

# A model for solar quiet day variation at low latitude from past observations using singular spectrum analysis

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Singular spectrum isolates significant principal components in a time series from the embedded noise. This tool-kit is used to reconstruct trend-free individual time series, formed by restricting the mean monthly hourly values of geomagnetic field to one hour at a time at a low latitude station Alibag (dipole latitude  $9.5^{\circ}\text{N}$ ). Each reconstructed component is extrapolated over the next 12 values using an autoregressive model based on Burg's maximum entropy algorithm. Details of a numerical approach to increase the reliability of extrapolation are highlighted. The extrapolated reconstructed components are then combined to generate predicted monthly values for each hour. The mean diurnal variation for any month obtained from the extrapolated individual hourly time series compares favorably with the observations. This approach to Sq(H) modelling incorporating both long and short term variations will be beneficial in the derivation of Dst index.

## 1. Introduction

Geomagnetic daily variations on quiet days, called Sq, are produced by the daytime dynamo currents in the E-region ( $\sim 110\text{ km}$ ) of the ionosphere. The observed quiet-day trend at the ground is a result of several parameters like ionospheric conductivity, winds, interior structure of the earth, influence of the moon etc. The day-to-day variability in Sq is rather spectacular and could be highly localized (Schlapp *et al* 1988; Butcher 1989). Apart from the unpredictable day-to-day variability, Sq amplitude changes fairly systematically with the season and with the phase of the solar cycle (Yacob and Rao 1966; Rastogi and Iyer 1976). The phase and amplitude also vary as a function of latitude (Arora *et al* 1980).

One of the common methods of modelling the global Sq is through spherical harmonic analysis (Matsushita 1967; Price and Wilkins 1963; Schmitz and Cain 1983). For this, data from a good network of global stations are needed. For the regional analysis of geomagnetic variations, Haines and Torta (1994) used Spherical Cap Harmonic Analysis (SCHA) to separate the external and internal parts. In deriving the Dst index, the quiet day component is eliminated using a

combination of a parabolic secular trend model and a double Fourier series model (Sugiura and Kamei 1991). The method of Natural Orthogonal Components (MNOC) has also been used successfully to derive a representative quiet day variation from the observations over several consecutive days at a given station or over the specific day at several stations (Golovkov *et al* 1978).

For several investigations in magnetospheric and space physics, it is essential to isolate a representative quiet-day component and, therefore, it becomes imperative to generate some useful models. It is considered that any methodology that can objectively provide a reasonable estimate of the Sq field at a given location which incorporates all the predictable facets like long term secular trend, solar cycle and other longer period variations, annual, seasonal and other identifiable variations like the Quasibiennial oscillations (Bhargava 1972; Rangarajan *et al* 1996) will prove extremely useful.

Towards achieving this goal, our attention was drawn to two interesting papers in recent years. Penland *et al* (1991) demonstrate that a low-order autoregressive (AR) model applied to the noise-free significant individual components isolated from a

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chaotic noisy time series through the method of Singular Spectrum Analysis (SSA), can be effectively combined to derive a high resolution noise-free spectrum with fewer number of AR coefficients. Keppenne and Ghil (1992) show how a reliable extrapolation of a time series can be obtained from a combination of the AR-models of individual principal components derived through SSA. In the present investigation an attempt has been made to generate a model for solar quiet day variation in H at a low latitude station (Alibag) from the past observations by using the technique of singular spectrum analysis following the guidelines of Penland *et al* (1991) and Keppenne and Ghil (1992). The predicted monthly pattern of Sq(H) is compared with observations to demonstrate the relative success of the effort. A modification for improving the reliability of extrapolation outside the range of observations is suggested. It may be emphasized that the location of Alibag is considered ideal for derivation of the equatorial Dst index and efforts are continued to improve the present Dst by including Alibag as the fifth station in the network (Sugiura and Kameji 1997).

## 2. Data and method of analysis

The data used in this analysis are the monthly mean hourly values of the H component of the geomagnetic

field for quiet days at a low latitude station Alibag (18°38'N; 72°52'E) for the 70-year period from 1924 to 1993. This homogeneous data derived from magnetograms and scaled hourly values are scrutinized for jumps/discontinuities etc. and error free, common base (38000 nT+), mean monthly hourly values are generated. 24 time-series, one for each hour, are used in the subsequent analysis.

