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Dispersive properties of ULF waves at surfaces with finite width and finite temperature

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The dispersion relation for surface eigen modes with a finite width and having finite temperature is derived. The model surface may be identified with the magnetopause or the plasmapause. The spectrum depends on the temperature and the width of the surface. Basically there are two distinct branches, one of which is in the higher frequency range than the other. The possible implications of these modes in geomagnetic micropulsation, have been presented and discussed.

1 Introduction

In nature as well as in laboratory the non-uniform plasma is often so structured that the plasma parameters undergo very sharp variations within a short distance. Such a system is generally described as a surface separating two uniform media. New wave modes then become available to the plasma with distinctive properties which can explain many observed wave properties in space plasma.

The concept of such 'surface waves' has been successfully applied by Roberts[1] in solar plasma. Hollweg[2] also showed that new modes appear if the tangential discontinuity in the solar wind is looked upon as a 'surface'. The work of Chen and Hasegawa[3] gives some explanation about the Pc-5 micropulsation as surface waves excited at the plasmapause in the terrestrial magnetosphere. Similarly, the works of Somasundaram and Uberoi[4] and Uberoi[5] deal with the compressional surface modes in the magnetopause.

In all these cases the surface is truly a mathematical surface with zero width. In nature, however, the discontinuity is established within a finite distance as if the surface possesses a finite width. Moreover, the plasma in this surface region can, sometimes, be considerably hotter than the two adjoining semi-infinite media. The question of temperature , which is really irrelevant in the concept of zero-width surface, becomes an important controlling factor when the surface has a finite width. Actually the analysis by Chen and Hasegawa[3] treats the surface to have finite extent. But they confine their interest only on very localized mode so that the temperature does not come into the picture. However, it is a matter of record that the value of β (the ratio of thermal pressure to the magnetic pressure) at the magnetoapuse is usually high[6], i.e $\beta \approx 1$ or sometimes even more than 5. Moreover, intense low frequency wave activities observed by spacecrafts[7-9] show limited but finite spatial extent near the magnetopause surface. Similarly, the plasmapause surface during geomagnetic strom is populated by hot plasma[10-12] which is the source of intense ring current.

Although the surface waves are known to play an important role in the absorption of energy[5,13] from an incoming or existing wave source, the finite β effect on its spectrum has not been explored yet. In fact, the spectral characteristics are rather important in the energy absorption process.

The aim of this study is to derive an expression for the dispersion relation for the eigen modes of the surface waves. The spectrum in general will depend on the various parameters like width, temperature of the surface and level of sharp change in the Alfven velocity across the surface. The possible relevance of these new modes in connection with the observed wave phenomena in the magnetosphere is also explored.

2 Surface model and derivation of the dispersion relations

Here the relevant plasma parameter which causes the discontinuity is considered to be the Alfven velocity V_a. With respect to the plasmapause surface the V_a value is much smaller on the magnetopause side than that on the earth side[14]. The plasmapause itself extends over a distance of a few hundreds of kilometers or more in the radial direction[14]. During the main phase of a geomagnetic storm this region is populated with high temperature plasma. Moreover, this transition region may be considerably wider than the hypothetical zero-width surface. For the purpose of mathematical derivation of the dispersion formula, this situation can be approximated by the following model as depicted in Fig. 1. A surface of width a and having a finite β with an Alfven velocity V_a is sandwiched between two cold plasma media with uniform Alfven velocity V_{a1} and V_{a3} respectively. The ambient magnetic fields at all the three regions are in the $\hat{z}$ direction. The variations perpendicular to the surface are along the radial $\hat{y}$ direction. Similar three-layer model was also analyzed by Wolfe *et al.*[15] in connection with the penetration of magnetopause waves into the inner magnetosphere. In their situation, the Alfven velocity in the surface zone is greater than that in both the side zones, thereby producing a hump in V_a profile as opposed to a dip as depicted in Fig. 1(b). Moreover, they did not consider the effect of finite β on the dispersion relation and the spectral character. As mentioned earlier the plasmapause during geomagnetic storm is populated by hot plasma. Therefore, we have to consider finite β case.

