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Deterministic retrieval of ionospheric phase screen from amplitude scintillations

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Abstract. In the phase screen theory of ionospheric scintillations the ionospheric irregularities are considered as pure phase objects, and the diffraction pattern produced on the ground, by radio waves propagating through this phase screen, is obtained from conventional Fresnel diffraction theory. In the present paper the inverse problem of retrieving phase variations in the screen from the diffraction pattern on the ground is treated by using the transport-of-intensity equation (TIE), which is derived from the Fresnel diffraction theory result. An approximate version of the TIE is solved to obtain the phase variation in the screen from the intensity distribution on the ground. It is seen that when the phase variations involve spatial scale lengths ~400 m, phase fluctuations of ~5 rad can be recovered from intensity variations on the ground, for a 300-MHz signal. For larger spatial scale lengths and higher frequencies this method can be used to retrieve larger phase variations in the screen.

1. Introduction

In most studies of the ionospheric scintillation phenomenon, statistical characteristics of fluctuations in amplitude and phase of transionospheric radio waves, recorded on the ground, are determined with the view of obtaining a statistical picture of the ionospheric irregularities which give rise to the scintillations. The framework for this is provided by a stochastic approach in theories of scintillations in which the ionospheric irregularities are assumed to constitute a random medium, characterized by its statistical properties [*Yeh and Liu*, 1982; *Rytov et al.*, 1989].

For a sufficiently thin layer of irregularities, amplitude fluctuations which occur within the layer are negligible, and the layer can be replaced by a phase-changing screen wherein the irregularities are treated as pure phase objects that impose a phase variation on the incident radio wave. The forward problem of calculating the fluctuations which occur in amplitude and phase of radio waves, on propagation through this phase screen to the reception plane, has usually been solved on the basis of the Huygens-Fresnel approximation [*Booker et al.*, 1950; *Ratcliffe*, 1956]. Further, in the "phase screen theory", phase variations in the screen are assumed to constitute a Gaussian random field with zero mean, and moments of the amplitude and phase variations in the reception plane are derived in terms of the correlation function of the phase variations in the screen. Thus, using this theory, it has been possible to deduce statistical properties of the irregularities such as mean square fluctuations of electron density and power spectra of the irregularities [*Rufenach*, 1975]. In general, such calculations can be carried out for only a few moments, and it is not possible to determine the actual distribution of phase fluctuations in the screen. A statistical theory of weak scintillations produced by a thick layer of irregularities is based on the Rytov solution for the complex amplitude [*Barabenenkov et al.*, 1971], which, in this case, satisfies a linearized version of the parabolic equation describing propagation of the waves under the forward scattering and Fresnel approximations. Strong scintillations have been treated theoretically by seeking solutions of the equations satisfied by the moments of the complex amplitude of the propagating radio wave [*Yeh et al.*, 1975; *Bhattacharyya et al.*, 1992]. These studies have yielded information about statistical properties of the iono-

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spheric irregularities through calculation of the S_4 index, power spectra, and the mutual coherence function using amplitude and phase scintillation data.

Adoption of a stochastic approach in the standard theories of ionospheric scintillations has proved to be useful in view of the complexity of the irregularity structures. However, the limitations of such an approach have also been considered in the past. For instance, ionospheric irregularities may be characterized by sharp gradients, implying a high degree of phase coherence of their Fourier spectral components, or they may be described as turbulence with little phase coherence in their spectral components, although the power spectra are identical in both cases. *Wernik et al.* [1980] have carried out a forward calculation of scintillations produced by ionospheric electron density variations with these two distinct characteristics by solving the parabolic equation satisfied by the wave field. This led to the idea of considering the phenomenon of ionospheric scintillation as scattering due to a fractal screen and thus obtaining information about the fractal nature of the ionospheric phase screen by studying the chaotic behavior of ionospheric scintillations [*Bhattacharyya*, 1990; *Wernik and Yeh*, 1994].

Statistical studies of scintillation data also require that the stationarity condition be satisfied by the data. This requirement is often difficult to satisfy, especially when the scintillations arise due to isolated irregularity patches. All these considerations point to the need for finding solutions of the inverse problem of deterministic retrieval of ionospheric phase variations, caused by the irregularities, from scintillation data. In the past, simultaneous phase and amplitude measurements have been utilized in the reconstruction of ionospheric irregularities by inverse propagation using Fourier optical techniques similar to those involved in holography [*Schmidt and Tauriainen*, 1975; *Stone*, 1976; *Liu and Yeh*, 1980; *Tauriainen*, 1982]. With these techniques it is possible to obtain the wave field distribution at the bottom of the irregularity slab. In order to simplify the problem further, in practice, the irregularity slab is replaced by a phase screen, and the ground wave pattern is used in inverse propagation to a height where the amplitude fluctuations become a minimum, which is then assumed to be the height of the phase screen. For a thin irregularity region (thickness ~30 km) considered by *Liu and Yeh* [1980], the retrieved phase variations in the equivalent phase screen yielded useful information about electron density fluctuations in the model equatorial bubble, which was used in obtaining the amplitude and phase variations on the ground. However, there may be a problem in using actual phase data in such reconstruction. This happens because whereas the beginning and end of intensity scintillations are clearly demarcated in actual data, for phase scintillations it is necessary to separate the background trend from the short-period variations in order to limit the extent of phase scintillation data. It can be seen from equation (6) of *Liu and Yeh* [1980] that it is only possible to compute the required one-dimensional Fourier transform of the wave field on the ground if the variations in the complex amplitude of the wave are limited to a finite range of x. The ground phase data used to obtain the backward propagated wave would have a long wavelength trend which has to be removed.