Each time series is decomposed into two parts: (i) the slowly varying secular trend; and (ii) the residual part derived by removing the secular variation. For estimating the secular variations a polynomial of fourth degree is fitted by the method of orthogonal polynomials (Kendall 1948). The sampling interval is taken as one year and the coefficients for each UT, specifying the secular trend are given in table 1. The residuals, i.e. observed values from which the secular trend has been removed, is then subjected to singular spectrum analysis (SSA) (Vautard *et al* 1992) and the reconstructed components are analyzed by maximum entropy method (MEM).

The details of the methodology is given by Vautard *et al* (1992) and the computational steps involved are described in detail by Rangarajan and Araki (1997). The eigen values obtained in the process are arranged in decreasing order. The relative magnitude of individual eigen value enables one to assess its significance as the percentage of the total variance accounted for by each of the reconstructed components (RC)

Table 1. 4th Degree coefficients computed by orthogonal polynomials.

Hour(UT)	$a_0$	$a_1$	$a_2$	$a_3$	$a_4$
1	-0.96247168E + 03	0.72560348E + 01	0.38487380E + 01	-0.10317020E + 00	0.70991530E - 03
2	-0.96174719E + 03	0.72871880E + 01	0.38481710E + 01	-0.10314260E + 00	0.70960767E - 03
3	-0.96059479E + 03	0.75744538E + 01	0.38349631E + 01	-0.10286250E + 00	0.70766237E - 03
4	-0.95914770E + 03	0.84596519E + 01	0.37925260E + 01	-0.10203760E + 00	0.70231972E - 03
5	-0.95795868E + 03	0.97077990E + 01	0.37449419E + 01	-0.10130590E + 00	0.69844781E - 03
6	-0.95597662E + 03	0.11004620E + 02	0.36945600E + 01	-0.10058030E + 00	0.69497607E - 03
7	-0.95314319E + 03	0.11408180E + 02	0.36893840E + 01	-0.10073400E + 00	0.69714279E - 03
8	-0.94886841E + 03	0.10743620E + 02	0.37276340E + 01	-0.10158250E + 00	0.70313702E - 03
9	-0.94206622E + 03	0.89527492E + 01	0.38112490E + 01	-0.10314260E + 00	0.71302999E - 03
10	-0.93369989E + 03	0.64084659E + 01	0.39268720E + 01	-0.10520200E + 00	0.72560058E - 03
11	-0.92670080E + 03	0.41106820E + 01	0.40268259E + 01	-0.10689280E + 00	0.73544210E - 03
12	-0.92426520E + 03	0.27039010E + 01	0.40863972E + 01	-0.10786280E + 00	0.74088288E - 03
13	-0.92627679E + 03	0.22803500E + 01	0.41005211E + 01	-0.10800440E + 00	0.74116117E - 03
14	-0.93160162E + 03	0.26698120E + 01	0.40804582E + 01	-0.10759960E + 00	0.73844980E - 03
15	-0.93889630E + 03	0.35368309E + 01	0.40371709E + 01	-0.10676380E + 00	0.73299522E - 03
16	-0.94545038E + 03	0.42831478E + 01	0.40000029E + 01	-0.10603450E + 00	0.72813238E - 03
17	-0.95068902E + 03	0.49032521E + 01	0.39696281E + 01	-0.10544600E + 00	0.72424271E - 03
18	-0.95343842E + 03	0.53323040E + 01	0.39462480E + 01	-0.10496500E + 00	0.72098378E - 03
19	-0.95428808E + 03	0.55690308E + 01	0.39335921E + 01	-0.10471090E + 00	0.71930193E - 03
20	-0.95505621E + 03	0.57578540E + 01	0.39237571E + 01	-0.10451300E + 00	0.71797607E - 03
21	-0.95589148E + 03	0.59012971E + 01	0.39162841E + 01	-0.10435540E + 00	0.71686908E - 03
22	-0.95595782E + 03	0.59372020E + 01	0.39134870E + 01	-0.10428040E + 00	0.71627938E - 03
23	-0.95560437E + 03	0.59564762E + 01	0.39125810E + 01	-0.10425840E + 00	0.71610103E - 03
24	-0.95422479E + 03	0.57897840E + 01	0.39224541E + 01	-0.10446620E + 00	0.71753992E - 03

