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APPLYING A LEAST SQUARE BASED TECHNIQUE TO PROTON MAGNETOMETER

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ABSTRACT

A proton magnetometer has been designed using a microcontroller. Microcontroller is used to implement a least square based signal processing technique to estimate the precession frequency of proton precessions. The dependence of accuracy on the signal to noise ratio and decay of the signal is discussed. The processing technique used to estimate the frequency is discussed. Results are compared with data from other standard magnetometers. Results show a considerable amount of increase in the measurement accuracy compared to conventional methods.

1. INTRODUCTION

A proton precession magnetometer (PPM) is used for measuring the geomagnetic field (Wienert[1], 1970). It involves measuring the sinusoidal oscillation frequency induced in the sensor coil by the precession of protons in the ambient geomagnetic field. The typical frequency for the magnetic field of the earth ranges from 1 kHz to 4 kHz. The protons are first polarized in a chosen direction with the help of a relatively large polarizing current. When the polarization current is withdrawn the proton's precession around the earth's magnetic field is initiated. The signal amplitude decays with spin- phase memory time constant T_2. This time constant depends on the liquid in which the sensor is immersed and on the gradient of magnetic field across the sensor. Even in a uniform field, this time rarely exceeds two seconds. Initial signal amplitude at the sensor output is few microvolts peak-peak.

An accuracy of 1 nT (nano Tesla) can be achieved using usual phase lock techniques. But to attain greater precision a different approach has to be adopted. In what. follows, we discuss a data processing technique that can be used to achieve better accuracy.

2. DEFINITION OF THE PROBLEM

The relation between the geomagnetic field and proton precession frequency is given by

$$B = gF_0 \tag{1}$$

Where B - Geomagnetic field in nT

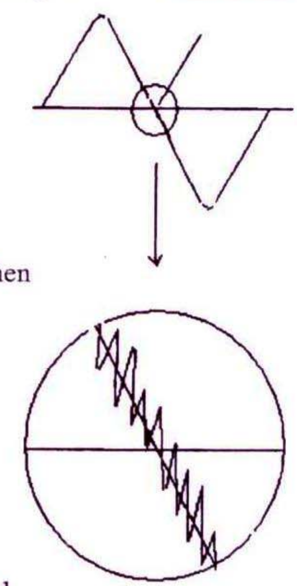
g - constant - 23.4874

F_0 - Proton Precession frequency

Zero Crossing

The frequency F_0 is estimated from the number of cycles of a higher frequency signal F_r measured during N number of signal cycles F_0. This involves recording the times of zero crossings of the signal voltage. Because of accompanying noise, an error ΔT introduced at zero crossing. Proton signal can be expressed as a sinusoidal wave with noise overriding it.

$$v = S.\mathrm{Sin}(\omega t) + n(t) \qquad (2)$$



Where S is signal amplitude, which decays with time as

$S = S_0 e^{-t/T_2}$, S_0 is amplitude at time zero

T_2 is transverse relaxation time constant

and n (t) is the random noise component

Let the signal S cross the threshold level Δv at time t_1. Then

$$\Delta v = S.\mathrm{Sin}\,\omega t_1 + n(t_1)$$

and

$$t_1 = \frac{1}{\omega}\mathrm{Sin}^{-1}\frac{\Delta v - n}{S}$$

Let us assume that the pure sine wave would have crossed the threshold ΔV at time t_2.

$$\Delta v = S.\mathrm{Sin}\,\omega t_2$$

$$t_2 = \frac{1}{\omega} Sin^{-1}\frac{\Delta v}{S}$$

The error ΔT introduced by the noise is then given by

$$\Delta T = t_1 - t_2 = \frac{1}{\omega R} \qquad (3)$$

Where $R = \dfrac{S}{n}$ is the signal to noise ratio, which decays with time as:

$$R = R_0 e^{-t/T_2}$$

Here R_0 is the signal to noise ratio at time zero. Thus measurement of period T_0 is accompanied by an error ΔT at every zero crossing of the signal. Error arising from the fluctuations of the zero crossings generated by the noise component has to be treated using special techniques. This error can be minimized with different processing techniques. Here we discuss the Least square technique.

3. LEAST SQUARE TECHNIQUE

In this method period of the sinusoidal signal is derived from N number of zero crossings. The period is determined from N values of tp (p=O,N), the observed times of the zero crossings. each having a variance of σ_p. If T_c is the exact time period then after p periods the relation between T_c and measured time t_p is given by

$$t_0 + pT_c = t_p$$

Where

$$T_p = pTo + \Delta t_p$$

$$p = 0, 1, 2 \ldots\ldots N$$

and

t_o = uncertainty in the timing of the first zero crossing.

The techniques used here, derive T_c in such a way that its variance is as small as possible. The standard technique used, attempts to minimize the mean square difference of the actual zero crossing times and those computed from the estimated fit. This is referred to as the least square method here.

