Pitch angle transport of electrons due to cyclotron interactions with the coherent chorus subelements

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[1] Chorus is a right-hand, circularly-polarized electromagnetic plane wave. Dayside chorus is a bursty emission composed of rising frequency "elements" with duration of ~0.1 to 1.0 s. Each element is composed of coherent subelements with durations of ~1 to 100 ms or more. Due to the coherent nature of the chorus subelements/wave packets, energetic electrons with pitch angles near the loss cone may stay in resonance with the waves for more than one wave cycle. The electrons could therefore be "transported" in pitch across a relatively large angle from a single wave-particle interaction. Here we study the cyclotron resonance of the energetic electron with the coherent chorus subelements. We consider a Gaussian distribution for the time duration of the chorus subelements and derive an expression for the pitch angle transport due to this interaction. For typical chorus subelement parameters, the average pitch angle diffusion coefficients ~(0.5–8.5) s^{-1} are found. Such rapid pitch angle scattering may provide an explanation for the ionospheric microbursts of ~0.1 to 0.5 s in bremsstrahlung x-rays formed by ~10–100 keV precipitating electrons. The model is applicable to the cases when $R = t_{tr}/\Delta t = [(\omega + \Omega/2)\ t_{tr}/\omega\tau] > 1$ and inhomogeneity factor $S = t_{tr}^2/t_{inh}^2 < 1$, where Ω is the electron cyclotron frequency in the ambient magnetic field, B_0, ω is the frequency of chorus, t_{tr} is the trapping time (or phase oscillation period), t_{inh} is the time for the passage through the resonance in the inhomogeneous magnetic field, and τ is the duration of the chorus subelement. For the typical parameters at $L = 5$, the energetic electrons having pitch angles of $\alpha \leq \pi/3$ can satisfy both the condition $R > 1$ and $S < 1$ for a range of chorus wave amplitudes.

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1. Introduction

[2] Chorus is a right-hand circularly-polarized electromagnetic plane wave. It was observed first on the ground in the local morning hours about more than 65 years ago [*Storey*, 1953; *Helliwell*, 1967, 1969]. It occurs in the frequency range from a few hundreds of Hertz to several kHz. When the data are converted to audio and played on a tape recorder or other audio device, its sound resembles the sound of chirping birds in the morning, giving it the name "dawn chorus".

[3] Early spacecraft observations showed that chorus occurs usually outside the plasmapause during the local morning and day time [*Gurnett and O'Brien*, 1964; *Burtis and Helliwell*, 1976; *Tsurutani and Smith*, 1974, 1977; *Burton and Holzer*, 1974; *Anderson and Maeda*, 1977; *Cornilleau-Wehrlin et al.*, 1978; *Koons and Roeder*, 1990]. Chorus is a bursty electromagnetic emission with element durations of ~0.1 to 1.0 s. Chorus elements are composed of a sequence of subelements [*Santolík et al.*, 2001, 2003, 2004; *Tsurutani et al.*, 2009; *Verkhoglyadova et al.*, 2009]. The subelements take the form of small wave packets with durations from 1 to 100 ms. Time average chorus amplitudes are usually in the range 0.01–0.05 nT or lower [*Tsurutani and Smith*, 1974; *Burtis and Helliwell*, 1976; *Meredith et al.*, 2001; *Andre et al.*, 2002]. However this includes inter-element and intersubelement spacings. Subelements have been found to have much higher intensities of ~0.2 to 0.3 nT [*Tsurutani et al.*, 2009; *Li et al.*, 2009]. Chorus elements sometimes have frequencies that increase or fall in time. Each element is similar to the other elements of the ~hr long chorus event. They generally sweep at the same rate. However, different events have different element frequency-time shapes. Dayside chorus waves are usually composed of rising tones. In the evening sector, chorus elements are often falling tones or emissions with flat frequency-time profiles [*Tsurutani and Smith*, 1974, 1977]. For our effort we assume that the frequency within the subelement is constant with time. If the element sweeps upward (downward) in time, we assume that the next subelement is at a higher (lower) frequency and is constant throughout the subelement interval, which is a

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reasonable assumption based on analysis of subelement structure within chorus elements from observations.