is given from the ratio of the individual eigen value to the sum of all the  $M$  eigen values.  $P = (\lambda_k / \sum_{k=1}^m \lambda_k) \times 100$  provides the relative percentage contribution of the  $k^{\text{th}}$  component to the total variance  $[(1)/(N-1)\sum (X_i - \bar{X})^2]$  of the system and one need not proceed beyond a value where the percentage variance accounted for tends to be small (say less than 1 or 2%). The maximum entropy method has been extensively used as a tool for high resolution spectrum analysis (see Marple 1987 for details and references). Keppenne and Ghil (1992) could successfully predict the southern oscillation index by a combination of noise reduction through SSA and low order autoregressive modelling by MEM. We first applied SSA to the time series to isolate significant spectral components and to eliminate noise. Low order AR model coefficients appropriate for the individual reconstructed component (RC) derived by MEM, are used to successively predict the future values of  $x_i$  from the past observations. For each RC, the  $t^{\text{th}}$  value outside the range of observations, is extrapolated from the previous  $M$  observations.

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \dots + \alpha_M X_{t-M} + \epsilon_t \quad (1)$$

where  $(\alpha_1, \alpha_2, \dots, \alpha_M)$  are the Prediction Error Filter coefficients derived using Burg's algorithm and  $\epsilon_t$  is the residual error (Burg 1968). The extrapolated values of each RC can then be combined by summation.

We note that the most dominant oscillation in the H-residuals is the 11-year sunspot cycle; we therefore adopt a window size of 144 months (much less than one-third of the data length, the upper limit, for  $M$  recommended by Vautard *et al* 1992). The eigen values, in decreasing order, provide an estimate of the significant spectral components in the time series as the largest eigen value corresponds to the maximum spectral density and for finite  $M$ , all eigen values fall between the maximum and minimum spectral densities (Vautard *et al* 1992). The elements of individual eigen vectors constitute the numerical weights of the optimal data-adaptive filter. The RCs, in decreasing order of significance, are generated using these filter weights and end points lost in the computation are reconstructed utilizing the formulae given by Vautard *et al* (1992). These generated RCs are then spectrally analyzed by MEM to estimate the AR coefficients utilizing the computer routine outlined in Ulrych and Bishop (1975). The RCs are essentially noise-free and, therefore, AR coefficients with an appropriate low order are enough to provide high resolution spectrum and these coefficients can be used to extrapolate the time series using equation (1).

Each component is extrapolated up to 12 values beyond 1993 covering the months of January to December 1994. The reliability of prediction will naturally decrease as one moves farther away in time as successive predicted values are used for later forecast rather than the actual observations. To minimize

the error due to this source, we have modified the approach of Keppenne and Ghil (1992) as follows:

The time series (for any given UT hour) – January 1924 to December 1993 – is used to generate 12 succeeding values representing the anticipated monthly mean values of H-component from January to December 1994 for that particular hour. Next, the series – January 1924 to November 1993 – truncated by one end point, is used to generate data for December 1993 to November 1994 and so on till we get predicted values for February 1993 to January 1994 from the data for January 1924 to January 1993. The 12 individual values in each extrapolation are weighed in decreasing order (12/12, 11/12, 10/12, ..., 1/12) to account for the relative reduced reliability. Thus, for the first analysis January 1994 will be weighed by unity, for the second run the same will have the weight 11/12 etc. and for the last run, a weight of only 1/12. These weighted values for each month are then added, averaged and normalized for unit weight. It is to be noted that the number of values averaged in this process decreases from 12 for the first month January (1994) to 1 for the last month December (1994). We find unambiguously that the stability of prediction increases significantly by this approach in comparison with direct prediction using all the data in one step. The window size and the choice of the number of significant RCs play a crucial role in the 'direct' prediction whereas their importance, particularly of the window size, is diminished in the 'average' prediction approach adopted here. We

