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Effect of ion temperature on ion-acoustic solitary waves in a magnetized plasma in presence of superthermal electrons

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Obliquely propagating ion-acoustic soliatry waves are examined in a magnetized plasma composed of kappa distributed electrons and fluid ions with finite temperature. The Sagdeev potential approach is used to study the properties of finite amplitude solitary waves. Using a quasi-neutrality condition, it is possible to reduce the set of equations to a single equation (energy integral equation), which describes the evolution of ion-acoustic solitary waves in magnetized plasmas. The temperature of warm ions affects the speed, amplitude, width, and pulse duration of solitons. Both the critical and the upper Mach numbers are increased by an increase in the ion temperature. The ion-acoustic soliton amplitude increases with the increase in superthermality of electrons. For auroral plasma parameters, the model predicts the soliton speed, amplitude, width, and pulse duration, respectively, to be in the range of (28.7–31.8) km/s, (0.18–20.1) mV/m; (590–167) m, and (20.5–5.25) ms, which are in good agreement with Viking observations. © *2013 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4776710]

I. INTRODUCTION

The particle velocity distributions in many physical situations, e.g., in the laboratory, space, and astrophysical plasmas are not Maxwellian. In space plasmas, particles typically possess distribution function with an enhanced high-energy tail. These distributions may be well modelled by a generalized Lorentzian (or kappa) distribution with spectral index κ. Such particle distributions may be the result of acceleration provided by the shocks and turbulent fields in space and laboratory plasmas. The κ distribution which is an empirical fit to the observed particle distributions was first suggested by Vasyliunas[1] to model solar wind data from OGO 1 and OGO 2 satellites. Since then, it has been widely used by various researchers to study linear and nonlinear phenomena in space and dusty plasmas. Other form of non-thermal particle distributions in the literature comprehensively studied is the Cairns type of distributions.[2] They used the distribution function to explain the ion-acoustic solitary waves observed by the Freja satellite.

Ion-acoustic solitary waves in unmagnetized and magnetized plasmas have been studied extensively for over several decades. Washimi and Taniuti[3] studied the propagation of ion-acoustic solitons in two-component plasma using small amplitude theory. Buti[4] showed that plasmas with two Maxwellian electron populations can support large amplitude rarefactive structures. Baboolal *et al.*[5] investigated the existence conditions of the large amplitude ion-acoustic solitons and double layers in a plasma consisting of two Boltzmann electrons (hot and cool) components and single cool ions. Reddy and Lakhina[6] studied small amplitude ion-acoustic double layers and solitons in auroral magnetized plasma consisting of hot and cold electrons and two ions (oxygen-hydrogen) species. Their analysis predicted either negative potential double layers, or negative potential solitons or positive potential solitons in distinct parametric regime. Reddy *et al.*[7] extended the analysis to include a number of ion beams. Cairns *et al.*[2] showed that it is possible to obtain both compressive and rarefactive solitons in a plasma consisting of non-thermal electrons (with excess energetic particles) and cold ions. They were able to explain the Freja satellite observations[8] of the solitary structures with density depletions. Baluku *et al.*[9] revisited the existence of ion-acoustic solitons and double layers. They not only confirmed the earlier results of Baboolal *et al.*[5] but also found that depending upon the cool to hot electron temperature ratio, positive polarity solitons can exist beyond the positive potential double layers. Berthomier *et al.*[10] studied the characteristics of solitary waves and weak double layers in a two-electron temperature auroral plasma and reproduced the structures observed by the Viking satellite.[11,12]

Ion-acoustic waves and solitons in two-component electron-ion magnetized plasma have been studied by Lee and Kan.[13] Using the reductive perturbation method, obliquely propagating ion-acoustic solitons in a magnetized plasma consisting of warm adiabatic ions and nonthermal electrons have been examined by Cairns *et al.*[14] Abbasi and Pajouh[15] studied the ion-acoustic solitons considering free and trapped electrons following kappa-distribution and ions following fluid dynamical equations. Lakhina *et al.*[16,17] studied large amplitude ion and electron acoustic waves in an unmagnetized multi-component plasma system consisting of cold background electrons and ions, a hot electron beam, and a hot ion beam. It was found that three types of solitary waves, namely, slow ion-acoustic, ion-acoustic, and electron-acoustic solitons can exist

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provided that the Mach numbers exceed the critical values. Choi *et al.*[18] studied the existence conditions for electrostatic solitary waves and double layers in a nonthermal electron plasma with heavy ions. They found that nonthermality of the electrons determines the existence of double layers.