[4] It has been shown that chorus is generated close to the geomagnetic equatorial plane or in minimum B field pockets [*Tsurutani and Smith*, 1974, 1977; *LeDocq et al.*, 1998; *Lauben et al.*, 2002; *Omura et al.*, 2008]. The equatorial chorus is generated by the loss cone instability [*Kennel and Petschek*, 1966; *Tsurutani and Lakhina*, 1997] due to anisotropic ~10–100 keV electrons [*Tsurutani et al.*, 1979]. In the source region, two frequency bands of chorus have been observed [*Tsurutani and Smith*, 1974; *Burtis and Helliwell*, 1976; *Goldstein and Tsurutani*, 1984], below and above one half of the local electron cyclotron frequency. These bands are separated by a narrow frequency interval of decreased (or no) intensity just at one half of the local electron cyclotron frequency [*Tsurutani and Smith*, 1974; *Santolík*, 2008]. *Tsurutani and Smith* [1974] suggested that the attenuation band at one-half the electron gyrofrequency may be due to the Landau damping by electrons that are traveling in the same direction as the waves, and having energy close to cyclotron resonance energy. From the ray tracing studies, *Bortnik et al.* [2006] found that Landau damping effects could produce a gap at one half of the electron cyclotron frequency, but generally only after the chorus had propagated a significant distance away from the magnetic equator. Recently, *Omura et al.* [2009] have tried to explain the gap in terms of the nonlinear damping of the longitudinal component of a slightly oblique whistler wave packet propagating along the inhomogeneous ambient magnetic field. On the other hand, *Bell et al.* [2009] have put forward a suggestion that the gap can be explained naturally by assuming the chorus to be excited in ducts of either enhanced (lower band) or depleted (upper band) cold plasma density.

[5] Emissions of equatorial chorus are often observed during periods of disturbed magnetospheric conditions [*Tsurutani and Smith*, 1974, 1977; *Inan et al.*, 1992; *Lauben et al.*, 1998; *Meredith et al.*, 2001], in association with anisotropic electrons at energies of 10–100 keV [*Burton*, 1976; *Anderson and Maeda*, 1977; *Tsurutani et al.*, 1979, 2006; *Kasahara et al.*, 2009]. Wave-particle interactions between electromagnetic "chorus" emissions and high energy electrons of E ~ 100 keV are believed to play an important role in the acceleration of electrons to relativistic energies [*Horne and Thorne*, 1998; *Summers et al.*, 1998, 2004; *Roth et al.*, 1999; *Meredith et al.*, 2002, 2003; *Horne et al.*, 2003, 2005a, 2005b]. Further, the gyroresonant interaction of chorus with ~10–100 keV electrons can also give rise to electron pitch angle scattering into the loss-cone, thereby leading to their precipitation into the atmosphere as "microbursts", and a net loss of energetic electrons from the outer radiation belt [*Anderson and Milton*, 1964; *Parks*, 1967; *Parks and Winckler*, 1969; *Nakamura et al.*, 2000; *Lorentzen et al.*, 2001a, 2001b; *Summers et al.*, 2007a, 2007b; *Thorne et al.*, 2005].

[6] Generally, the studies on wave-particle interactions involving whistler mode chorus treat the chorus waves as incoherent broadband emissions and then apply quasi-linear diffusion theory to obtain the saturation amplitudes and diffusion coefficients, [e.g., *Kennel and Petschek*, 1966; *Albert*, 2005; *Horne et al.*, 2005b; *Summers*, 2005]. Strictly speaking, as the VLF chorus consists of coherent, discrete narrowband emissions, the usual quasi-linear diffusion theory can not provide an accurate description of the electron-VLF chorus interaction [*Inan et al.*, 1978; *Albert*, 2002; *Trakhtengerts et al.*, 2003; *Omura and Summers*, 2006]. Due to the coherent nature of the chorus subelements/wave packets, the energetic electrons near the loss cone may stay in resonance with the waves for more than one wave cycle. The electrons could therefore be "transported" in pitch across a relatively large angle from a single wave-particle interaction.