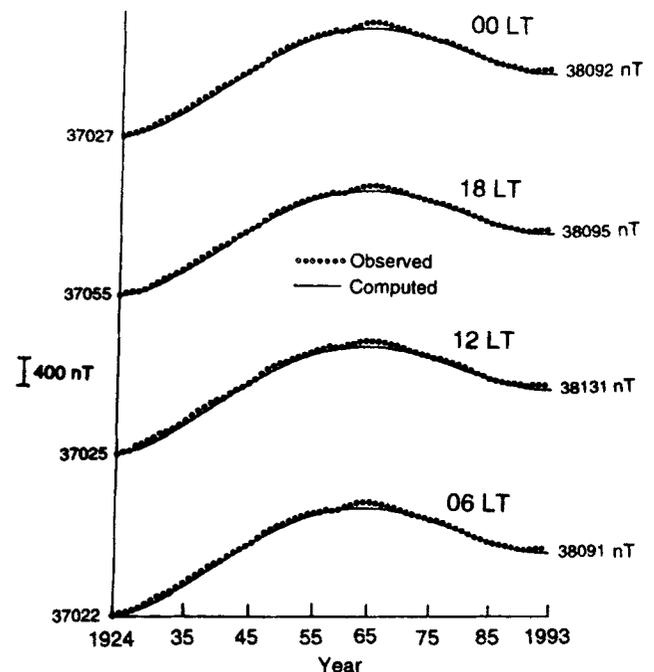


Figure 1. Observed annual mean values of horizontal component (H) for quiet days at Alibag from 1924 to 1993 together with their best-fitting curves.