These studies on ion-acoustic solitary waves in unmagnetized or magnetized plasmas have concentrated mainly on Maxwellian distribution of electrons or Cairns type distributions. So far, very few studies have examined the ion-acoustic solitary waves with highly energetic kappa-distributed electrons. For example, Saini and Kourakis[19] examined the existence of arbitrary amplitude ion-acoustic solitary waves in an unmagnetized plasma consisting of ions and excess superthermal electrons (modelled by a kappa-type distribution) and an electron beam. Sultana *et al.*[20] examined ion-acoustic solitary waves in magnetized plasmas with kappa-distributed electrons and fluid cool ions. They employed psuedo Sagdeev potential approach and used quasi-neutrality condition. They studied the effect of obliqueness and superthermality of electrons on the ion-acoustic solitary waves. The effect of superthermal electrons modeled by a Lorentzian velocity distribution function has been studied on obliquely propagating linear and nonlinear ion-acoustic waves in a electron-ion magnetized plasma.[21] They used the small amplitude theory to study the nonlinear ion-acoustic waves. In both of the above mentioned study, effect of finite ion temperature on solitary structures was not considered. In this paper, we extend the work of Sultana *et al.*[20] to include finite ion temperature, and our model consists of kappa-distributed electrons and warm ions following fluid dynamical equations. The paper is organized as follows: In Sec. II, theoretical model is presented and results are discussed in the following section, and possible application to space plasma is brought out.

II. THEORETICAL MODEL

We consider a magnetized collisionless plasma consisting of fluid, adiabatic warm ions and hot electrons having kappa distribution. The plasma is considered to be immersed in uniform ambient magnetic field, $\mathbf{B}_0 = B_0\hat{z}$ where $\hat{z}$ is the unit vector along the z axis and ion-acoustic waves are propagating in the x–z plane, which means that there are no variations along y-axis $(i.e., \frac{\partial}{\partial y} = 0)$.

The normalized set of governing equations of ion-acoustic mode in such a two-component magnetized plasma is given by

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_x)}{\partial x} + \frac{\partial (n_i v_z)}{\partial z} = 0, \tag{1}$$

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_z \frac{\partial v_x}{\partial z} = -\frac{\partial \phi}{\partial x} - 3\sigma n_i \frac{\partial n_i}{\partial x} + R v_y, \tag{2}$$

$$\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_z \frac{\partial v_y}{\partial z} = -R v_x, \tag{3}$$

$$\frac{\partial v_z}{\partial t} + v_x \frac{\partial v_z}{\partial x} + v_z \frac{\partial v_z}{\partial z} = -\frac{\partial \phi}{\partial z} - 3\sigma n_i \frac{\partial n_i}{\partial z}. \tag{4}$$

Here, n_i, v_x, v_y, v_z represents density and the x, y, z-components of the velocity of the warm ion fluid, respectively, and ϕ is the electrostatic potential, $R = \Omega_i/\omega_{pi}; \Omega_i = eB_0/m_i c$ is the ion cyclotron frequency and $\omega_{pi} = \sqrt{4\pi N_0 e^2/m_i}$ is the ion plasma frequency, and $\sigma = T_i/T_e$ is the ion to electron temperature ratio. It may be emphasized here that for simplicity, we have taken adiabatic index $\gamma = 3$ in the momentum equation, which is valid for one degree of freedom. However, in a magnetized plasma, $\gamma = 5/3$ can be taken by considering three degrees of freedom. Since, in our analysis, we have treated equation of motion and continuity equation exactly, the results are not expected to change qualitatively by taking $\gamma = 5/3$ instead of $\gamma = 3$.