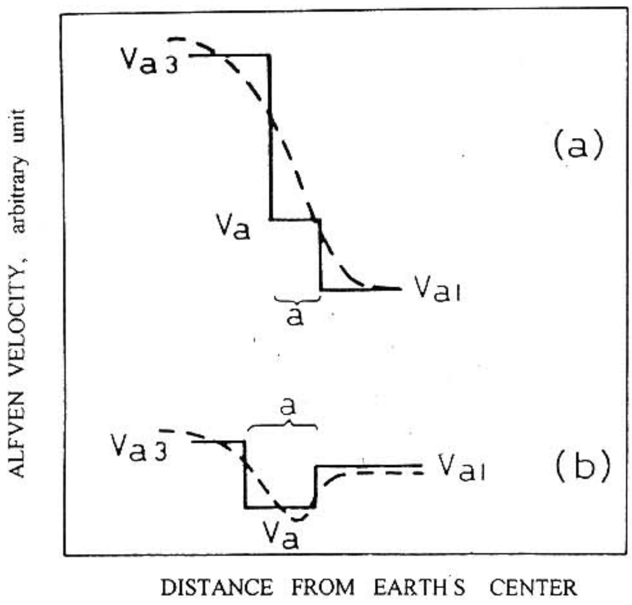

Fig. 1—Surface model depicting the steep change in the Alfven velocity (The dashed line is the actual[10] profile and the solid line is the model.)

We study the response of this system to a perturbation with frequency ω and wave vectors $\mathbf{k}_\parallel$ and $\mathbf{k}_\perp$, respectively. In the limit of frequencies much smaller than the ion cyclotron frequency, the one-fluid model of the magneto-hydrodynamic picture of the plasma seems to be an adequate description. The fluid displacement $\xi_y(y)$ in the y direction as a function of y will obey the differential equation[3]

$$\frac{d}{dy}\left\{\frac{\varepsilon\alpha B_0^2}{\varepsilon-\alpha B_0^2 k_\perp^2}\,\xi_y'\right\}+\varepsilon\,\xi_y=0 \qquad (1)$$

where,

$$\varepsilon = k_\parallel^2\, B_0^2\,(\psi-1)$$

$$\alpha = 1+\frac{\beta\psi}{\psi-\beta}$$

Here, ψ is the normalized frequency, $\omega^2/k_\parallel^2 V_a^2$, and all lengths are scaled by $\mathbf{k}_\parallel^{-1}$.

We seek the solutions in the three regions separately, each with a constant V_a. These have to be matched at the two boundaries y=0 and y=a. In the regions on both sides of the surface, the solutions have to be of the form $\sim e^{\lambda_1 y}$ for $y\le 0$ and $e^{-\lambda_3 y}$ for $y\ge 0$. Between y=0 and y=a, the solution is the linear combination of $e^{\pm\lambda y}$, where

$$\lambda_{1,3}=\sqrt{k^2-h_{1,3}\psi} \qquad (2)$$

$$\lambda=\sqrt{k^2-\frac{\psi^2}{(\beta+1)\psi-\beta}} \qquad (3)$$

$$h_{1,3}=\frac{V_a^2}{V_{a1,3}^2} \qquad (4)$$

The parameter λ can be further written as

$$-\lambda^2=\frac{(\psi-a_1)(\psi-a_2)}{(\beta+1)(\psi-a_3)} \qquad (5)$$

$$a_{1,2}=(\beta+1)\frac{k^2}{2}\left\{1\pm\sqrt{1-\frac{4\beta}{k^2(\beta+1)^2}}\right\} \qquad (6)$$

$$a_3 = \frac{\beta}{\beta+1} \quad \ldots (7)$$

As we are interested in the surface mode, $\lambda_{1,3}$ are taken to be positive and real for real ω. However, λ admits either real or imaginary values.