In the present study, deterministic phase retrieval from amplitude data alone is attempted. The idea used here was first suggested by *Teague* [1983] and has in recent times been utilized successfully in imaging phase objects using hard X rays [*Nugent et al.*, 1996]. To test the utility of the method, amplitude variations in the plane of the receiver due to propagation of a radio wave through isolated ionospheric irregularities, considered as pure phase objects, are obtained in a forward calculation described in section 2. The inverse problem of retrieving the phase variations in the ionosphere from the amplitude variations in the reception plane is treated in section 3. Section 4 contains a discussion of the results and their utility.

2. Amplitude Variations Due to Isolated Irregularities

Diffraction patterns produced on the ground when radio wave signals traverse isolated irregularities in the ionosphere have been studied in the past by many workers [*Titheridge*, 1971; *Davies and Whitehead*, 1977; *Heron*, 1979]. In all these studies the irregularities have been assumed to be pure phase objects. In the middle and low latitudes, occasionally there have been observations of isolated patches of amplitude fluctuations. These "ringing" type of amplitude scintillation events were modeled in terms of diffraction produced by isolated, field-aligned irregularities with sufficiently small extension in the vertical direction such that the irregularities could be considered to simply constitute a phase-changing screen. Further, one such irregularity was assumed to give rise to a Gaussian-shaped phase variation in the screen. The scale size associated with these ringing irregularities was considered to be in the range 100-1000 m [*Davies*, 1990].

The pattern of amplitude variation on the ground, for plane waves of wavelength λ, normally incident on a phase-changing screen located at $z = 0$, are obtained in a forward calculation wherein phase variations are imposed on the incident wave by a set of three closely spaced irregularities, as considered by *Davies* [1990]. These irregularities are assumed to give rise to phase variations in the x direction only. Each irregularity produces a Gaussian phase variation:

$$\phi_n(x) = \phi_{0n} \exp[-(x - x_n)^2 / d_n^2] \qquad (1)$$

With the assumption of forward scattering, complex amplitude of the wave at a point (x, y, z) on the ground at a distance z below the phase screen, can be obtained from Fresnel diffraction theory as

$$u(x, y, z) = \frac{iA_0}{\lambda z} \int\int \exp\Big\{ -i\phi_0(x') - \frac{i\pi}{\lambda z} \cdot [(x - x')^2 + (y - y')^2] \Big\} dx'\, dy' \qquad (2)$$

where A_0 is the amplitude of the incident wave and $\phi_0(x')$ is the phase variation produced by the screen, which here is given by

$$\phi_0(x) = \sum_{n=1}^{3} \phi_n(x) \qquad (3)$$

With $A_0 = 1$, the inphase and quadrature components of the wave received at (x, y, z) are given by

$$X = 1 - \frac{2}{(\lambda z)^{1/2}} \int_{-\infty}^{\infty} \sin\left[\frac{\phi_0(x')}{2}\right] \cdot \sin\left[\frac{\phi_0(x')}{2} + \frac{\pi}{\lambda z}(x - x')^2 - \frac{\pi}{4}\right] dx' \qquad (4)$$

$$Y = \frac{2}{(\lambda z)^{1/2}} \int_{-\infty}^{\infty} \sin\left[\frac{\phi_0(x')}{2}\right] \cdot \cos\left[\frac{\phi_0(x')}{2} + \frac{\pi}{\lambda z}(x - x')^2 - \frac{\pi}{4}\right] dx' \qquad (5)$$

respectively. Intensity of the received wave is then obtained from

$$I = X^2 + Y^2 \qquad (6)$$

For the purpose of comparison with earlier results, location and scale sizes of the irregularities are identical to those considered by *Davies* [1990]. Thus the following values for z, x_n, and d_n are assumed in the forward calculation: $z = 400$ km, $x_1 = 13.9$ km, $x_2 = 15.0$ km, $x_3 = 15.9$ km, $d_1 = 400$ m, $d_2 = 410$ m, and $d_3 = 390$ m. Three different cases are considered for the strength of the irregularities:

Case 1 $\quad \phi_{01} = -0.467$ rad; $\phi_{02} = -0.560$ rad; $\phi_{03} = -0.373$ rad

Case 2 $\quad \phi_{01} = -4.670$ rad; $\phi_{02} = -5.600$ rad; $\phi_{03} = -3.730$ rad

Case 3 $\quad \phi_{01} = -9.340$ rad; $\phi_{02} = -11.20$ rad; $\phi_{03} = -7.460$ rad

For radio waves of frequencies 300 MHz ($\lambda = 1$ m) and 60 MHz ($\lambda = 5$ m), the pattern of amplitude variations on the ground caused by the irregularities in case 1 is shown in Figures 1a and 1b, respectively. Amplitude fluctuations on the ground on a 300-MHz signal due to the irregularities in cases 2 and 3 are depicted in Figures 2a and 2b. For a given phase variation in the screen the patterns in Figure 1 show increased depth of amplitude fading and spreading out of the pattern on the lower-frequency signal. Figure 2 shows that for a given signal frequency, as the irregularity strength increases, there is a corresponding increase in the depth of amplitude fading and a decrease in the spatial scale sizes associated with the pattern of amplitude variations on the ground, which would be converted to temporal variations at higher frequencies if the irregularities drifted across the signal path. It is also seen from Figures 1 and 2 that the extent of the pattern on the ground is basically controlled by the signal wavelength, as is evident from equations (4) and (5). Hence, with increasing strength of the irregularities, there is no corresponding increase in the spread of the ground pattern. Dimensions of the first Fresnel zone, $\sqrt{2\lambda z}$, for signals of wavelengths 1 m and 5 m, are 894 m and 1.99 km, respectively. The diffraction pattern for the phase of the wave in the reception plane also exhibits some of the characteristics described above for the pattern of amplitude fluctuations. For case 1, the ground diffraction patterns for the phases of received signals of various wavelengths have been obtained by *Davies* [1990]. In the present study, only amplitude variations will be used in the reconstruction of the irregularities, as described in section 3.