[7] The interaction of energetic electrons with coherent whistler waves in a dipole magnetic field has been studied either by the direct analysis of the equation of motions [*Gendrin*, 1974; *Inan et al.*, 1978; *Omura and Summers*, 2006; *Omura et al.*, 2007, 2008; *Bortnik et al.*, 2008, and references therein] or by a Hamiltonian formulation [*Albert*, 2001, 2002]. The nonlinear particle motion involves a separatrix in phase space and the particle trapping can occur in the so-called "trapping potential" which is affected by the inhomogeneity of the geomagnetic field [*Omura and Summers*, 2006]. The trapping frequency essentially determines the type of behavior of the particle motion. *Albert* [2002] has shown that when R, the ratio of phase oscillation period (or trapping time) at resonance to the time-scale for passage through resonance (or resonant interaction time), is large (i.e., $R > 1$), the phase at resonance is effectively random, resulting in pitch angle and energy diffusion. For $R \sim 1$ phase trapping can occur which leads to large rates of systematic pitch angle and energy change. However, for $R < 1$ phase bunching can occur.

[8] Here we study the cyclotron resonance of the energetic electron with the coherent chorus waves. We consider a power law distribution for the time duration of the chorus elements and derive an expression for the pitch angle transport due to this interaction. The model is simple and straightforward, captures main physics of this interaction for the case of $R > 1$, i.e., when the phase trapping time is larger than the resonant interaction time.

2. Wave-Particle Cyclotron Resonant Interactions

[9] The wave-particle cyclotron resonance condition is given by:

$$\omega - k_{\parallel} v_{\parallel} = n\Omega/\gamma \tag{1}$$

where ω is the wave frequency, $k_{\parallel}$ and $v_{\parallel}$ are the parallel (to $\boldsymbol{B_0}$) components of the wave vector, $\boldsymbol{k}$, and the particle velocity vector, $\boldsymbol{v}$, respectively, Ω is the electron cyclotron frequency, $n = (0, \pm 1, \pm 2, \ldots)$ is the harmonic number, and $\gamma = (1 - v^2/c^2)^{-1/2}$ is the relativistic factor. In the expression for γ, v is the particle speed and c is the speed of light.

[10] For positive (negative) values of n, equation (1) represents the normal (anomalous) resonant condition [*Tsurutani and Lakhina*, 1997]. For normal ($n = 1, 2, \ldots$) Doppler-shifted cyclotron resonance, the waves and particles are traveling in opposite directions along the magnetic field. The waves will be Doppler-shifted up to the electron cyclotron frequency or its harmonics. The more energetic the particles, the lower the frequency of the waves needed for resonance, and vice-versa, the higher the wave frequency, the less energetic the particles for resonance [*Tsurutani and Lakhina*, 1997].

[11] For the fundamental normal electron cyclotron resonance ($n = 1$), the expression in (1) can be divided by $k_{\|}$ and one gets the resonant particle velocity:

$$v_{\|R} = v_{ph}(1 - \Omega/\omega\gamma) \tag{2}$$

[12] Here v_{ph} is the parallel wave phase speed. From this expression it can be noted that the lower the wave phase speed, the lower the speed of the resonant electrons. The higher the wave phase speed, the higher the parallel speed of the resonant electrons. Further, using v_{ph} obtained from the cold plasma whistler wave dispersion relation in (2), we get the parallel kinetic energy of electrons in cyclotron resonance with the chorus wave, $E_R = mv_{\|R}^2/2 \propto B_0^2/8\pi N$, where N is the cold plasma density. Therefore, strong interaction is expected to occur near the magnetic equator or in dayside minimum B pockets where the minimum in $B_0^2/8\pi N$ permits resonance with the lower-energy, and thus higher phase space density, part of the electron distribution.

[13] The overall particle pitch angle scattering rates due to incoherent electromagnetic or electrostatic waves have been derived by *Kennel and Petschek* [1966] and have empirically been shown to be valid for the rate of scattering of electrons in the outer magnetosphere. Since the VLF chorus consists of coherent, discrete narrowband emissions [*Inan et al.*, 1978; *Omura and Summers*, 2006], the energetic electrons can stay in gyroresonance with the wave for more than a wave cycle and may undergo large pitch angle changes in a single chorus subelement-electron encounter. In this paper we derive pitch angle "transport" (scattering?) rates from simple physical arguments [*Tsurutani and Lakhina*, 1997].