The normalizations used in analysis are as follows: densities are normalized with respect to the equilibrium density $N_0 = N_{0e} = N_{0i}$, temperatures by the electron temperature, T_e, time by the inverse of ion plasma frequency, ω_{pi}^{-1}, velocities by the acoustic speed $c_s = \sqrt{T_e/m_i}$, lengths by the electron Debye length, $\lambda_{de} = \sqrt{T_e/4\pi N_{0e} e^2}$, and the electrostatic potential, ϕ by T_e/e.

We consider the kappa-distribution function for hot superthermal electrons, which is given by Thorne and Summers,[22]

$$f_{oe}(v) = \frac{N_{0e}}{\pi^{\frac{3}{2}}\theta^3} \frac{\Gamma(\kappa)}{\sqrt{\kappa}\,\Gamma\left(\kappa - \frac{1}{2}\right)} \left(1 + \frac{v^2}{\kappa\theta^2}\right)^{-(\kappa+1)}, \tag{5}$$

where κ is the superthermality index, $\Gamma(\kappa)$ is the gamma function and the modified thermal speed is given by $\theta^2 = (2 - 3/\kappa)T_e/m_e$, and N_{0e} is the unnormalized equilibrium electron density. In order that the thermal speed θ to be physical (i.e., real), we require $\kappa > \frac{3}{2}$. The Maxwell-Boltzmann equilibrium can be obtained in the limit $\kappa \to \infty$. The distribution of electrons in the presence of non-zero potential can be found by replacing v^2 by $v^2 - e\phi/T_e$ in Eq. (5). Thus, the number density of electrons can be obtained by integrating distribution function f_{oe} given by Eq. (5) over velocity space and in the normalized form can be written as

$$n_e = \left[1 - \frac{\phi}{\kappa - \frac{3}{2}}\right]^{-\kappa + \frac{1}{2}}. \tag{6}$$

In order to study the nonlinear propagation of ion-acoustic waves, we now look for the solution of Eqs. (1)–(6). First task is to transform these equations which depend on x, z, and t to a single variable, $\xi = \alpha x + \beta z - Mt$, where $M = V/c_s$ (here, V is the speed of the soliton) is the Mach number, α, β are the directions of cosines along x and z directions, i.e., $\alpha = k_x/k = \sin\theta; \beta = k_z/k = \cos\theta$, respectively and satisfy the relation $\alpha^2 + \beta^2 = 1$. The transformed Eqs. (1)–(4) can be written as

Continuity equation

$$-M\frac{\partial n_i}{\partial \xi} + \alpha \frac{\partial (n_i v_x)}{\partial \xi} + \beta \frac{\partial (n_i v_z)}{\partial \xi} = 0. \tag{7}$$

Equation of motion, the x, y, and z-components can be written as

$$L_M \frac{\partial v_x}{\partial \xi} + \alpha \frac{\partial \phi}{\partial \xi} + 3 n_i \alpha \sigma \frac{\partial n_i}{\partial \xi} - R v_y = 0, \tag{8}$$

$$L_M \frac{\partial v_y}{\partial \xi} + R v_x = 0, \tag{9}$$

$$L_M \frac{\partial v_z}{\partial \xi} + \beta \frac{\partial \phi}{\partial \xi} + 3\beta n_i \sigma \frac{\partial n_i}{\partial \xi} = 0, \tag{10}$$

where $L_M = (-M + \alpha v_x + \beta v_z)$. Equations (7)–(10) cannot be solved analytically due to the complexity introduced by the magnetic field. However, under the assumption that the quasi-neutrality condition, $n_e = n_i$ is satisfied, Eqs. (7)–(10) can be integrated. Using the appropriate boundary conditions (namely, $n_{e,i} \to 1$, $v_{x,z} \to 0$, $\phi \to 0$, and $d\phi/d\xi \to 0$ at $\xi \to \pm\infty$) and some algebraic manipulations, the energy integral for such a system can be written as