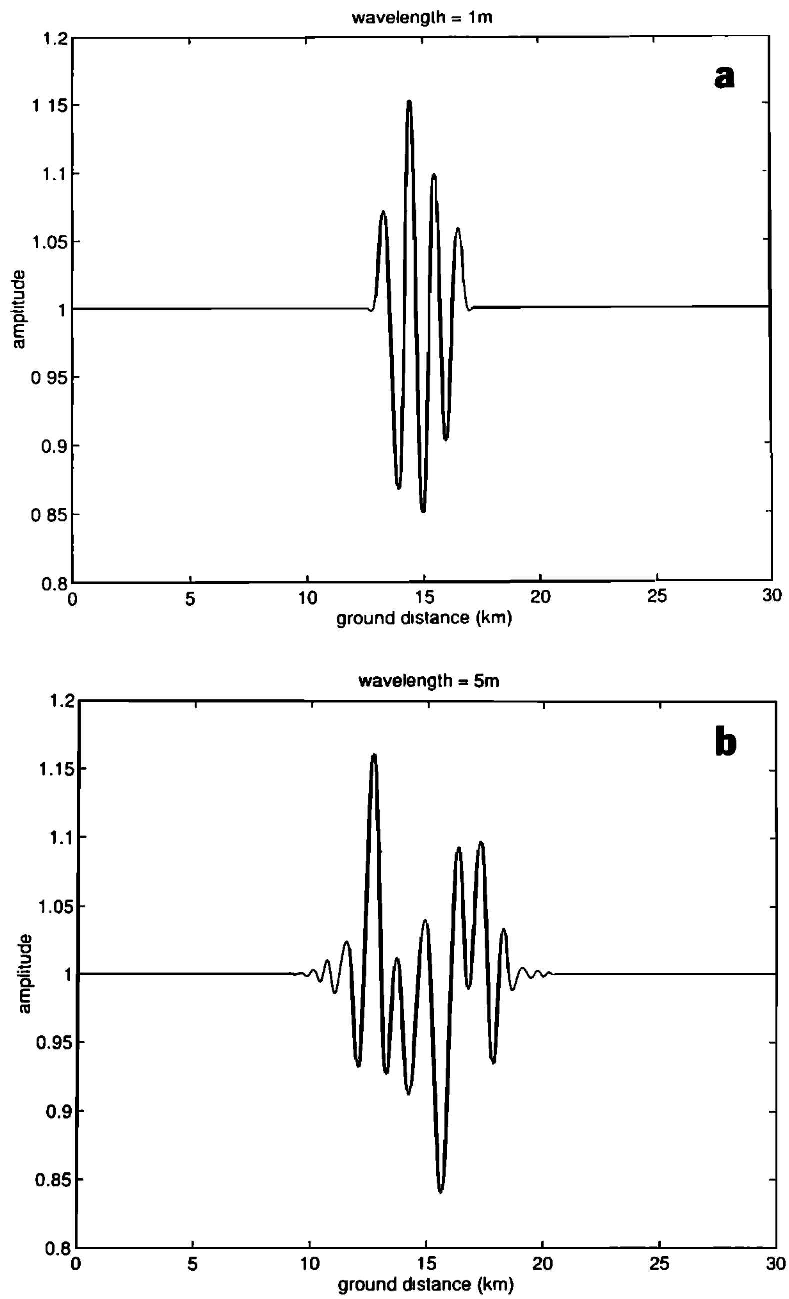


Figure 1. Diffraction patterns of amplitude for radio waves of wavelengths (a) 1 m and (b) 5 m, produced by a set of three irregularities with sizes and positions given in section 2 and strengths described in case 1.

3. Inverse Source Problem

For a plane wave of unit amplitude propagating in the z direction, a component of the wave electric field can be written as a complex entity:

$$u_z(\vec{r}) = u(\vec{r}) \exp[-ikz] \tag{7}$$

where $k = 2\pi / \lambda$. When this wave encounters a phase screen located in the $z = 0$ plane, in its propagation path, conventional Fresnel diffraction theory yields