3. Pitch Angle Transport

[14] The change in the particle pitch angle, α (the angle between the particle velocity vector $\boldsymbol{v}$ and $\boldsymbol{B_0}$), of electrons which are in cyclotron resonance with the chorus can be obtained from the identity: $\tan\alpha = v_\perp/v_\|$ (where $v_\perp$ is the perpendicular component of the particle velocity with respect to the magnetic field direction) as

$$\Delta\tan\alpha = \sec^2\alpha\Delta\alpha = \Delta v_\perp/v_\| - (v_\perp/v_\|^2)\Delta v_\| \tag{3}$$

which yields the relation

$$\Delta\alpha = \frac{v_\|}{v^2}\Delta v_\perp - \frac{v_\perp}{v^2}\Delta v_\| \tag{4}$$

Now, the equation of motion of a charged particle in a chorus wave electromagnetic field can be written as

$$\frac{d\mathbf{v}}{dt} = \frac{q}{m}(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \tag{5}$$

where q and m are the charge and mass of the particle, and $\mathbf{E}$ and $\mathbf{B}$ are the chorus wave electric and magnetic fields, respectively. For simplicity, we consider the parallel propagating whistler waves and nonrelativistic electron energies. From the above equation and Maxwell's equations, we can write the change in the parallel and perpendicular components of the particle velocity as a result of interaction with the chorus wave. This is written as

$$\begin{aligned}\Delta v_\| &= -\frac{q}{m}v_\perp B\sin\phi\Delta t\\ \Delta v_\perp &= \frac{q}{m}\left(v_\| - \frac{\omega}{k}\right)B\sin\phi\Delta t\end{aligned} \tag{6}$$

where ϕ is the angle between the $\mathbf{v}_\perp$ and $\mathbf{B}$, and Δt is the interaction time between the electrons and the chorus subelement. To leading order, we have

$$\frac{d\phi}{dt} = \Omega - \omega + kv_\| \tag{7}$$

From the above equation, it is clear that if $v_\| = v_{\|R}$, the electron resonates with the wave, and sees a constant phase, ϕ, of the wave field.

[15] From (4) and (6), we get

$$\Delta\alpha = \frac{B}{B_0}\Omega\sin\phi\left[1 + \frac{\omega\cos^2\alpha}{\Omega - \omega}\right]\Delta t \tag{8}$$

It is clear that $\Delta\alpha$ depends on the wave amplitude B, the interaction time, Δt, and ϕ, the initial phase angle between $v_\perp$ and the wave magnetic field B. Also, depending on the phase angle ϕ, the change in the pitch angle due to the interaction of an electron with the coherent chorus subelement could be positive or negative.

[16] Since for a given energetic electron distribution, different electrons will encounter the coherent wave at different phase angle ϕs, the resulting $\Delta\alpha$s would be randomly positive or negative with different magnitudes, just like the Brownian motion. Then, we may calculate the pitch angle diffusion [*Kennel and Petschek*, 1966; *Tsurutani and Lakhina*, 1997]:

$$D = \frac{\langle(\Delta\alpha)^2\rangle}{2\Delta t} \tag{9}$$

3.1. Calculation of Interaction Time, Δt

[17] Let τ be the time duration of a chorus subelement. Then its scale size will be $X = v_g\tau$, where $v_g = \partial\omega/\partial k$ is the group velocity of the chorus wave packet. Note that the relative speed between an electron and the chorus wave, $V_s = v_g - v_{\|R} = [v_g - v_{ph}(1 - \Omega/\omega)]$. Here we have taken the resonant electrons as nonrelativisitic and made use of equation (2).

[18] Then, the time of cyclotron resonant interaction between the electrons and the chorus wave can be estimated as:

$$\Delta t = \frac{X}{V_s} = \frac{\omega\tau}{\omega + \Omega/2} \tag{10}$$

Here, we have made use of the cold plasma dispersion relation for the whistler waves. Then, we have

$$\Delta\alpha = \frac{B}{B_0}\frac{\omega\sin\phi}{\left(\frac{\omega}{\Omega} + \frac{1}{2}\right)}\left[1 + \frac{\omega\cos^2\alpha}{\Omega - \omega}\right]\tau \tag{11}$$