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + \psi(\phi, M) = 0, \tag{11}$$

where

$$\psi(\phi, M) = R^2 \frac{\psi_1(\phi, M)}{\psi_2(\phi, M)} \tag{12}$$

is the Sagdeev pseudo potential, where

$$\begin{aligned}
\psi_1(\phi,M) = {} & \left(1+\frac{\beta^2}{M^2}\right)\left[1-\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-\kappa+3/2}\right] + (M^2+\beta^2)\left[1-\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{\kappa-1/2}\right] + \left[1-\beta^2\left(\frac{\kappa-\frac{1}{2}}{\kappa-\frac{3}{2}}\right)\right]\phi \\
& - \frac{\beta^2}{2M^2}\left[1-\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-2\kappa+3}\right] - \frac{M^2}{2}\left[1-\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{2\kappa-1}\right] \\
& + \sigma\left\{\frac{\beta^2}{M^2}\left[1-\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-\kappa+3/2} + \left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-4\kappa+3} - \left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-3\kappa+3/2}\right]\right. \\
& + \beta^2\left[1-\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{\kappa-1/2}\right] + \left(\frac{\beta^2-3}{2}\right)\left[1-\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-2\kappa+1}\right] + \left(1+\frac{\sigma\beta^2}{M^2}\right)\left[1-\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-3\kappa+3/2}\right] \\
& \left. - \frac{\sigma\beta^2}{2M^2}\left[1-\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-6\kappa+3}\right]\right\}
\end{aligned}$$

and

$$\psi_2(\phi,M) = \left[1 - M^2\left(\frac{\kappa-\frac{1}{2}}{\kappa-\frac{3}{2}}\right)\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{2\kappa-2} + 3\sigma\left(\frac{\kappa-\frac{1}{2}}{\kappa-\frac{3}{2}}\right)\left(1-\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-2\kappa}\right]^2.$$

It must be pointed out that Eq. (11) will give solitary wave solutions when the Sagdeev potential satisfies the following conditions: $\psi(\phi,M)=0$, $\mathrm{d}\psi(\phi,M)/\mathrm{d}\phi=0$, $\mathrm{d}^2\psi(\phi,M)/\mathrm{d}\phi^2<0$ at $\phi=0$; $\psi(\phi,M)=0$ at $\phi=\phi_0$, and $\psi(\phi,M)<0$ for $0<|\phi|<|\phi_0|$, where ϕ_0 is the maximum amplitude of the solitons. From Eq. (12), it is easy to prove that $\psi(\phi,M)=0$ and $d\psi(\phi,M)/d\phi=0$ at $\phi=0$. It is to be noted that the Sagdeev potential given in Eq. (12) contains the effect of ion temperature through σ terms. For $\sigma=0$, we recover Eq. (24) of Sultana *et al.*[18]

It is important to analyze the second derivative of the Sagdeev potential, i.e., $\mathrm{d}^2\psi(\phi,M)/\mathrm{d}\phi^2$, which has to be negative at the origin, $\phi=0$. Thus, the above condition yields

$$\left.\frac{d^2\psi(\phi,M)}{d\phi^2}\right|_{\phi=0} = \frac{R^2(M^2-M_0^2)}{M^2(M^2-M_1^2)} < 0, \tag{13}$$

where

$$M_0 = \beta\left(3\sigma + \frac{\kappa-\frac{3}{2}}{\kappa-\frac{1}{2}}\right)^{1/2} \tag{14}$$

and

$$M_1 = \left(3\sigma + \frac{\kappa-\frac{3}{2}}{\kappa-\frac{1}{2}}\right)^{1/2}. \tag{15}$$

It must be pointed out here that $M_1 \geq M_0$ since $\beta=\cos\theta \leq 1$. Thus, the soliton condition $\mathrm{d}^2\psi(\phi,M)/\mathrm{d}\phi^2<0$

at $\phi = 0$ (i.e., inequality (13)) is satisfied when $M_0 < M < M_1$ for $\beta \neq 1$. It is interesting to note that M_0 and M_1 are critical and upper limit of the Mach number, respectively. Also, it is to be mentioned that for $\beta = 1$, i.e., at parallel propagation ($\theta = 0$) inequality (13) cannot be satisfied because both critical Mach number (M_0) and upper limit (M_1) of the Mach number will coincide with each other, therefore soliton solution will not be possible. Thus, M_1 may be called as the maximum Mach number beyond which soliton solutions are not possible. Further, upper limit of the Mach number does not depend on the angle of propagation.