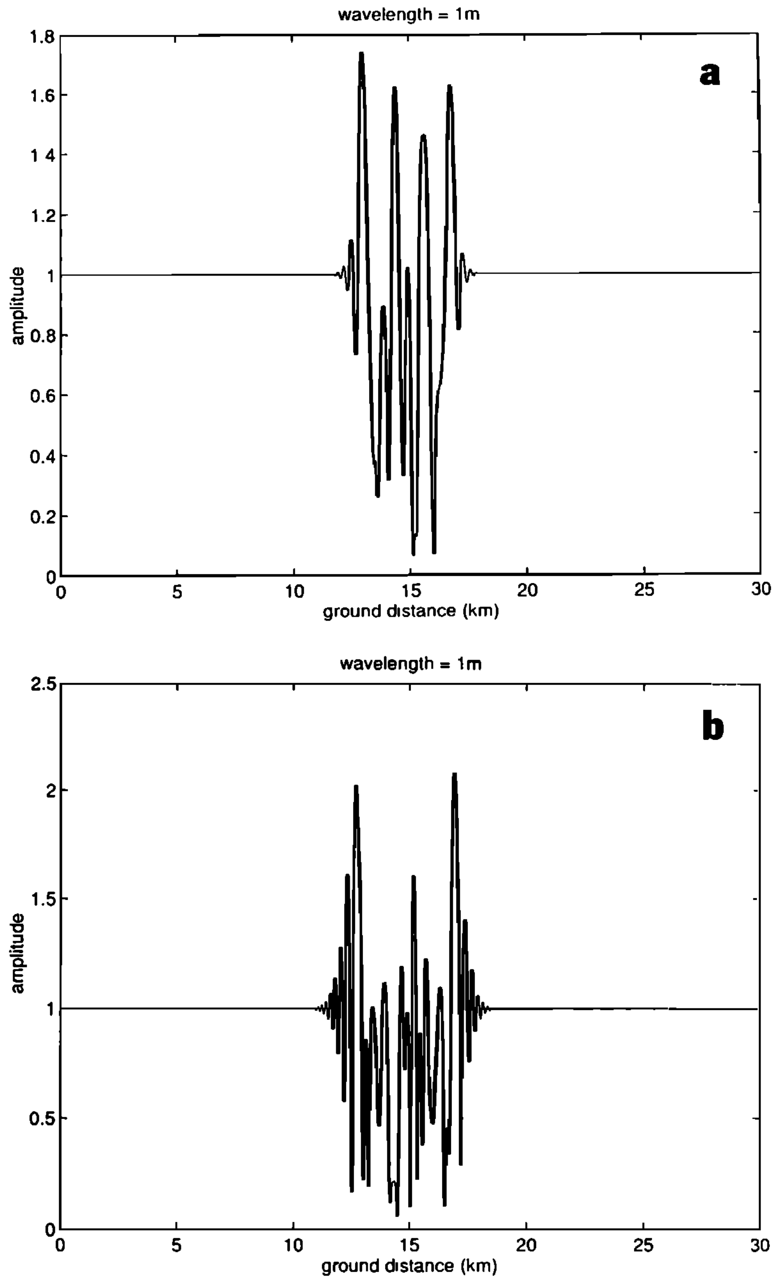


Figure 2. Diffraction patterns of amplitude for radio waves of 1-m wavelength, produced by the set of three irregularities with strengths described in (a) case 2 and (b) case 3.

the following for the complex wave amplitude at a distance z from the screen:

$$u_z(\vec{r}) = \frac{i}{\lambda z} e^{-ikz} \int\int \exp\Big\{ -i\phi_0(\vec{\rho}') - i\frac{\pi}{\lambda z}|\vec{\rho} - \vec{\rho}'|^2\Big\} d^2\rho' \qquad (8)$$

where $\phi_0(\vec{\rho}')$ is the phase variation imposed by the screen. A special case of this was considered in section 2 in equation (2), where ϕ depends only on x. This Fresnel result satisfies exactly the parabolic equation

$$\left[-i\frac{\partial}{\partial z} + \frac{1}{2k}\nabla_\perp^2 + k\right] u_z(\vec{r}) = 0 \qquad (9)$$

where $\nabla_\perp^2 = [(\partial^2/\partial x^2) + (\partial^2/\partial y^2)]$. The complex wave amplitude at a point may be expressed in terms of the intensity $I(\vec{r})$ and phase $\phi(\vec{r})$, which are real quantities:

$$u_z(\vec{r}) = [I(\vec{r})]^{1/2} \exp[-i\phi(\vec{r})] \tag{10}$$

From equation (9) multiplied by $u_z^*(\vec{r})$, the complex conjugate of equation (9) multiplied by $u_z(\vec{r})$ is subtracted to yield the transport-of-intensity equation (TIE):

$$k\frac{\partial I}{\partial z} = -\vec{\nabla}_\perp \cdot (I\vec{\nabla}_\perp \phi) \tag{11}$$

Addition of the two resultant equations above leads to the following equation:

$$2kI^2\frac{\partial \phi}{\partial z} = \frac{1}{2}I\nabla_\perp^2 I - \frac{1}{4}(\nabla_\perp I)^2 - I^2(\nabla_\perp \phi)^2 + k^2 I^2 \tag{12}$$

From (11) and (12) it is seen that, in general, ϕ and I satisfy a set of coupled nonlinear differential equations. Thus, if both ϕ and I are unknown, it is simpler to seek a solution for the complex amplitude from the parabolic equation (9), as is done in the forward problem yielding the Fresnel diffraction result given in (8) when ϕ_0 is known. In the inverse source problem considered here a solution for $\phi_0(\vec{\rho})$ is required when the intensity distribution in the reception plane is known through measurement. In this case equation (11) is the obvious choice for seeking a solution since it becomes a linear equation for ϕ when the distribution of intensity is known.

Here a one-dimensional phase screen is assumed with phase variations along the x direction only. Thus equation (11) reduces to

$$k\frac{\partial I}{\partial z} = -\frac{\partial}{\partial x}\left(I\frac{\partial \phi}{\partial x}\right) \tag{13}$$

Integration of the left-hand side of equation (13) from $z = 0$, where the phase screen is located, to the ground yields the change in intensity distribution measured on the ground, $I(\vec{\rho}, z)$, from the uniform intensity I_0 of the wave incident at $z = 0$. For the right-hand side, some approximations are made. First, it is assumed that

$$\left(\frac{\partial I}{\partial x}\right)\left(\frac{\partial \phi}{\partial x}\right) \ll I\frac{\partial^2 \phi}{\partial x^2} \tag{14}$$

At $z = 0$, ϕ varies with x, but $I = I_0$ is independent of x. Hence sufficiently close to the phase screen it is expected that the assumption will hold because lateral variations in I are gradually built up as the wave propagates beyond the phase screen. It is a basic property of diffraction that sharp edges in the diffracting screen, which would be equivalent to large gradients in ϕ, are blurred in the image at the reception plane due to diffraction. Hence the left-hand side of the inequality (14) is expected to be smaller than the right-hand side, the ratio of the two being governed by the magnitude of phase variation in the phase screen and the length scales associated with this variation. The range of applicability of this approximation will be seen in some case studies with the model irregularities considered in section 2. With assumption (14) the integrand on the right-hand side has only one term: $I\partial^2\phi/\partial x^2$. Now the second assumption is made, namely, that this term varies slowly with z so that it may be treated as a constant in carrying out an integration over z. To see the extent of validity of this assumption, this term is expanded as follows:

$$I\frac{\partial^2 \phi}{\partial x^2} = I_0\frac{d^2\phi_0}{dx^2} + \frac{\partial}{\partial z}\left(I\frac{\partial^2 \phi}{\partial x^2}\right)\bigg|_{z=0} z + \cdots \tag{15}$$