Since the effective pressure across the boundaries has to be continuous, another quantity to match at both the boundaries is the pressure perturbation defined by[3]

$$\tilde{p} = \frac{\alpha B_0^2 \varepsilon}{\varepsilon - \alpha B_0^2 k_\perp^2} \xi'_y \quad \ldots (8)$$

Applying the above mentioned boundary conditions one gets the dispersion equation as

$$e^{2\lambda a} = \frac{(S_1-\lambda)(S_3-\lambda)}{(S_1+\lambda)(S_3+\lambda)} \quad \ldots (9)$$

$$S_{1,3} = \frac{(\psi-1)\lambda_{1,3}}{(\beta+1)(h_{1,3}\psi-1)} \quad \ldots (10)$$

The Eqs (9) and (10) will be reduced to those given by Wolfe *et al.*[15] if we put $\beta = 0$.

Before attempting any numerical calculations from the Eqs (9) and (10), let us first discuss the feasibilities of the existence of the eigen values. This will depend on the numerical values of $h_{1,3}$ or in other words on the nature of the Alfven velocity profile in the neighbourhood of the surface. First we consider the case of gradual transition from small V_{a1} in region I to large V_{a3} in region III through intermediate V_a in the surface region as depicted in Fig. 1(a). In this case $h_1>1$ whereas $h_3<1$. For such a situation the eigen values, if exist at all, must be limited by

$$\psi_{\max} = h_1^{-1} k^2$$

where $k^2 = 1 + k_\perp^2$. There are two sets of eigen values, one corresponding to real values for λ and the other for imaginary λ.

In the case of real λ, since the left hand side of Eq. (9) is greater than one, we must have either S_1 or S_3 negative. The S_1 negative mode is bounded by $h_1^{-1} \le \psi \le 1$. This is basically a low frequency mode. For large value of $\mathbf{k}_\perp$

$\lambda \simeq \lambda_1 \simeq \lambda_3 \simeq k^2$

The solution of Eq. (8) will exist when

$$-S_1 \simeq \lambda_1 \text{ or, } \psi \simeq \frac{\beta+2}{1+h_1(\beta+1)}$$

Similarly, for real λ the $S_3<0$ mode is bounded by $1<\psi<h_3^{-1}$ provided this range is below $\psi_{\max}$. Therefore, this mode will not admit very low $\mathbf{k}_\perp$. The dispersion curves for these modes are shown in Fig. 2 for $h_1=2$, $h_3=0.5$ and $\beta = 0.5$, respectively, with different surface width a as labelled.

The low frequency branch ($S_1<0$) shows a maximum in the region $\mathbf{k}_\perp \simeq \mathbf{k}_\parallel$ and, therefore, zero group velocity. The very low frequency region is highly dispersive, frequency decreasing with increasing surface width for fixed $\mathbf{k}_\perp$. In contrast the other mode ($S_3<0$) is less dispersive and the frequency generally increases with increasing a for a given $\mathbf{k}_\perp$. The high $\mathbf{k}_\perp$ parts of both the branches are virtually dispersion less with almost zero group velocity. Therefore, signal generated in a certain region will hardly propagate in the azimuthal direction.