It may be pointed out that Saini *et al.*[23] carried out the analysis of ion-acoustic solitons in an unmagnetized plasma with superthermal electrons and ions without considering the effect of finite ion temperature. Further, for $R = 0$ and $\sigma = T_i/T_e = 0$, we will not recover their results simply because they have not considered quasi-neutrality condition. Instead, they used Poisson equation whereas we have assumed quasi-neutrality condition in order to obtain analytical expression for the energy integral given by Eq. (11). Therefore, our analysis is valid only for short wavelength perturbations and does not allow for $R = 0$ case. To recover the unmagnetized case of Saini *et al.*,[23] one has to put $R = 0$ in Eqs. (2), (3), (8), and (9) and redo the above analysis and use the Poisson equation instead of quasi-neutrality condition.

III. NUMERICAL RESULTS

In this section, we present our numerical results obtained from the model by solving Eqs. (11) and (12), respectively. At the outset, it must be made clear that we recover all the analytical and numerical results of Sultana *et al.*[18] when thermal effects of ions are removed, i.e., $\sigma = 0$ case. Figure 1 shows the variation of Sagdeev potential $\psi(\phi, M)$ versus electrostatic potential ϕ for different values of the Mach numbers as shown on the curves and for $\sigma = T_i/T_e = 0.0$ and 0.01. The dashed (– – –) and solid (—) curves are corresponding to $\sigma = T_i/T_e = 0.0$ and $\sigma = 0.01$, respectively. The parameters chosen are the same as of Figure 7(a) of Sultana *et al.*,[20] namely superthermality index, $\kappa = 2$, $\beta = \cos\,\theta = 0.9$, ratio of cyclotron to plasma frequency of the ions, $R = \Omega_i/\omega_{pi} = 0.5/1.4$, which corresponds to $\Omega_i = 0.5$ and $\omega_{pi} = 1.4$ in Sultana *et al.*[18] Here, we study the effect of ion temperature on ion-acoustic solitary waves, therefore, we chose ion to electron temperature ratio $\sigma = T_i/T_e = 0.01$. We would like to mention here that in Sultana *et al.*[20] paper, $\Omega_i = 0.5$ and $\omega_{pi} = 1.4$ are taken as dimensionless parameters, which is not the right way to choose these frequencies as they are not the normalized parameters. It is the ratio $R = \Omega_i/\omega_{pi}$ of these two frequencies, which is a dimensionless parameter. In order to compare our results with Sultana *et al.*,[20] we have kept R as same ratio as in their paper. It can be seen from the figure that amplitude of the ion-acoustic solitary waves increases with Mach number and with the inclusion of warm ions, the range of Mach number for which soliton solutions are obtained also widens with lower (critical Mach number) and upper (maximum Mach number) Mach numbers shifting to the higher side. It is also clear from the analytical expressions for M_0 and M_1 given by Eqs. (14) and (15), respectively. A comparison of our results of Figure 1 for M = 0.56 with Sultana *et al.*[20] Figure 7(a) for M = 0.56 curves shows significant reduction in amplitude of the solitary waves due to the inclusion of finite temperature of the ions. We think that the inclusion of finite ion temperature gives rise to dispersive effects, which tends to suppress the effects due to nonlinearity, thus, leading to smaller ion-acoustic soliton amplitudes.

Figure 2 shows the variation of ion-acoustic soliton amplitude, ϕ with ξ for the parameters, namely $\beta = 0.8$, M = 0.8, $\sigma = 0.01$, and $R = \Omega_i/\omega_{pi} = 0.2/1.3$ (same parameters as Figure (8) of Sultana *et al.*[20] for various values of