For evaluating the second term on the right-hand side of equation (15), use is made of equation (13) and the one-dimensional version of equation (12), keeping in mind that at $z = 0$, $\partial I/\partial x = 0$. Thus the following expression is obtained for the second term:

$$\frac{\partial}{\partial z}\left(I\frac{\partial^2 \phi}{\partial x^2}\right)\bigg|_{z=0} z = -\frac{I_0 z}{k}\left[\left(\frac{d\phi_0}{dx}\right)\left(\frac{d^3\phi_0}{dx^3}\right) + 2\left(\frac{d^2\phi_0}{dx^2}\right)^2\right] \tag{16}$$

In order that this term be negligible compared with the first term on the right- hand side of equation (15), the following condition must be satisfied:

$$\frac{z}{k}\left|\left(\frac{d\phi_0}{dx}\right)\left(\frac{d^3\phi_0}{dx^3}\right) + 2\left(\frac{d^2\phi_0}{dx^2}\right)^2\right| \ll \left|\frac{d^2\phi_0}{dx^2}\right| \tag{17}$$

which requires that

$$\frac{\lambda z}{2\pi d^2}\phi_0 \ll 1\,, \tag{18}$$

where d represents a typical horizontal scale length associated with the irregularities. Under the above assumptions the TIE for a one-dimensional phase screen yields the following simple differential equation for ϕ_0:

$$\frac{I(x, z)}{I_0} = 1 - \frac{\lambda z}{2\pi}\frac{d^2\phi_0}{dx^2} \tag{19}$$

As far as irregularities of finite extent are concerned this equation can be solved for $\phi_0(x)$ with the appropriate initial conditions. To demonstrate the use of (19) in the deterministic retrieval of phase variations in the ionospheric phase screen from amplitude variations measured on the ground, intensity fluctuations produced by the model irregularities considered in section 2 will be utilized.

It can be seen from Figures 1 and 2 that for irregularities of effectively finite extent, length of the patch of amplitude variations on the ground is greater than or equal to the effective size of the irregularities. Here the term "effective" is introduced to indicate that even for irregularities such as those considered in section 2, in practical terms their extent is finite in that measureable amplitude fluctuations in the reception plane are only produced by a region of the phase screen where phase variations differ significantly from zero. Thus the following boundary conditions are used in solving equation (19):

$$\phi_0 = 0; \qquad \frac{d\phi_0}{dx} = 0 \qquad \text{at} \qquad x = x_0 \tag{20}$$

where x_0 is chosen to be well before the point where significant amplitude fluctuations are seen. The solution of equation (19) is now given by

$$\phi_0(x) = \frac{2\pi}{\lambda z}\int_{x_0}^{x}\int_{x_0}^{x'} [1 - I(x'')]\, dx''\, dx' \tag{21}$$

The integrals in (21) can be evaluated approximately from intensity values at equally spaced points along the x axis. In the present calculations the ground intensity is sampled at intervals of 12.5 m. Assuming the drift speed of the irregularities across the signal path to be typically of the order of 100 m/s, this sampling distance is converted to a sampling interval of approximately 0.12 s for intensity scintillation data recorded by a receiver on the ground. Scintillation observations are generally carried out at sampling intervals of $\leq$0.1 s, which should be adequate to resolve phase variations on scale legths ~400 m in the ionosphere, provided the conditions required for application of this method are satisfied, as will be seen from the results obtained here. The ability of the technique to reproduce a given phase gradient is limited by the condition expressed in (18), which needs to

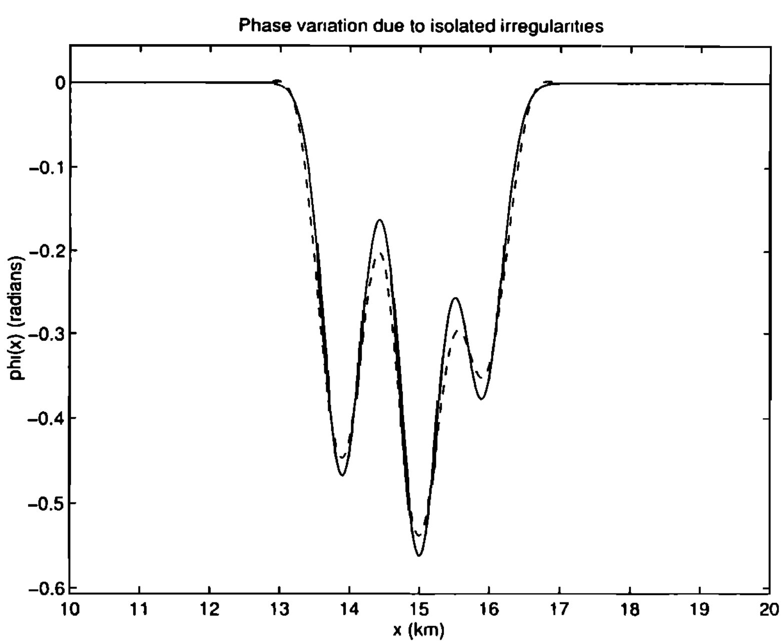


Figure 3. Phase variation in the phase screen retrieved from amplitude fluctuations on a 1-m wavelength signal, shown in Figure 1a, with z = 400 km (true height), is depicted by the dashed curve. Solid curve shows the actual phase variation in the screen.