There is a third mode for which λ assumes

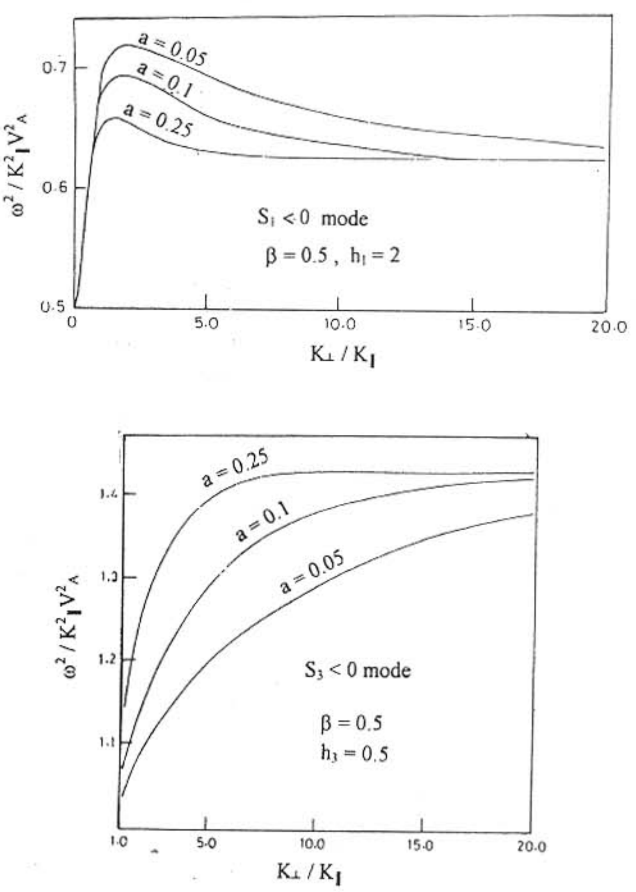

Fig. 2—Eigen modes corresponding to the model Fig. 1(a)

imaginary values (not shown in the figure) which must lie between $a_3<\psi<a_2$ But we know,

$\lim_{k_\perp \to 0} a_2 = \beta$ and $\lim_{k_\perp \to \infty} a_2 = a_3 = \frac{\beta}{\beta+1}$.

Moreover, the eigen frequency must be less than $h_1^{-1}k^2$. Thus the solution will exist only for relatively higher $\mathbf{k}_\perp$ and, therefore, closer to $\frac{\beta}{\beta+1}$.

The V_a profile near the surface may undergo a small dip which can be modelled as an asymmetric square-well profile as shown in Fig. 1(b). The corresponding spectra with h_1=0.25 and h_3=0.5 are shown in Fig. 3. In this case, the two branches are identified as negative S_3 and negative S_1, S_3 branches, the former having higher frequencies than the latter.

For real values of λ there may be a mode for which both S_1 and S_3 are negative. In that case, ψ has to be greater than both h_1^{-1} and h_3^{-3} with $\psi<1$. But this is not possible if any of h_1 or h_3 is fractional. Thus, this low frequency mode is not accessible to a V_a profile which increases gradually across the surface [Fig. 1(a)] or there is a dip in V_a [Fig. 1(b)]. In the latter case, the high frequency part ($\psi>1$), however, can exist as shown in the Fig. 3. It may be noted that if there is a hump in the V_a profile as considered by Wolfe *et al.*[15] in the magnetopause case, this forbidden low frequency ($\psi<1$) mode with both $S_{1,3}$ negative would be an allowed eigen mode.

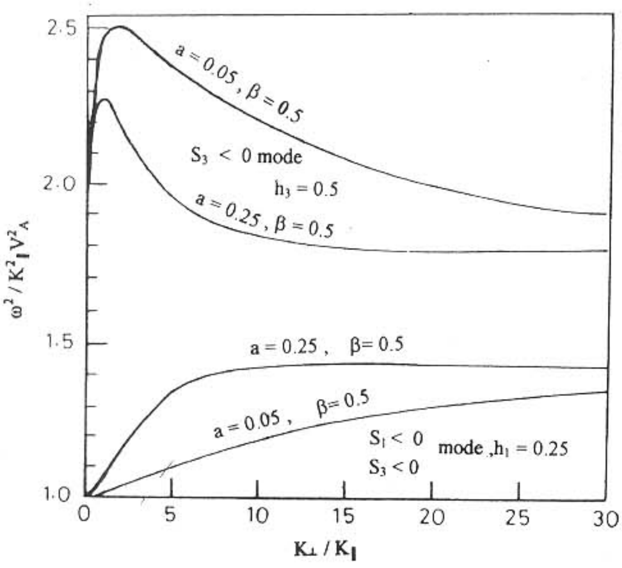

Fig. 3—Eigen modes corresponding to the model Fig. 1(b)

