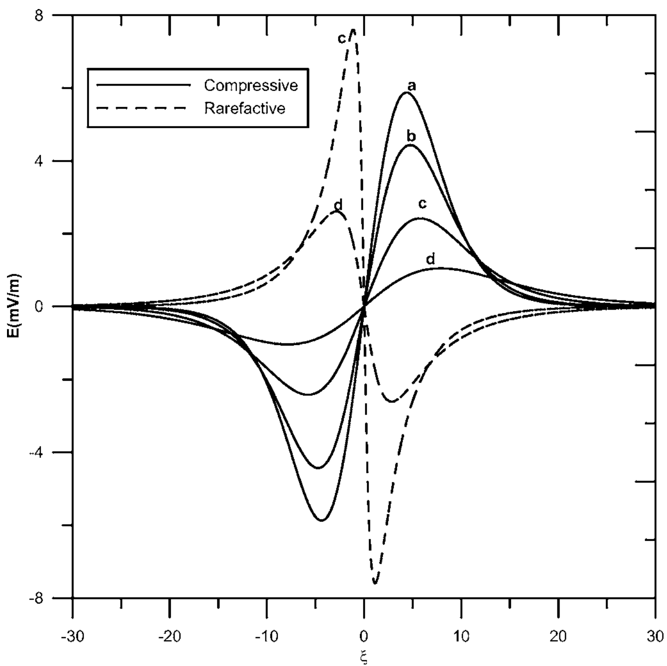

FIG. 4. Variation of electric field E with ξ for different electron beam velocities: (a) u_0=0.3, (b) u_0=0.4, (c) u_0=0.5, and (d) u_0=0.55. Here T_l=5 eV, n_0=0.1 cm^{-3}, and other parameters are same as used in Fig. 3.

compressive EA solitary structures. The model introduces, for the first time, a feature of arbitrary amplitude EASWs; i.e., the coexistence of rarefactive and compressive electron acoustic solitary modes for specific beam velocity ranges. We would like to mention that Baboolal *et al.*[32] have pointed out the possibility of the coexistence of compressive and rarefactive ion acoustic solitons in plasmas containing double-Maxwellian electrons and two ion species.

The plasma sheet boundary layer of the Earth's magnetotail is found to contain multiplasma species; e.g., cold electrons, electron beams, and ions/ion beams. This is, therefore, one of the situations in which the model developed in this paper may be applicable. Although the model is simplified in the sense that hot ions are considered as stationary, it may be usefully employed whenever the relative drift speed between the cold and hot ions is small compared to effective electron acoustic speed. For illustrative purposes, we consider the temperatures of the cold and hot ions as T_l=5 eV and T_h =500 eV, respectively.

The typical electrical fields associated with the electron acoustic solitary structures are plotted in Fig. 2 (for a three-component plasma) and Fig. 4 (for a four-component plasma). For a three-component plasma, only one type of ions with T_l=5 eV is considered. In this case the electric field is associated with the rarefactive EASWs (cf. Fig. 2). The maximum electric field is found to be in the range of 0.5–3 mV/m for the electron beam speed in the range of u_0=0.3–0.55. In Fig. 4, we have shown the electric fields for a four-component plasma, for the two cases: (a) when only a compressive mode exists and (b) when both compressive and rarefactive modes coexist. The maximum electric field associated with the compressive and rarefactive EASWs is found to be in the range of 1–2 mV/m and 2–8 mV/m, respectively, when both the modes coexist (cf. curves c and d). It is found that when there exist only the compressive EASW modes, the electric field are in the range of 4–5 mV/m (cf. curves a and b). The velocities of both compressive and rarefactive solitons are calculated from the Mach number, i.e., Ma=V/C_{eff} relationship, which can be rewritten as

$$V = \text{Ma}\left[\frac{(T_l/m_e)}{(\mu+\nu\beta)}\right]^{1/2}. \tag{11}$$

Using the Earth's plasma sheet boundary layer parameters in the above equation, the soliton velocity is calculated to be 1000 km/s in the case of a three-component plasma, while in the case of a four-component plasma it is found to be 2300 km/s.

We have presented a simple one-dimensional model for the electron acoustic (EA) solitons. Using particular boundary conditions, we have restricted our study for EA solitons, which supports the bipolar electric field structures. Different boundary conditions may supports EA double layers and that may helpful to understand tripolar electric field structures. For simplicity, we have not considered ion drift velocities. It might be necessary to include an ion beam in the analysis to bring the model closer to observations. This is beyond the scope of the present paper. It will be included in further work on electron-acoustic solitary waves.

The present study could have an interesting implication for data interpretation. The presence of the bipolar pulses of opposite initial polarity in data is commonly explained by assuming identical potential, either positive or negative, structures traveling in opposite directions. Our results provide a good alternative possibility of the structures traveling in the same direction but with opposite potentials.

ACKNOWLEDGMENTS

This work was done under the CAWSES-India project, "Boundary layer waves and ring current dynamics." A.P.K., S.V.S., and R.V.R. would like to thank ISRO, Bangalore for financial support. G.S.L. would like to thank the Council of Scientific and Industrial Research, Government of India, for the support under the Emeritus Scientist Scheme. F.V. thanks the Fonds voor Wetenschappelijk Onderzoek (Vlaanderen) for a research grant.