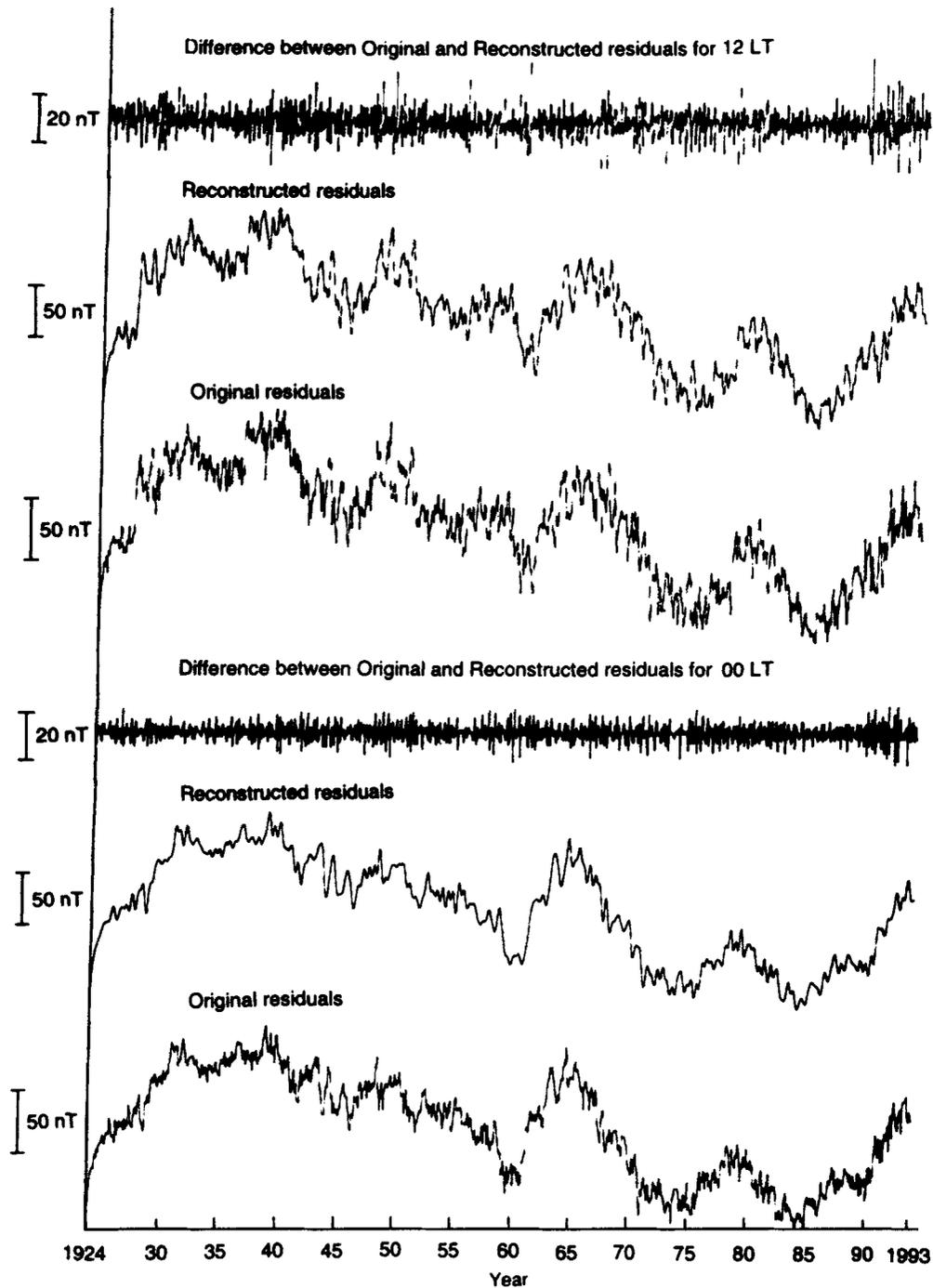


Figure 2. Residuals of horizontal component (H) monthly values for quiet days at Alibag, together with reconstructed residuals and their differences between original and reconstructed residuals from 1924 to 1993 for local noon (12 LT) (top) and local midnight (00 LT) (bottom).

retain for forecast, all such RCs whose cumulative variance is about 95%.

### 3. Results and discussion

Typical secular trends, derived from the fourth degree polynomials, along with the observed annual mean values are shown in figure 1 for representative local hours close to dawn, dusk, noon and midnight.

Though all the curves appear to be parallel to each other, the listed coefficients of table 1 suggest clear differences in detail. Yacob and Sen (1969) had carried out a comparative study of the secular trend of H field at Alibag centered on observations at noon and midnight for the period 1906 to 1967. They observed the presence of cyclic components in both the midday and midnight series, with prominent amplitudes from 1930 to 1955. For most of the solar cycles during the period, the variation is nearly in phase with the sunspot cycle