The degree of compression for each mode can be calculated from the expressions

$$\frac{b_z}{b_x} = \frac{k_\perp^2 - \lambda^2}{\mathbf{k}_\perp \mathbf{k}_\parallel}$$

$$\frac{b_x}{b_y} = \frac{k_\perp^2}{\lambda^2} \xi_y' / \xi_y$$

where b_z and b_x are the magnetic perturbations in the directions of $\mathbf{k}_\parallel$ and $\mathbf{k}_\perp$, respectively. The parameter b_y is, therefore, perturbation in the third direction. In the regions I and III, ξ_y'/ξ_y is just λ_1 and λ_3, respectively. In the surface region one can calculate the solution, ξ_y in terms of $\xi_y(0)$ and then evaluate ξ_y'/ξ_y. The relative variations of different components of the field are summarized in Table 1.

3 Discussion

The importance of surface waves in space plasma has two aspects. If a broad band of frequencies are incident on the surface, only the surface eigen modes are likely to transmit them across it. The day side Pc3-4 micropulsations are thought to be generated in the solar wind. But they are observed even at very low latitudes on the ground. Therefore, they must have penetrated across the magnetopause as well as the plasmapause at the inner magnetosphere. The sudden compression of the magnetopause also should generate surface eigen modes. When a sudden compression is transmitted across the magnetosphere it has to pass through the plasmapause discontinuity also. A new set of eigen modes are then likely to be generated.

The second aspect is that the waves generated by any other mechanism will be absorbed in the eigen mode part of the spectrum, if these modes have damping rather than instability. In this paper the stability properties are not addressed.

Generally, surfaces are supposed to be ideally very thin. But the storm time plasmapause region is, indeed, populated by hot plasma and the observed Pc3-5 waves are generally attributed to be generated by drift mirror instability or by bounce resonance. But the plasmapause is a known discontinuity in Alfven speed profile. Thus, it is expected to respond to the type of

Table 1—Relative variations of different components of magnetospheric fields under various conditions

Variations of perturbations			Conditions
Region I	Region II	Region III	
$b_x >> b_y \approx b_z$	$b_z >> b_y \approx b_x$	$b_z >> b_y \approx b_x$	for $\mathbf{k}_\perp < \mathbf{k}_\parallel$, $S_1 < 0$ mode
$b_x > b_y > b_z$	$b_z >> b_x > b_y$	$b_y > b_x > b_z$	for $\mathbf{k}_\perp < \mathbf{k}_\parallel$, $S_1 < 0$ mode
$b_x \approx b_y >> b_z$	$b_x > b_y \approx b_z$	$b_x \approx b_y >> b_z$	for $\mathbf{k}_\perp > \mathbf{k}_\parallel$, $S_1 < 0$ mode
$b_z \approx b_x >> b_y$	$b_x > b_y > b_z$	$b_x \approx b_y >> b_z$	for $\mathbf{k}_\perp > \mathbf{k}_\parallel$, $S_3 < 0$ mode
$b_x > b_z > b_y$	$b_z >> b_x > b_y$	$b_x \approx b_y >> b_z$	for $\mathbf{k}_\perp > \mathbf{k}_\parallel$, λ imaginary mode