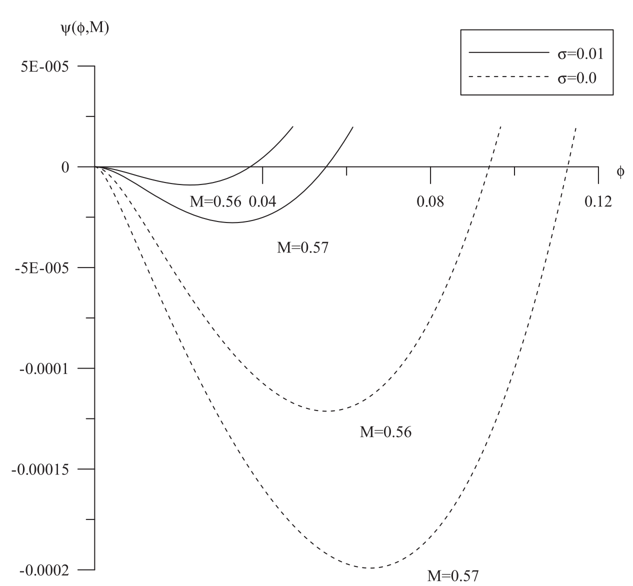

FIG. 1. Variation of the Sagdeev potential $\psi(\phi, M)$ with respect to ϕ for various values of Mach number, M for $\kappa = 2.0$, $R = \Omega_i/\omega_{pi} = 0.5/1.4$, and $\beta = \cos\,\theta = 0.9$. The dashed (- - -) and solid (——) curves are corresponding to $\sigma = T_i/T_e = 0.0$ and $\sigma = 0.01$, respectively.

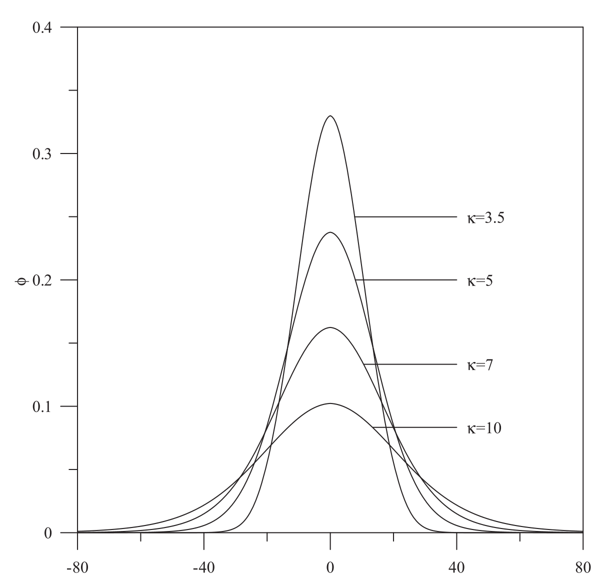

FIG. 2. Variation of normalized real potential, ϕ with respect to ξ for various values of superthermality index, κ for Mach number, M = 0.8, $R = \Omega_i/\omega_{pi} = 0.2/1.3$, $\sigma = 0.01$, and $\beta = 0.8$.

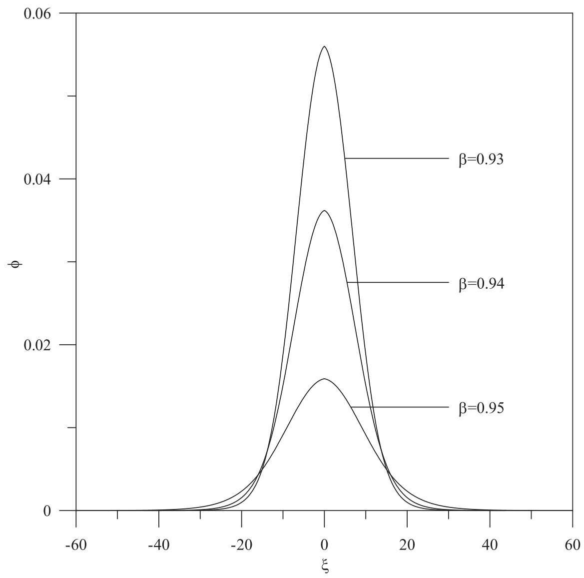

FIG. 3. Variation of normalized real potential, ϕ with respect to ξ for various values of β for $\kappa = 3$, M = 0.76, R $= \Omega_i/\omega_{pi} = 0.5/1.5$, and $\sigma = 0.01$.