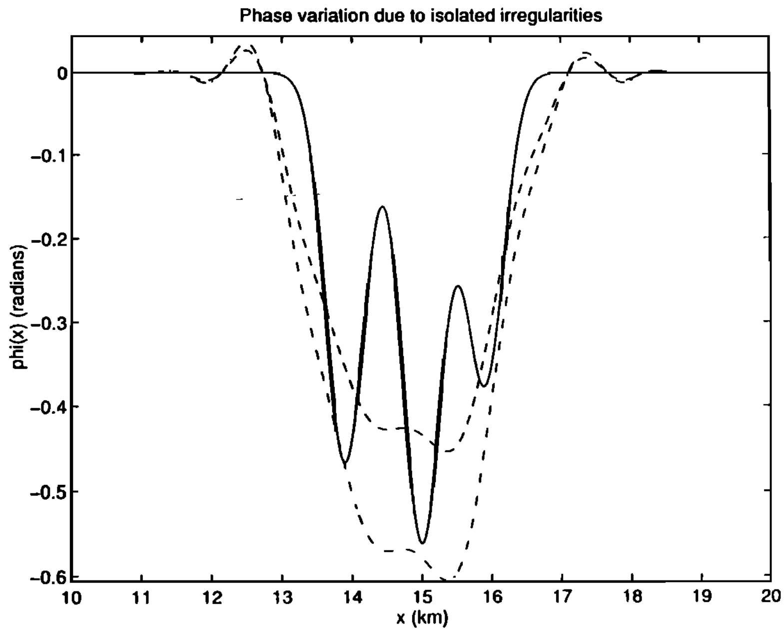


Figure 4. Phase variation in the phase screen retrieved from amplitude fluctuations on a 5-m wavelength signal, shown in Figure 1b, with z = 400 km (dashed curve) and z = 300 km (dash-dotted curve). Solid curve shows the actual phase variation in the screen.

be satisfied for the underlying assumptions to hold. Thus, for a given magnitude of phase variation, data on a higher-frequency signal would be able to reproduce steeper phase gradients. The phase variations imposed on radio waves of wavelengths 5 m and 1 m, due to the irregularities considered in section 2, are retrieved from the computed amplitude fluctuations shown in Figures 1 and 2. In the deterministic retrieval of phase variation in this manner, height of the phase screen has to be provided as an input. It is clear from (21) that the percentage error in the computed $\phi_0(x)$ at any point due to uncertainty in the value of z used in the retrieval scheme is the same as the percentage error in z. The phase $\phi_0(x)$ retrieved from intensity fluctuations on a signal of wavelength 1 m, caused by the irregularities considered in case 1 of section 2, is plotted along with the actual phase variation in Figure 3. In this case the actual value of z (400 km) has been used in the determination of $\phi_0(x)$. For the results shown in Figure 4, amplitude fluctuations on a radio wave of wavelength 5 m shown in Figure 1b are used in the retrieval of $\phi_0(x)$. Two different values of z have been used, one being the true value and the other equal to 300 km. Amplitude variations on a signal of wavelength 1 m, caused by irregularities considered in case 2 of section 2, which are depicted in Figure 2a, have been used in the reconstructions shown in Figure 5. The two curves plotted here, besides that for the actual phase variation, correspond to screen heights of 400 km (true height) and 500 km. It is seen that phase variations $\gg$1 rad can also be retrieved to some extent with this method. For even larger phase variations, results deteriorate significantly, as can be seen from Figure 6. In this case, amplitude fluctuations on a 1-m wavelength signal, caused by phase variations considered in case 3 of section 2 and depicted in Figure 2b, have been used to retreive the phase. It is seen that for phase variations ~10 rad on a 300-MHz signal, information about spatial variations on short scale lengths (<1 km) cannot be recovered by the present method. Hence, in such cases, deterministic phase retrieval carried out here can only serve to provide the lowest-order estimate.

Amplitude scintillations recorded on a radio wave signal transmitted from a geostationary satellite and received on the ground are temporal variations caused by the movement of the diffraction pattern on the ground due to drift of the irregularities across the signal path. In order to recover the ground diffraction pattern from recorded amplitude scintillations

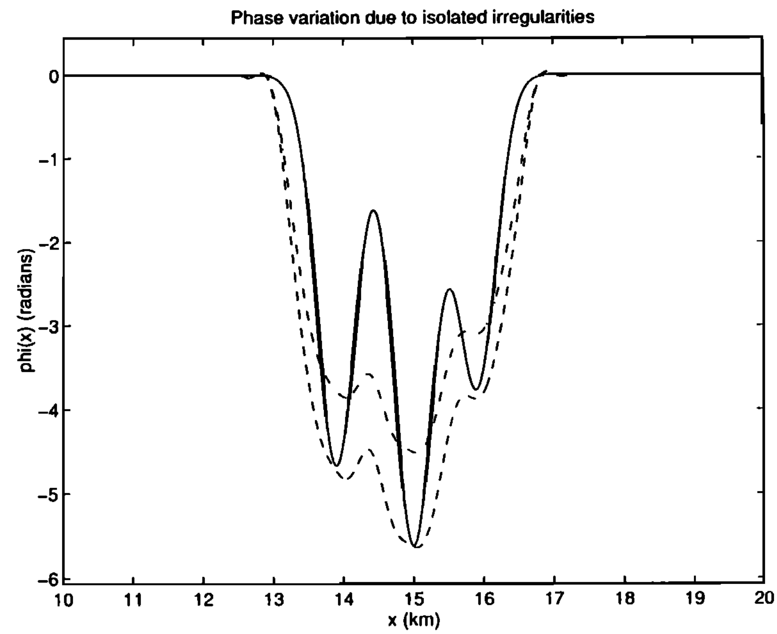


Figure 5. Phase variation in the phase screen retrieved from amplitude fluctuations on a 1-m wavelength signal, shown in Figure 2a, with $z = 400$ km (dashed curve) and $z = 500$ km (dash-dotted curve). Solid curve shows the actual phase variation in the screen.