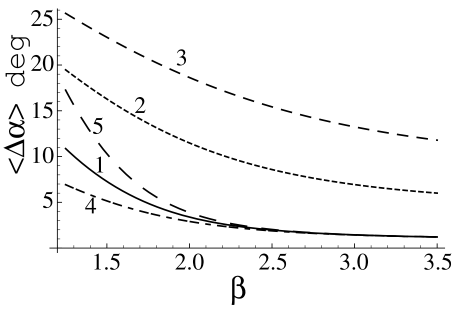

Figure 1. Variation of maximum value of the average change in the pitch angle $\langle\Delta\alpha\rangle$ (assuming the initial phase angle ϕ = 90°) versus the power law index β for chorus wave amplitude B = 0.2 nT, ambient magnetic field B_0 = 125 nT, and chorus frequency $\omega = 0.25\ \Omega$. Here, the electron pitch angle is taken as $\alpha = \pi/3$. For the curves 1, 2 and 3, $\tau_{\max}$ = 100 ms, and $\tau_{\min}$ = 1 ms, 5 ms and 10 ms, respectively. For the curves 4 and 5, $\tau_{\min}$ = 1 ms, and $\tau_{\max}$ = 50 ms and 200 ms, respectively. Here, $\tau_{\min}$ and $\tau_{\max}$ denote the minimum and the maximum value of the time duration, τ, of chorus subelements which follow the power law distribution.

Then, the pitch angle diffusion coefficient can be written as

$$D = \frac{B^2}{4B_0^2}\frac{\omega\Omega}{\left(\frac{\omega}{\Omega}+\frac{1}{2}\right)}\left[1+\frac{\omega\cos^2\alpha}{\Omega-\omega}\right]^2\tau \tag{12}$$

[19] Let us consider the chorus subelement/wave packet duration τ = 10 ms, chorus magnetic wave amplitude B = 0.2 nT, ambient magnetic field B_0 = 125 nT, and chorus frequency $\omega = 0.25\ \Omega$ as observed by *Tsurutani et al.* [2009]. Then, for electron pitch angle $\alpha = \pi/3$ and taking $\sin\phi = 1$, we get $\Delta t = \tau/3 = 3.3$ ms, $\Delta\alpha = 7.2°$ and $D = 1.2\ \mathrm{s}^{-1}$.

3.2. Distribution of Chorus Subelement Time Duration

[20] The above derivation assumes implicitly that the time duration of the chorus subelements, τ, is constant. In actual practice, the chorus sub-elements have time durations spanning a range of time periods ~1 ms to several 100s ms [*Santolík et al.*, 2004]. *Santolík et al.* [2007] have found a power law type of distribution for the chorus subelements time duration, i.e, $P \propto \tau^{-\beta}$, where P is the probability density and β = (2–3) is the spectral index.

[21] The probability distribution of time duration of chorus subelements will have effects on the pitch angle transport and diffusion coefficient calculated above. In this case, the average pitch angle transport and average diffusion coefficient can be written as:

$$\langle\Delta\alpha\rangle = \frac{B}{B_0}\frac{\omega\sin\phi}{\left(\frac{\omega}{\Omega}+\frac{1}{2}\right)}\left[1+\frac{\omega\cos^2\alpha}{\Omega-\omega}\right]\int\tau P(\tau)d\tau \tag{13}$$

and

$$\langle D\rangle = \frac{B^2}{4B_0^2}\frac{\omega\Omega}{\left(\frac{\omega}{\Omega}+\frac{1}{2}\right)}\left[1+\frac{\omega\cos^2\alpha}{\Omega-\omega}\right]^2\frac{\int\tau^2P(\tau)d\tau}{\int\tau P(\tau)d\tau} \tag{14}$$

In Figures 1 and 2 we have shown the variations of $\langle\Delta\alpha\rangle$ and $\langle D\rangle$ when the chorus subelements have a power law distribution of time durations versus β, the power law index, for various values of $\tau_{\min}$ and $\tau_{\max}$. Here, $\tau_{\min}$ and $\tau_{\max}$ represent the minimum and maximum value of the subelement time duration, respectively, such that the power law distribution is valid over the range $\Delta\tau = (\tau_{\max} - \tau_{\min})$.