## Principal components of 'H' for Quiet days at Alibag for 12 LT

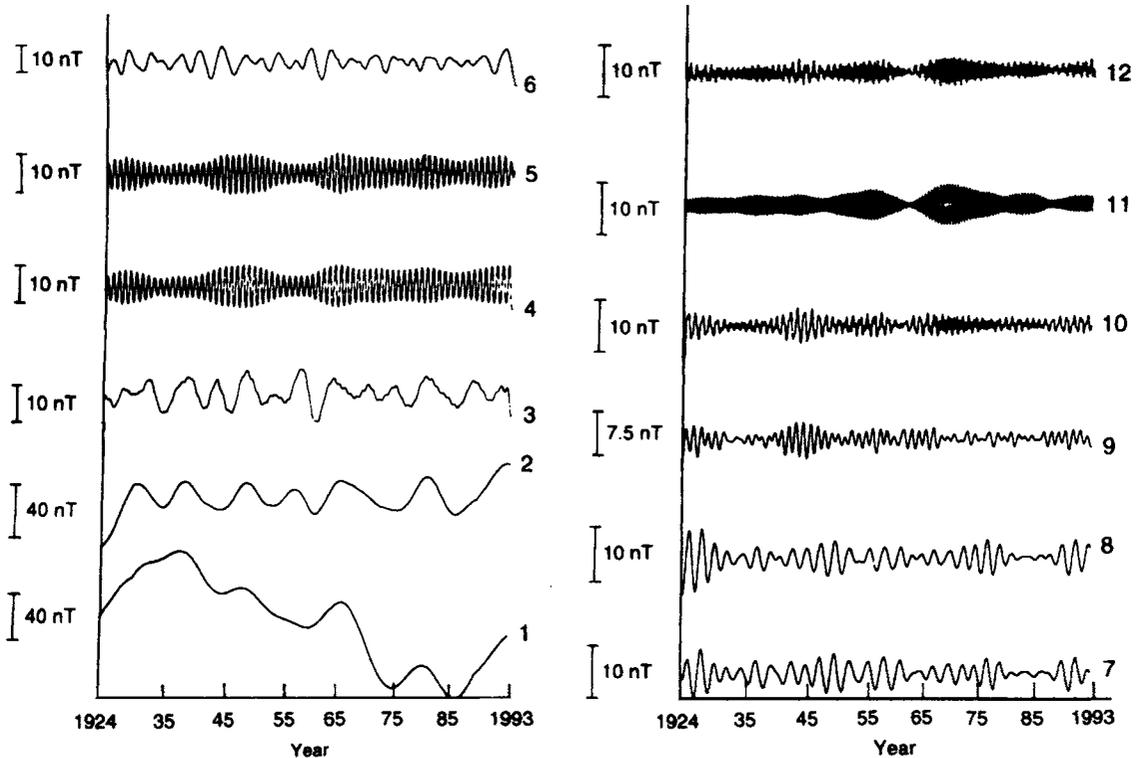


Figure 3. First 12 principal components corresponding to 840 mean monthly values of (H) for quiet days at Alibag from 1924 to 1993 for local noon (12 LT).

and its amplitude for the midday series is almost twice that for the midnight series. For prediction purposes, we feel it is necessary to estimate the long term trend for each hour separately rather than taking one set of coefficients for the polynomial for all the hours.

Observed monthly mean values for each hour are next detrended, subtracting the appropriate values computed from the fourth degree polynomials. Typical residuals so obtained for local noon (12 LT) and midnight (00 LT) are shown in figure 2. From the nature of the singular spectrum of the residuals, we find that the first 12 eigen values account for about 95% of the total variance; the remaining fraction is accounted for by several subsequent components none of which is significant and hence, we generate 12 reconstructed components from each time series using the corresponding eigen vectors as data-adaptive filters.

A typical example of the reconstructed series (840 values each) from the estimated constituents of the local midday time series is shown in figure 3. The efficiency of the methodology can be gauged from the fact that the addition of the 12 component series match very closely the original series of residuals; (see figure 2). The difference between the original and reconstructed values, in figure 2, is clearly white noise and is not amenable to modelling. The amplitude of the noise is much higher for the noon series, as can be

expected due to the highly variable ionospheric contributions.

The percentage variance accounted for by the first 12 components indicate that a longer period variation with a periodicity of about 80–100 years superposed with other minor oscillations is the most dominant feature (see figure 3 curve 1 for e.g.) followed by the 11-year cycle and its harmonics. Annual and semi-annual variations are other important components. Some unusual quasi-periodic fluctuations like  $\sim 44$  mon,  $\sim 15$  mon, quasibiennial oscillations (26 to 31 mon) etc. are also significantly present in many of the series. Interestingly some of these have been under active investigation in recent years. Solar cycle periodicity in the geomagnetic field has been studied extensively and there is still the unresolved aspect about whether it is wholly of external origin (Allredge 1976). A 44-month periodicity in the equatorial Dst index has been recently analyzed (Rangarajan and Araki 1997) and its presence in some other parameters like sector polarity of IMF and Ap index has also been inferred earlier (Gonzalez and Gonzalez 1987; Gonzalez *et al* 1993). Similarly a 1.3-year fluctuation in solar wind speed during the epoch 1986–1993 has been identified by Paularena *et al* (1995) and its proxy is also seen clearly in the Ap index of geomagnetic activity (Rangarajan and Iyemori 1997). A quasibiennial oscillation (QBO) of the geomagnetic

disturbance field at low latitudes with variable amplitude was detected by Rangarajan (1985) based on the earlier analysis by Sugiura and Poros (1977). Recently, QBO has been identified in the ionospheric parameters (Olsen 1994) and the modified ionospheric currents could lead to the presence of QBO signal in the quiet day magnetic field. A most noteworthy feature of figure 3 is the insignificant amplitude of the semiannual oscillation during the epoch 1957–1960 when solar activity was at its highest ever but one does not see a general dependence of the amplitude of either 6- or 12-month oscillation on the solar activity.

#### 4. Sq(H) prediction approach

We have established that the cumulative sum of the reconstructed components for any given hour reproduces, to within noise limits, the original time series. Therefore, it should be logically possible to generate through a suitable AR model, successive values of the reconstructed component and from their sum, estimate the next several values (say 12 values January to December 1994) of the particular hour. If these pre-

dicted values are augmented by the appropriate terms derived from extrapolation of the secular trend using the coefficients listed in table 1, we can have a realistic estimate of the 12 monthly values for any individual UT hour. From these predicted values for all the 24 hours, we generate the diurnal variations of solar quiet day variation for each month (January to December 1994). These are shown in figure 4 along with the observed diurnal patterns for these months derived from 5 IQ days for each month.