superthermality index, κ. Here, also the reduction in the ion-acoustic soliton amplitude for $\sigma \neq 0$ is quite appreciable. The soliton amplitude increases with increase in superthermality, i.e., decrease in κ and solitons become narrower (soliton width decreases). These results are in agreement of the findings of Saini *et al.*[23] and Sultana *et al.*[20]

Figure 3 shows the variation of the ion-acoustic soliton amplitude, ϕ with for the parameters, $\kappa = 3$, M = 0.76, $\sigma = 0.01$, and $R = \Omega_i/\omega_{pi} = 0.5/1.3$ (same parameters as Figure (9) of Sultana *et al.*[18]) for various values of obliqueness, β. It is clear from the figure that the amplitude of the solitons increases with the increasing obliquity, i.e., in increasing angle of propagation, θ (decreasing β). It is important to note that with increasing β values as one approaches $\beta = 1$ case, the soliton solution will not exist. This is because of both critical and upper Mach numbers coincide with each other at $\beta = 1 (\theta = 0)$ [refer to Eqs. (14) and (15)].

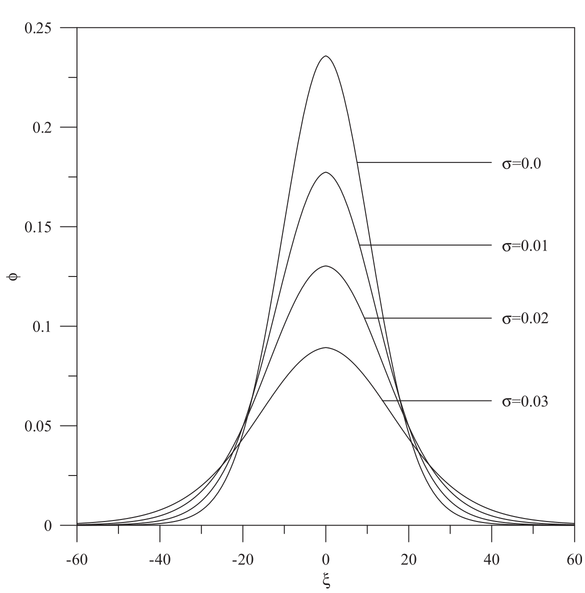

FIG. 4. Variation of normalized real potential, ϕ with respect to ξ for various values of σ for $\kappa = 3$, M = 0.7, R $= \Omega_i/\omega_{pi} = 0.2/1.3$, and $\beta = 0.8$.

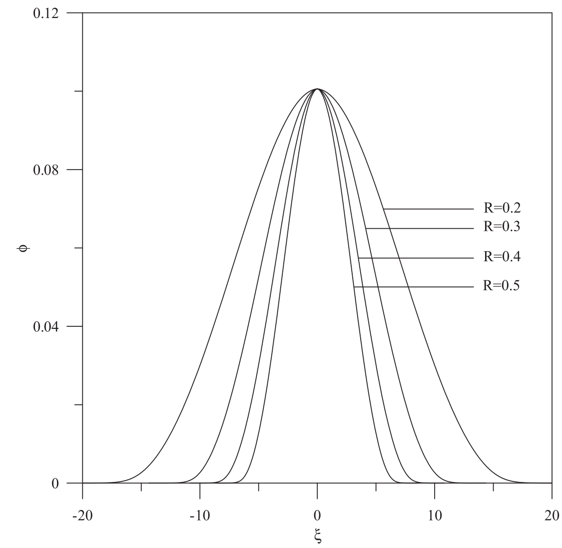

FIG. 5. Variation of normalized real potential, ϕ with respect to ξ for various values of R $= \Omega_i/\omega_{pi}$ for $\sigma = 0.01$ for $\kappa = 2$, M = 0.6, and $\beta = 0.9$.

In Figure 4, amplitude ϕ of the ion-acoustic soliton versus ξ is shown for the parameters: $\kappa = 3$, M = 0.7, $\beta = 0.8$, and $R = \Omega_i/\omega_{pi} = 0.2/1.3$, and for various values of the ion to electron temperature ratio, $\sigma = T_i/T_e$ as depicted on the curves. It is important to note that amplitude of the soliton is maximum for cool ions ($\sigma = 0$), and it decreases as the ion to electron temperature increases. Also, width of the soliton increases with the increase in σ values.