[22] Figure 1 shows that the maximum value (assuming the initial phase angle ϕ = 90°) of the average (over chorus subelement time durations) pitch angle transport, $\langle\Delta\alpha\rangle$, decreases as the power law index β increases. An increase in $\tau_{\min}$, while keeping $\tau_{\max}$ fixed, results in the increase of $\langle\Delta\alpha\rangle$ (cf. curves 1, 2, and 3). On the other hand, an increase in $\tau_{\max}$, while keeping $\tau_{\min}$ fixed, leads to an increase in $\langle\Delta\alpha\rangle$ only for small values of $\beta \le 2$ (cf. curves 4, 1 and 5). Figure 2 shows that the average diffusion coefficient, $\langle D\rangle$, decreases with an increase of β (cf. curves 1, 2, 3, 4 and 5). Further, increases in $\tau_{\min}$ (while $\tau_{\max}$ = constant) (curves 1, 2, and 3) and $\tau_{\max}$ (while $\tau_{\min}$ = constant) (curves 4, 1 and 5) lead to enhancements in $\langle D\rangle$.

3.3. Limitations of the Model

[23] The model developed here is based on first order cyclotron resonance effects. The second and higher order effects, such as phase space bunching and trapping [*Inan et al.*, 1978; *Omura and Summers*, 2006; *Bortnik et al.*, 2008, *Albert*, 2001, 2002], and the relativistic turning acceleration [*Omura et al.*, 2007] are not included. This limits the applicability of the model to the cases where $R > 1$, where R is the ratio of phase oscillation period (or trapping time) at resonance to the time-scale for passage through resonance (or resonant interaction time). Now, the trapping frequency ω_{tr} is given by $\omega_{tr} = (kv_\perp\Omega_w)^{1/2}$ [*Omura et al.*, 2008], where $\Omega_w = eB/m$ is the electron cyclotron frequency in the wave magnetic field, B. Then, the trapping time $t_{tr} = 2\pi/\omega_{tr} = 2\pi/(kv_\perp\Omega_w)^{1/2}$. Therefore, we can express the applicability condition as $R = t_{tr}/\Delta t = [(\omega + \Omega/2)\ t_{tr}/\omega\tau] > 1$.

[24] In Figure 3, we have shown the trapping time, t_{tr}, versus the normalized magnetic field of the chorus wave, B/B_0 for the parameters at L = 5. We have taken B_0 = 250 nT

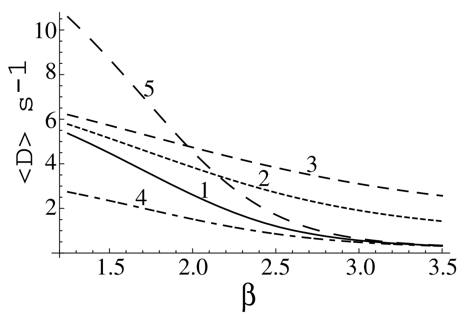

Figure 2. Variation of average diffusion coefficient, $\langle D\rangle$, versus the power law index, β, for chorus wave amplitude B = 0.2 nT, ambient magnetic field B_0 = 125 nT, and chorus frequency $\omega = 0.25\ \Omega$. Here, the electron pitch angle is taken as $\alpha = \pi/3$. For the curves 1, 2 and 3, $\tau_{\max}$ = 100 ms, and $\tau_{\min}$ = 1 ms, 5 ms and 10 ms, respectively. For the curves 4 and 5, $\tau_{\min}$ = 1 ms, and $\tau_{\max}$ = 50 ms and 200 ms, respectively.

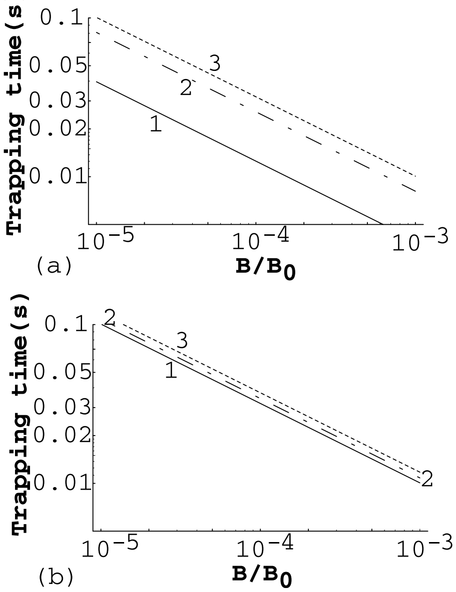

Figure 3. Variation of trapping time t_{tr} versus normalized chorus wave magnetic field, B/B_0, for the parameters at $L = 5$, with ambient magnetic field $B_0 = 250$ nT, and electron density, $n_0 = 10\ \mathrm{cm}^{-3}$. The cold plasma dispersion relation is used for the calculations. (a) Variations of t_{tr} for different values of the electron pitch angles. Here, $\omega/\Omega = 0.25$, and curves 1, 2, and 3 are for the electron pitch angle of $\alpha = \pi/3$, $\pi/8$, and $\pi/12$, respectively. The electrons have resonant energy of $E_R = 25$ keV. (b) Variations of t_{tr} for the case of $\alpha = \pi/12$ and for different values $\omega/\Omega = 0.25$, 0.35, and 0.45 for curves 1, 2, and 3, respectively. The corresponding resonant energies are $E_R = 25$ keV, 12 keV, and 6 keV for the curves 1, 2 and 3.