surface described in this article. There are observational evidences by Barfield and Lin[16] and Anderson[17] of Pc5 pulsations associated with ring currents where the Alfven velocity undergoes sharp change. It has also been suggested by Sutcliffe[18] that the mechanism of Pi2 pulsations may either be the surface modes at the plasmapause or be cavity resonances as shown by Saito and Matsushita[19]. Again spacecraft observation by Takashahi[20] shows that the compressional Pi2 waves are associated with a small radial component with small azimuthal wave number (i.e. $\mathbf{k}_\perp \leq \mathbf{k}_\parallel$). These may be the present type of surface modes of the $S_1 \leq 0$ branch of the spectrum. It is interesting to note that Pi2 pulsations at geostationary orbits often show strong azimuthal perturbations as evident from the work of Sakurai and McPherron[21]. In the framework of the present theory, negative S_1 mode in the region I, which indeed will be geostationary orbit with respect to plasmapause, does show similar property.

It may also be noted that simultaneous existence of Pc1 and Pc5, i.e. both high and low frequency hydromagnetic waves in the magnetosphere have been explained by Namikawa *et al.*[22] in terms of high β plasma near the plasmapause. The present theory also predicts two distinct branches in the surface mode spectrum. The negative S_1 frequency band is three times smaller than the negative S_3 band if we take $\beta = 0.5$ and $a = 0.25$. When β becomes as high as 1 it is conceivable that the ratio of the frequencies of these two bands may become as high as even 10. Thus, hot surface can support both high frequency Pc1 or Pc2 and the low frequency Pc5 waves. It may be pointed out here that although Pc1 is, generally, argued to be excited by ion-cyclotron waves (ω comparable to ion gyro-frequency) hydromagnetic surface waves with small ω may also be in the Pc1 or Pc2.

It is curious to note that the observed spectrum of the ULF waves are, generally, discrete in nature as shown by Samson and Rostoker[23] although there is actually a low level power at a wide band of frequencies in the magnetosphere as explained by Takashahi and McPherron[24]. Since surface waves satisfy only a narrow band condition it is more than possible that many of the observed micropulsation events are actually plasmapause-related surface waves.

Another characteristic of the present type of eigen modes is the variation of the polarization pattern with radial distance. In fact the observational Pc5 pulsations at a latitudinal chain of ground stations by Oksman *et al.*[25] (which may be translated as radial stations at the equatorial plane in the plasmasphere and magnetosphere) do indeed indicate that the waves might be due to surface modes at the plasmapause.

Looking at the dispersion curves we note the interesting feature of these modes that the group velocity in the azimuthal direction is rising for low $\mathbf{k}_\perp$ values and falling for high $\mathbf{k}_\perp$ values. In between the group velocity is zero as is for very high $\mathbf{k}_\perp$. Therefore, one of the distinctive feature of these waves should be different wave packets being detected with finite time lag. In fact, observations by Iyemori and Hayashi[26] at low altitude satellites at high latitudes do show such events originating at plasmapause.

One of the characteristics of the surface modes is that the polarization properties are different at different zones on the two sides of the surface. If we look at the localized modes (i.e. $\mathbf{k}_\perp >> \mathbf{k}_\parallel$), the waves are compressional at the surface location, but of toroidal type on the two sides of the surface with imaginary λ which has a rather narrow spectrum. Moreover, there is a mode where the surface waves are highly compressive, generally, for low $\mathbf{k}_\perp$ values unlike the drift mirror waves which have high $\mathbf{k}_\perp$.

Just as the drift mirror waves or the cavity modes can be coupled with the local field line resonances, physically it is natural to conclude that so do the surface modes. Therefore, there is a possibility of the plasmapause surface waves to excite the low lying field lines corresponding to low latitudes on the ground. The requirement for such condition is that the decay length, λ_3^{-1}, on the earth side of the surface should be large. Since $\lambda_3^2=k^2-\omega^2/V_{a3}^2$, large decay length will mean small $\mathbf{k}_\perp$, which exists in the present model. Thus, the low latitude ground stations may actually sense the plasmapause surface modes although these are supposed to be radially localized by the very definition of the surface. This coupling mechanism needs to be investigated more thoroughly.

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