The soliton amplitude ϕ versus ξ is plotted in Figure 5 for various values of $R = \Omega_i/\omega_{pi}$ as shown on the curves for other chosen parameters: $\sigma = 0.01$ for $\kappa = 2$, M = 0.6, and $\beta = 0.9$. It is obvious from Figure 5 that the maximum amplitude of the ion-acoustic soliton is not affected with the increase in R values. However, the width of the solitons decreases. These results are in agreement of Sultana *et al.*[20].

IV. CONCLUSION

We have examined the nonlinear properties of ion-acoustic waves in magnetized plasma consisting of warm ions and superthermal electrons having kappa distribution. This is an extension of the work of Sultana *et al.*[20] by including finite ion temperature. They had studied the ion-acoustic solitons in a magnetized plasma with kappa distributed electrons and cold ions. In our analysis, we have shown that the presence of warm ions affects the speed, amplitude, width, and pulse duration of solitons. The lower and upper limit of the Mach numbers gets affected by the presence of warm ions and both critical (M_0) as well as upper (M_1) Mach numbers are pushed to the higher side. The ion-acoustic soliton amplitude increases with the increase in superthermality of electrons. An increase in the magnetic field, i.e., R value, reduces the width of the ion-acoustic solitons without affecting their amplitudes.

Viking satellite reported observation of ion-acoustic solitary waves in the auroral region of the Earth's magnetosphere.[10] Here, we apply our theoretical model to the observed

TABLE I. Variation of the soliton velocity (V), electric field (E), soliton width (W), and pulse duration (τ) with respect to $\sigma = T_i/T_e$ for $\beta = 0.9$, $\kappa = 2.0$, total equilibrium electron density $N_{0e} = 2\,\mathrm{cm}^{-3}$, ambient magnetic field, $B_0 = 10^4$ nT, and $R = \Omega_i/\omega_{pi} = 1/1.86$. The values of B_0, R, and $\sigma = 0.0246$ (last row) corresponds to the parameters observed by Viking satellite in the auroral region.[10]

σ	V ($\mathrm{km\,s^{-1}}$)	E ($\mathrm{mV\,m^{-1}}$)	W (km)	$\tau = W/V$ (ms)
0.0	25.9–28.8	0.01–33.7	1.27–0.14	49–4.8
0.01	27.1–30.1	0.04–25.8	0.87–0.15	32.1–5
0.02	28.2–31.3	0.17–21.6	0.6–0.16	21.1–5.15
0.0246	28.7–31.8	0.18–20.1	0.59–0.17	20.5–5.25

parameters, namely, ambient magnetic field, $B_0 = 10^4$ nT, ion temperature, $T_i = 0.64$ eV, electron temperature, $T_e = 26$ eV, ion cyclotron frequency, $\Omega_i = 1000$ rad/s, and ion plasma frequency, $\omega_{pi} = 1860$ rad/s. For the above mentioned parameters, the ratio of cyclotron to plasma frequency of the ions comes out to be $R = 1/1.86$ and $\sigma = T_i/T_e = 0.0246$. The above mentioned parameters are taken from Ref. 10. Table I summarizes our results for different values of σ. For the auroral region parameters observed by Viking,[10] the soliton speed, amplitude, width, and pulse duration comes out to be in the range of (28.7–31.8) km/s, (0.18–20.1) mV/m, (590–167) m, and (20.5–5.25) ms, respectively (refer to last row of Table I). The observations[8] from Viking satellite have shown soliton speed, maximum electric field, width, and pulse duration, respectively, to be $\approx(5-50)\,\mathrm{km/s}$, $\approx(10-40)\,\mathrm{mV/m}$, few hundred meters and ≈20 ms. Thus, results obtained through our theoretical model are in good agreement of the Viking satellite observations. It is also clear from Table I that with increasing σ values, the range of soliton speeds increases, however, range of amplitude, width, and pulse duration decreases.

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