and electron density $n_0 = 10\ \mathrm{cm}^{-3}$ [*Bortnik et al.*, 2008]. We use the cold plasma dispersion relation to calculate the wave number, and use (2) to calculate the resonant parallel velocity. In Figure 3a, the curves 1, 2, and 3 are for the electron pitch angle of $\alpha = \pi/3$, $\pi/8$, and $\pi/12$, respectively, for the case of $\omega/\Omega = 0.25$. The trapping time decreases as the amplitude of the chorus magnetic field, B/B_0, increases. On the other hand, the decreases in the electron pitch angles lead to an increase in the trapping time (see curves 1, 2 and 3). Figure 3b shows the variation of the trapping time t_{tr} versus B/B_0 for electron pitch angle of $\alpha = \pi/12$, and for the normalized wave frequency $\omega/\Omega = 0.25$, 0.35, and 0.45 for the curves 1, 2, and 3, respectively. The trapping time increases with increasing value of the wave frequency ω/Ω (see curves 1, 2, and 3).

[25] From Figure 3, we notice that the lowest value of the trapping time is 5 ms for $B/B_0 = 6.0 \times 10^{-4}$, $\alpha = \pi/3$, and $\omega/\Omega = 0.25$ corresponding to curve 1 of Figure 3a. If the chorus subelement time duration is taken as $\tau = 10$ ms, this would give $R = 1.5$. The other curves shown in Figures 3a and 3b would yield even higher values of R. Therefore, there is a wide regime of parameters (corresponding to pitch angles of $\alpha \le \pi/3$ which give $R > 1$ or even $R \gg 1$) where our simple model can be applicable.

[26] In this paper we have considered the ambient magnetic field as uniform. It is suggested that the inhomogeneity of the ambient magnetic field plays a crucial role in the chorus generation mechanism [see *Omura et al.*, 2008, and the references therein] as it directly affects the resonance process between the electrons and the chorus subelements. By considering the ambient magnetic field as uniform, we have implicitly taken the trapping period, t_{tr} to be smaller than the t_{inh}, the time for the passage through the resonance in the inhomogeneous magnetic field. We will briefly discuss the limitations imposed by this assumption. The inhomogeneity factor $S = t_{tr}^2/t_{inh}^2$, which describes the relative importance of these two time scales, can be written as [*Dysthe*, 1971; *Inan et al.*, 1978; *Omura et al.*, 2007]:

$$S = \frac{1}{2\omega_{tr}^2}\left[3 + \frac{(\Omega - \omega)\tan^2\alpha}{\omega}\right]\left(v_{\parallel R}\frac{d\Omega}{dz}\right) \tag{15}$$

Now, the electron cyclotron frequency near the equator can be approximated [*Helliwell*, 1967; *Dysthe*, 1971] by

$$\Omega(z) = \Omega_0\left(1 + \frac{4.5z^2}{L^2R_E^2}\right) \tag{16}$$

On substituting for $d\Omega/dz$ using equation (16), and then taking $z = l_0$, where $l_0 = 2(L^2R_E^2v_{\parallel R}/4.5\Omega_0)^{1/3}$ is the length of the interaction region [*Helliwell*, 1967], we get

$$S = \frac{9}{(4.5)^{1/3}}\left[3 + \frac{(\Omega - \omega)\tan^2\alpha}{\omega}\right]\left(\frac{v_{\parallel R}}{LR_E}\right)^{4/3}\frac{\Omega_0^{2/3}}{\omega_{tr}^2} \tag{17}$$

Taking the parameters at L = 5, as done for Figure 3, $\omega = 0.25\Omega$ and $\alpha = \pi/3$, it is found that S equals 1 at $B/B_0 = 5 \times 10^{-5}$, and it decreases monotonically as B/B_0 increases. Furthermore, increases in ω/Ω and α tend to reduce the value of B/B_0 corresponding to $S = 1$. Thus, for $B/B_0 \ge 5 \times 10^{-5}$, S would be ≤ 1, and our model would be applicable provided $R > 1$.