[1]H. Matsumoto, H. Kojima, T. Miyatake, Y. Omura, M. Okada, I. Nagano, and M. Tsutui, Geophys. Res. Lett. **21**, 2915 (1994).
[2]J. R. Franz, P. M. Kintner, and J. S. Pickett, Geophys. Res. Lett. **25**, 1277 (1998).
[3]J. S. Pickett, J. D. Menietti, D. A. Gurnett, B. Tsurutani, P. Kintner, E. Klatt, and A. Balogh, Nonlinear Processes Geophys. **10**, 3 (2003).
[4]S. D. Bale, P. J. Kellogg, D. E. Larson, R. P. Lin, K. Goetz, and R. P. Lepping, Geophys. Res. Lett. **25**, 2929 (1998).
[5]S. R. Bounds, R. F. Pfaff, S. F. Knowlton, F. S. Mozer, M. A. Temerin, and C. A. Kletzing, J. Geophys. Res. **104**, 28709 (1999).
[6]A. Mangeney, C. Salem, C. Lacombe, J.-L. Bougeret, C. Perche, R. Manning, P. J. Kellogg, K. Goetz, S. J. Monson, and J.-M. Bosqued, Ann. Geophys. **17**, 307 (1999).
[7]J. S. Pickett, S. W. Kahler, L.-J. Chen *et al.*, Nonlinear Processes Geophys. **11**, 183 (2004).
[8]J. S. Pickett, L.-J. Chen, S. W. Kahler, O. Santolik, D. A. Gurnett, B. T. Tsurutani, and A. Balogh, Ann. Geophys. **22**, 2515 (2004).
[9]I. B. Bernstein, J. M. Greene, and M. D. Kruskal, Phys. Rev. **108**, 546 (1957).

Downloaded 18 Sep 2008 to 203.193.153.35. Redistribution subject to AIP license or copyright; see http://pop.aip.org/pop/copyright.jsp

[10]L. Muschietti, I. Roth, R. E. Ergun, and C. W. Carlson, Nonlinear Processes Geophys. **6**, 211 (1999).
[11]L. J. Chen, J. Pickett, P. Kinter, J. Franz, and D. Gurnett, J. Geophys. Res. **110**, A09211 (2005).
[12]D. Jovanovic and P. K. Shukla, Phys. Rev. Lett. **84**, 4373 (2000).
[13]Y. Omura, H. Matsumoto, T. Miyake, and H. Kojima, J. Geophys. Res. **101**, 2685 (1996).
[14]M. V. Goldman, M. M. Oppenheim, and D. L. Newman, Geophys. Res. Lett. **26**, 1821 (1999).
[15]K. Watanabe and T. Taniuti, J. Phys. Soc. Jpn. **43**, 1819 (1977).
[16]B. D. Fried and R. W. Gould, Phys. Fluids **4**, 139 (1961).
[17]N. Dubouloz, R. Pottelette, M. Malingre, and R. A. Treumann, Geophys. Res. Lett. **18**, 155 (1991).
[18]R. L. Mace, S. Baboolal, R. Bharuthram, and M. A. Hellberg, J. Plasma Phys. **45**, 323 (1991).
[19]S. V. Singh, R. V. Reddy, and G. S. Lakhina, Adv. Space Res. **28**, 1643 (2001).
[20]S. V. Singh and G. S. Lakhina, Planet. Space Sci. **49**, 107 (2001).
[21]S. V. Singh and G. S. Lakhina, Nonlinear Processes Geophys. **11**, 275 (2004).
[22]S. G. Tagare, S. V. Singh, R. V. Reddy, and G. S. Lakhina, Nonlinear Processes Geophys. **11**, 215 (2004).
[23]M. Berthomier, R. Pottelette, M. Malingre, and Y. Khotyainsev, Phys. Plasmas **7**, 2987 (2000).
[24]R. L. Mace and M. A. Hellberg, Phys. Plasmas **8**, 2649 (2001).
[25]W. F. El-Taibany, J. Geophys. Res. **110**, A01213 (2005).
[26]F. Verheest, T. Cattaert, and M. Hellberg, Space Sci. Rev. **121**, 299 (2005).
[27]G. K. Parks, M. McCarthy, R. J. Fitzenreiter *et al.*, J. Geophys. Res. **98**, 15509 (1993).
[28]T. G. Onsager, M. F. Thomsen, R. C. Elphic, J. T. Gosling, R. R. Anderson, and G. Kettmann, J. Geophys. Res. **98**, 15509 (1993).
[29]K. Takahashi, and E. W. Hones, Jr., J. Geophys. Res. **93**, 8558 (1988).
[30]G. K. Parks, L. J. Chen, M. McCarthy *et al.*, Geophys. Res. Lett. **25**, 425 (1998).
[31]C. A. Cattell, J. Dombeck, J. R. Wygant *et al.*, Geophys. Res. Lett. **26**, 425 (1999).
[32]S. Baboolal, R. Bharuthram, and M. Hellberg, J. Plasma Phys. **44**, 1 (1990).

Downloaded 18 Sep 2008 to 203.193.153.35. Redistribution subject to AIP license or copyright; see http://pop.aip.org/pop/copyright.jsp