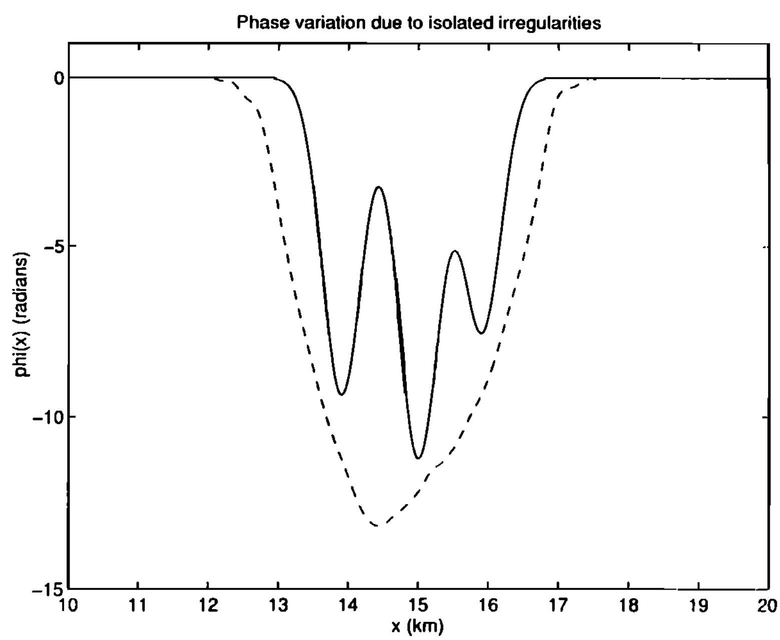


Figure 6. Phase variation in the phase screen retrieved from amplitude fluctuations on a 1-m wavelength signal, shown in Figure 2b, with $z = 400$ km (dashed curve). Solid curve shows the actual phase variation in the screen.

it is necessary to have information about the drift speed of the irregularities transverse to the signal path. Spaced receiver measurements of amplitude scintillations can be used to estimate the average drift speed of the irregularities across the signal path [*Wernik et al.*, 1983]. In the case of radio waves transmitted from an orbiting satellite, velocity of the satellite determines the speed at which spatial variations are converted to temporal variations. An approximate value for the effective height z of the phase screen can be obtained from the power spectra of weak to moderate intensity scintillations. Since irregularities of scale lengths of the order of the Fresnel dimension $d_F = \sqrt{2\lambda z}$ contribute the most to intensity scintillations, an estimate of the Fresnel frequency, $\Omega_F = 2\pi V/d_F$, from the power spectra of weak to moderate intensity scintillations can yield an approximate value of z, provided V, the average drift speed of the irregularities transverse to the signal path, is known. As discussed above, V can be determined from spaced receiver measurements of amplitude scintillations when the signal is transmitted from a geostationary satellite.

4. Conclusion

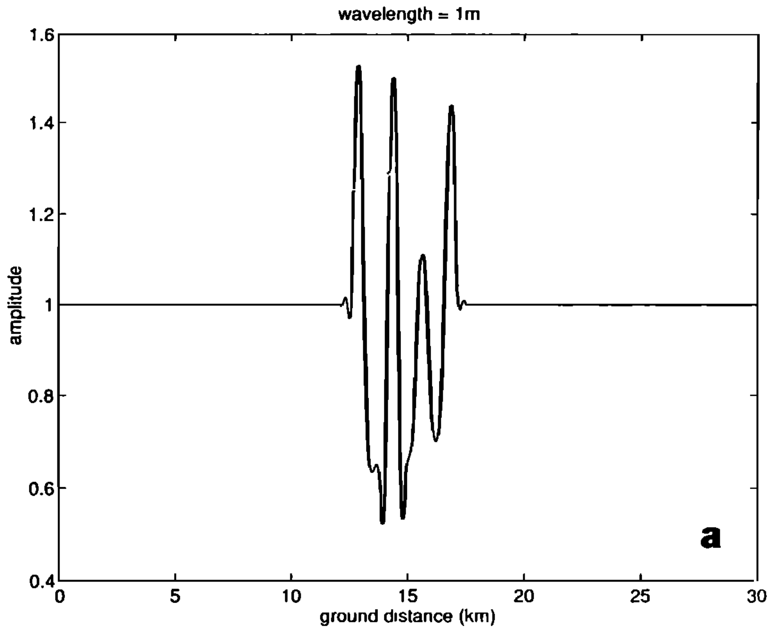
The basic idea of this paper is to investigate whether radio waves can be used in the imaging of isolated ionospheric irregularities, treated as pure phase objects, through Fresnel diffraction, as is being done in imaging phase objects using hard X rays [*Nugent et al.*, 1996]. There have been earlier attempts at reconstruction of ionospheric irregularities using simultaneously recorded amplitude and phase scintillation data, which have also been based on a phase screen approach [*Liu and Yeh*, 1980; *Tauriainen*, 1982]. Use of both amplitude and phase scintillation observations in the reconstruction of the phase screen does not require as many assumptions as are made here, and also information about the screen height is not required. However, as mentioned in the introduction, a long-term trend in the phase needs to be treated properly. In order to compute the Fourier transform of the wave field on the ground, as required for the reconstruction, it is necessary to filter out the long-term trend. This procedure needs justification. The method studied here uses only amplitude or intensity fluctuation data and can be very simply used to obtain a lowest-order estimate of variations in the total electron content (TEC) along the signal line of sight, due to the presence of ionospheric irregularities. Phase scintillations are not used here since the equation involving phase fluctuations is nonlinear. It may be possible to use this method to remove any ambiguity introduced in the reconstruction of the phase screen by the backward propagation technique, due to the application of a filter to the measured phase variations. This can be done by retrieving the short scale length phase variations in the screen independently from intensity variations close to the screen, where the present method should yield accurate results by virtue of the condition given in equation (18). These phase variations may be compared with the phase variations obtained by backward propagation of the wave field on the ground. In this manner, the present method could be useful in the unambiguous determination of short scale length variations of phase in the irregularity screen.