The predicted patterns, particularly for the initial six months are in very good agreement lending credibility to our method of estimating the quiet time diurnal pattern at low latitudes. The distortions are rather striking towards the last 5 to 6 months with phase difference being more pronounced, rather than much difference in the amplitude. The main reason for this discrepancy could well be the use of extrapolated values in successive prediction, as explained earlier. However, there is another strong candidate for the relative failure of the prediction of the phase of diurnal variation in the latter half of the year. A perusal of the mean seasonal variation of the  $\Delta H$  field on quiet days at Alibag, shown in figure 5 (redrawn from Bhargava

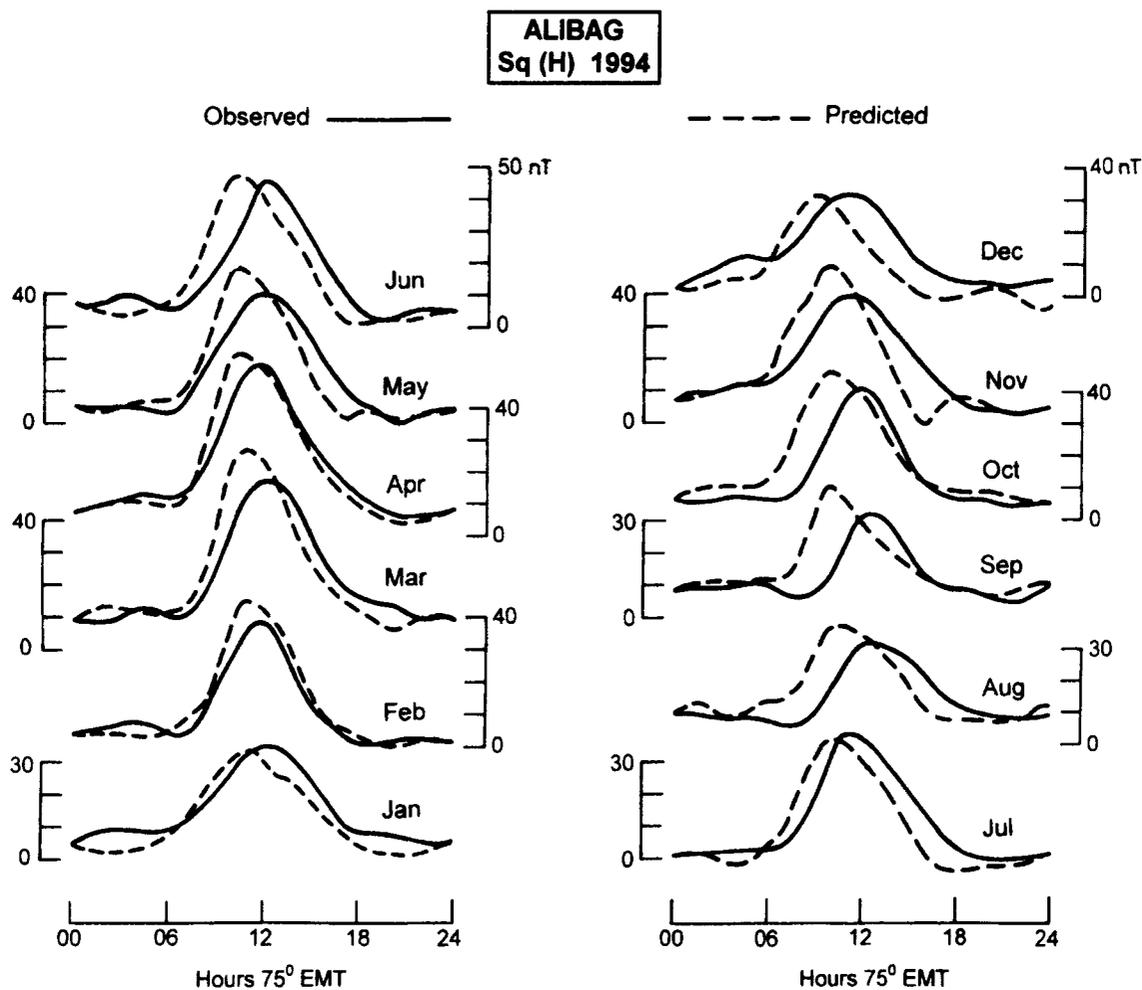


Figure 4. Observed and predicted diurnal variations of H at Alibag for the months January to December 1994.