4. Discussion

[27] Chorus is a right-hand, circularly-polarized bursty electromagnetic emission in the frequency range from hundreds of Hertz to several kHz with element duration of ~0.1 to 1.0 s. A chorus element is composed of a number of "subelements" or "packets". The time duration of chorus subelements (from 1 ms to 100s ms or so) has a power law distribution [*Santolík et al.*, 2004].

[28] Chorus is generated close to the geomagnetic equatorial plane or in minimum B field pockets by energetic electrons having anisotropic distributions. Since chorus subelements consist of coherent, discrete narrowband emissions, the energetic electrons can stay in gyroresonance with the wave for more than a wave cycle and may undergo large pitch angle changes in a single encounter. The changes in the pitch angles due to encounters of electrons with the chorus subelements can be randomly positive or negative, similar to Brownian type motion. This leads to electron pitch angle diffusion. Here we have derived pitch angle "transport" rates or diffusion coefficients from simple physical arguments for two cases, namely, (1) for a single chorus subelement with a given (or a constant) time duration, and (2) when the chorus subelement time durations have a power law distribution.

For case 1, electrons within ~7.2° of the loss-cone can be transported into the loss-cone by a single encounter with the chorus subelement wave, and the diffusion coefficient $D \sim 1.2\ s^{-1}$. For case 2 and considering $\beta \sim 1.5$–3.0, the maximum change in the average pitch angle $\langle\Delta\alpha\rangle$ can be ~2°–20° (cf. Figure 1) and the average diffusion coefficients are $\langle D\rangle \sim (0.5$–$8.5)\ s^{-1}$ (cf Figure 2).

[29] Since we have considered the case of uniform magnetic field, our results may apply only to the electrons trapped near the equatorial region. In actual practice, electrons may spend a lot of time away from the wave (the equator) region, and the effects due to bounce average of electrons could make the diffusion coefficient lower. The bounce average effects arising due to an inhomogeneous ambient magnetic field will be studied separately.

[30] *Summers* [2005] has derived formulae for the the quasi-linear diffusion coefficients for the charged particle in cyclotron resonance with field-aligned electromagnetic waves of any mode and general spectral density. Considering the ratio of wave magnetic energy density to the magnetic energy of the ambient field, i.e., $R_W = B^2/B_0^2 = 8.5 \times 10^{-8}$, and Gaussian form of the wave frequency spectrum, it was found that 100 keV electrons in cyclotron resonance with the whistler wave can have pitch angle diffusion coefficient $D \sim 10^{-3}\ s^{-1}$ [see *Summers*, 2005, Figure 1]. In our case, we have $R_W = 2.56 \times 10^{-6}$ which is about a factor of 30 higher than the value used by *Summers* [2005]. Since the pitch angle diffusion coefficient is directly proportional to R_W as seen by equations (33) and (36) of *Summers* [2005] and our equations (12) and (14), the scaled quasi-linear pitch angle diffusion coefficient will be about ~0.03 s^{-1}. This is much smaller than the pitch angle diffusion coefficients ~(0.5–8.5) s^{-1} obtained here.

[31] This rapid pitch angle diffusion or transport may be an explanation for ionospheric microburst structure. Microbursts are ~0.1 to 0.5 s bursts of ~10–100 keV electrons precipitating into the auroral ionosphere. As a chorus element propagates across the magnetic equator, the interaction between the coherent subelements and electrons will pitch angle scatter the resonant electrons, some into the loss cone and others away from the loss cone. The overall effect of the scattering process will be a precipitation "burst" of energetic electrons with parallel velocities that are in resonance with different frequency subelement components within the riser element. This concept could be tested by identifying a simultaneous microburst-chorus element event and determining if the microburst velocity dispersion matches the sweep of the chorus element riser. The quasi-linear or incoherent diffusion theory [*Kennel and Petschek*, 1966; *Summers*, 2005] cannot explain rapid bursts of precipitation such as microbursts. Faster scattering times are required which the theory developed in this paper can give.

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