It is seen from the results obtained in section 3 that the method may be used for retrieving phase variations up to 5 rad on a radio wave of 1-m wavelength, when irregularities of spatial scale length ~400 m are present. A phase variation of 5 rad on this wavelength implies a deviation ΔN_T in the TEC of around 2×10^{15} el/m^2. Deviations in TEC that may be more than an order of magnitude larger than this value are encountered in the ionosphere when plasma instabilities evolve nonlinearly. In support of the present method, it may be stated that since (13) is obtained from the first-order expansion of the conventional Fresnel diffraction theory result,

$$u_z(\vec{r}) = \exp\left[-ikz - i\frac{\lambda z}{4\pi}\nabla_\perp^2\right]u_0(\vec{r}) \qquad (22)$$

if the scale length of phase variations in the screen increases, validity of the present method can be extended to larger phase variations. Thus, in the absence of small-scale (<500 m) irregularities, much larger deviations in the TEC can be retrieved using this method because then the larger-scale irregularity acts as a lens. This can be seen from Figure 7, where amplitude fluctuations obtained in case 2 but with spatial scales of the irregularities increased by a factor of $\sqrt{2}$, shown in Figure 7a, are used for retrieving the phase variations $\phi_0(x)$ in the screen. Comparison of Figure 7b, where the true height, $z = 400$ km, has been used in the calculation, with Figure 5 shows the greater accuracy of the result obtained in the former case. Likewise, the range of validity of the method is extended when amplitude fluctuations on higher-frequency signals are considered.

In the present paper, radio waves have been con-



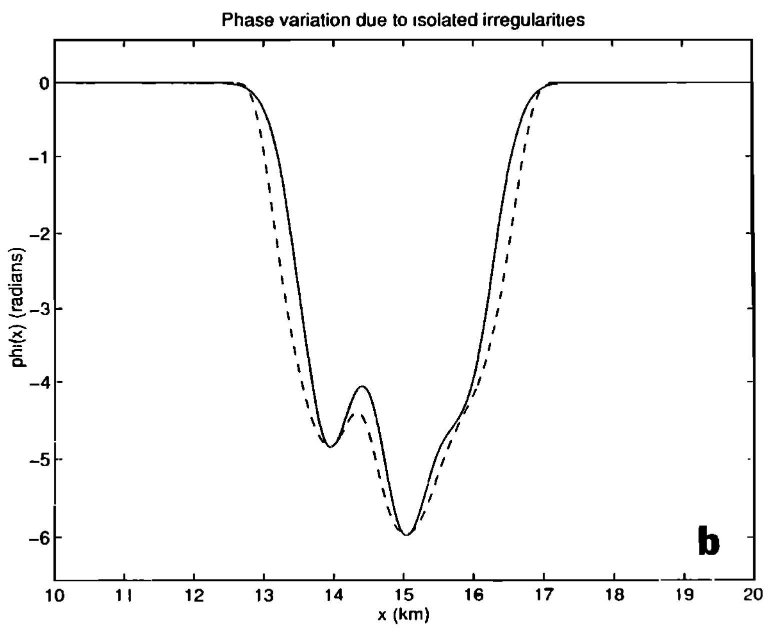


Figure 7. (a) Diffraction pattern of amplitude for radio waves of 1-m wavelength produced by a set of three irregularities with positions as given in section 2, strengths as described in case 2 and sizes a factor of $\sqrt{2}$ larger than those given in section 2. (b) Phase variation in the phase screen retrieved from amplitude fluctuations shown in Figure 7a, with z = 400 km, is depicted by the dashed curve. Solid curve shows the actual phase variation in the screen.

sidered to be normally incident on the phase screen. The formalism used here can be modified without difficulty for the case of oblique incidence. Then the deviations in the TEC along slanted paths are obtained. Also, the basic equation (13) can be modified to treat spherical waves [*Wilkins et al.*, 1996] so that intensity fluctuations recorded on the ground on radio waves transmitted from a satellite orbiting above the ionospheric irregularities can be used in the retrieval of phase variations in an effective phase screen. It is then possible to envisage a scenario where amplitude scintillations recorded on a radio wave signal transmitted from an orbiting satellite, by a series of suitably placed receivers on the ground, can be used in a tomographic reconstruction of two-dimensional irregularities, since the deviation in the TEC along crisscrossing slanted paths can be obtained in this manner. It is well known that the occurrence of phase scintillations on the received signal, due to the presence of ionospheric irregularities, may interfere with the differential Doppler technique used to obtain the electron content along the signal path, which is required in radio tomography of the ionosphere [*Bernhardt et al.*, 1997]. Thus for large-scale tomographic reconstruction of the ionosphere, phase information in the absence of severe scintillations is used. However, if only the density irregularities, which grow in the ambient ionosphere under certain conditions, are to be reconstructed tomographically, information about phase variations on short scale lengths is required. Deterministic retrieval of phase variations caused by the irregularities themselves would be useful for this purpose.

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