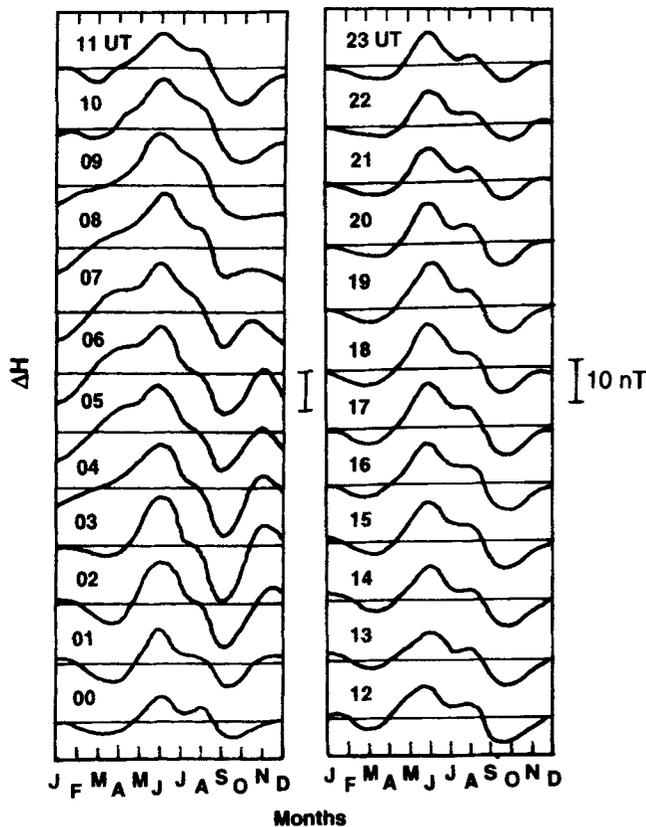


Figure 5. Seasonal variation of the quiet-day mean monthly horizontal intensity at Alibag for each hour (UT) (redrawn from Bhargava 1972).

1972) brings out clearly the large hour-to-hour variability in the latter half of the year and the significant reduction in Sq(H) amplitude in September. While this figure is an average pattern covering 46 years between the period 1924 and 1969, the features also undergo solar cycle modulation. It is thus possible that the relative phase difference between the observations and predictions of the diurnal pattern from August onwards can be the result of the irregular solar cycle variability of the quiet day field in those months. Their prediction will be more reliable if the data up to July are used for analysis and in extrapolation. The August – December part falls in the first six estimates, as we find that if we confine the extrapolation to just six values outside the range of observations, a greater degree of reliability is noticed.

## 5. Conclusions

By the effective use of the combination of three different techniques – estimation of secular trend through orthogonal polynomials, identification and isolation of principal components of the trend-free series and extrapolation of individual components by autoregressive models and cumulative additions of the extrapolated components, we have been able to derive a reasonably good model for the solar quiet day varia-

tion in the low latitudes at least six months ahead. In studies related to geomagnetic disturbances or in the efforts to quantify the solar wind plasma-induced geomagnetic variations we need to eliminate the influence of the variability associated with quiet time conditions. For example in deriving the Dst index, K index, Ap index etc., a reference level corresponding to an objective estimate of the Sq(H) is eliminated (see Mayaud 1980; Rangarajan 1989 for details). The present approach provides a reliable way towards this and appears particularly useful when we are interested in near real time or quick look indices like Dst, AE and Kp.

At Alibag in the Indian zone, the seasonal variation in Sq(H) is marked by a sharp depression in September followed by a fairly steep rise in the next two months which is also strongly influenced by the solar activity. This apparently can cause some difficulty in profitable prediction beyond six months. It will be worthwhile if the present exercise is repeated with the data of other Dst stations like San Juan, Honolulu, Hermanus and Kakoika to see how far into the future we can extrapolate, for deriving a usable diurnal pattern of the Sq(H) at the Dst stations.

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