S^2, The sum of the squares of the difference between the observed time and fitted value of the zero crossings is given by

$$S^2 = \sum_0^N \frac{(t_0 + pT_c - tp)^2}{\sigma_p^2}$$

Differentiating the above expression, with respect to T_c and t_0 and solving for T_c we get

$$T_c = \frac{S_A S_E - S_B S_D}{S_A S_C - S_B^2} \qquad (5)$$

Where $S_A = \sum_{p=0}^{N} \frac{1}{\sigma_p^2}$, $S_B = \sum_{p=0}^{N} \frac{p}{\sigma_p^2}$, $S_C = \sum_{p=0}^{N} \frac{p^2}{\sigma_p^2}$, $S_D = \sum_{p=0}^{N} \frac{tp}{\sigma_p^2}$, $S_E = \sum_{p=0}^{N} \frac{p\,tp}{\sigma_p^2}$

Variance of T_c is given by $\sigma(T_c) = \sqrt{\frac{S_A}{S_A S_C - S_B^2}}$

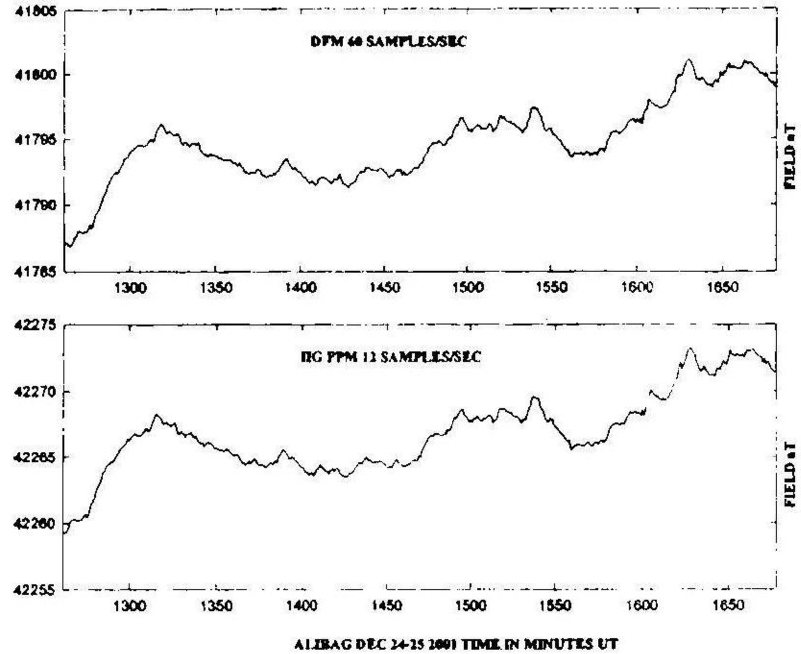


Fig.1 : Plot of F the total magnetic field variation at Alibag from the PPM and the corresponding value derived from fluxgate magnetometer developed by the Danish Meteorological Observatory with 0.1 nT. The offset in the figures is because of different location of the instruments.

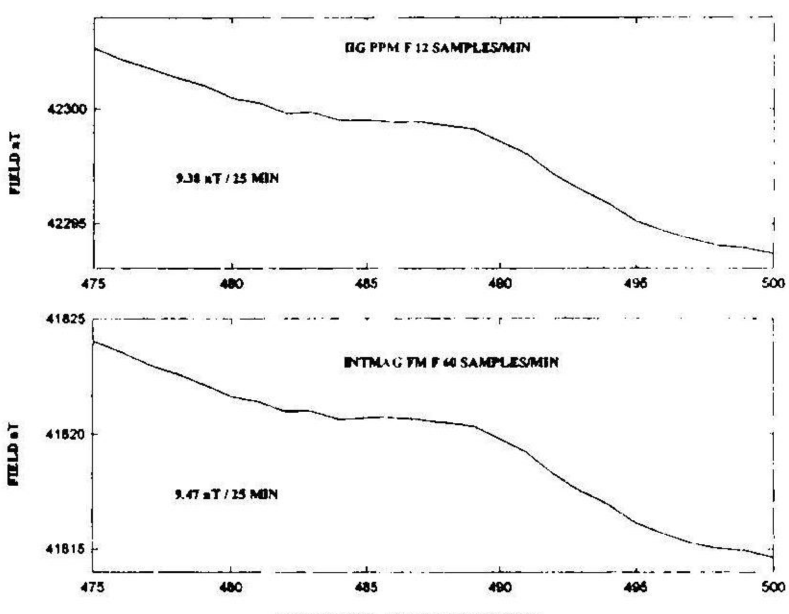


Fig.2 : Same as Figure 1 but confined to a shorter period of rapid changes to demonstrate the authenticity of the short period variations of the PPM

The above expressions provided by Hancke[2] (1990) can be retrieved if it is assumed that the variance in the estimates of the time of zero crossings remains same right through the measurement cycle.

4. RESULTS

The instrument was operated at the Alibag observatory and the results were compared with a standard fluxgate magnetometer developed by the Danish Meteorological Observatory with 0.1 nT accuracy. Figures 1 and 2 provide a good account of the sensitivity of the PPM. These plots show field variations at Alibag (18.64 N, 72.87 E geographic co-ordinates). The number of signal cycles of the proton precession utilized was 1000. The signal to noise ratio was 10 in the beginning and the signal decay time T_2 was around 1.5 seconds ensuring that over the entire measurement period the signal to noise ratio was above 6. The remarkable similarity in the long period variations and trends (Figure 1) and in the short period rapid changes (Figure 2) bring out very effectively the authentic response of the PPM to changes even of the order of 0.1 nT. The offset between the two instruments is due to the different locations.

5. CONCLUSION

Applying least square techniques, using microcontroller, the accuracy and the sensitivity of the PPM measurements improve considerably. With ten second sampling, the PPM estimates based on least square algorithms can generate values reliable up to 0.1 nT.

REFERENCES

Books : [1] Wienert K.A., Notes on geomagnetic observatory and survey practice, United Nations Educational, Scientific, and Cultural Organisation, 1970.

Journals : [2] Hancke G.P., Transactions on Instrumentation and measurement, Vol. 39, No 6, December 1990.