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Ionospheric propagation of magnetohydrodynamic disturbances from the equatorial electrojet

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Abstract. The propagation features of magnetohydrodynamic (MHD) disturbances along the ionosphere are considered from a theoretical point of view. Special attention is paid to the possibility of ionospheric propagation of disturbances produced by variations in the equatorial electrojet. The possible MHD modes within the E layer include oscillations that have damping scales of about effective ionospheric skin depth or less. The existence of a large-scale compressional surface mode at small inclinations of the geomagnetic field along the E layer is shown. The propagation of this mode, called the gyrotropic surface mode, takes place in a diffusive-like way along the E layer. Its damping scale is much greater than the ionospheric skin depth, and it may reach several hundred to thousand kilometers. The apparent propagation velocity of the gyrotropic mode at the near-equatorial latitudes is determined by height-integrated Cowling conductivity; its value is estimated to be about $20-100$ km/s for the Pc3-4 frequency range. We suggest that the equatorial electrojet may contribute to the ULF geomagnetic variations observed at near-equatorial stations.

1. Introduction

At present there is no general understanding of the propagation features of low-frequency ($10^{-3}-10^{-1}$ Hz) magnetohydrodynamic (MHD) disturbances along the ionosphere. Some authors [e.g., *Webster et al.*, 1989], following the early idea of *Rostoker* [1965], believe that the propagation of ULF disturbances along the ionosphere is similar to electromagnetic wave propagation in a conductive slab and is limited by a skin-length $\delta_P = (2/\mu_0\omega\sigma_P)^{1/2}$, determined by Pedersen conductivity, σ_P . On the other hand, in a series of papers [*Sorokin*, 1987, 1988; *Sorokin and Yashenko*, 1988], it was suggested that along the E layer of the ionosphere, where Hall conductivity $\sigma_H \gg \sigma_P$, long-range propagation of a specific MHD mode (called a gyrotropic wave) becomes possible. Further researches have shown that in a real ionosphere a gyrotropic mode can propagate at high and middle latitudes only in a diffusive way [*Borisov*, 1988; *Mazur*, 1988]. However, the latter results cannot be applied to the ionosphere with a small inclination of the geomagnetic field.

The goal of the present paper is to supplement the existing physical picture of ionospheric MHD propagation by the analysis of wave features of the near-equatorial ionosphere. The physics of ULF waves at near-equatorial latitudes remains to be poorly understood. It is widely believed that the only feature of ULF waves in this region is the equatorial enhancement of geomagnetic variations caused by a strip with high Cowling conductivity. However, many observations have shown that even this simple effect does not always reveal itself, and, when it does, it does not do so in all frequency bands [*Stuart and Hunter,* 1975; *Sarma et al.,* 1991].

More intriguing is the old idea of *Saito* [1983], who suggested that the near-equatorial ionosphere not only passively amplifies spreading currents but may actively generate some ULF geomagnetic disturbances. This suggestion appears to have stemmed from the existence of an additional maximum in the diurnal distribution of Pc3 pulsations, observed at stations close to the dip equator only. Also, it is worth noting that along the meridional profiles of low-latitude stations, the propagation of ULF signals outward from the equator has been reported [*Yumoto et al.,* 1992; *Rao,* 1995]. All these experimental facts lead to the hypothesis that the ULF fluctuations of the equatorial electrojet may induce geomagnetic disturbances, which propagate further along the ionosphere. These fluctuations may be caused by the turbulence of the equatorial atmosphere and ionosphere, the incidence of compressional waves from the magnetosphere, the acoustic impact of earthquakes, tropical thunderstorms, etc. Here we will give a basic analysis of the MHD propagation along the ionospheric layer at regions of low inclinations of the geomagnetic field. Also, we will present the overall picture of ionospheric propagation of such MHD disturbances at all latitudes.

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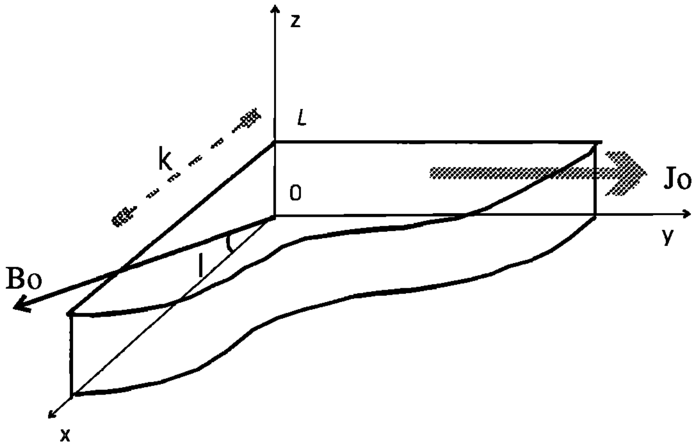

Figure 1. Schematic plot of the model of near-equatorial ionosphere. Direction of the equatorial electrojet is indicated by the thick arrow.

2. Model of the Near-Equatorial Ionosphere and the Basic Equations

For our theoretical analysis we use the multilayer model of the ionosphere (Figure 1). In this model, the X axis of the rectangular coordinate system is directed to the north, the Y axis is directed to the west, and the Z axis is upward. The conductive layer with anisotropic conductivity is bounded at $z = 0$ and $z = l$. Within the layer ($0 < z < l$), the Hall and Pedersen conductivities, σ_H and σ_P, are constant; and the field-aligned conductivity of ionospheric plasma $\sigma_{\parallel} \to \infty$. The geomagnetic field $\mathbf{H}_0$ lies in the plane ZX with inclination I (in the Northern Hemisphere $I < 0$). It is assumed that any fluctuations of the geomagnetic field are induced by the oscillating part of an external current (electrojet), $\mathbf{J}_0(x, z, t)$, which flows along the dip equator (Y axis). This current is assumed to be a given function of time and spatial coordinates, and MHD disturbances produced by it have no reverse influence on a source. Some dragging of neutrals by ions becomes noticeable only at timescales about ν_{ni}^{-1} (where ν_{ni} is the frequency of collisions of neutrals with ions), which is about several hours in the terrestrial ionosphere.

Taking into account that the field-aligned electric component $E_{\parallel} \to 0$ when $\sigma_{\parallel} \to \infty$, we have for the disturbed transverse electric field $\mathbf{E}_{\perp}$

$$(\nabla \times \nabla \times \mathbf{E}_{\perp})_{\perp} + \mu_o \partial_t \hat{\sigma}_{\perp} \mathbf{E}_{\perp} = -\mu_o \partial_t \mathbf{J}_0 \qquad (1)$$

where $\hat{\sigma}_{\perp}\mathbf{E}_{\perp} = \sigma_P \mathbf{E}_{\perp} + \sigma_H (\mathbf{H}_0 \times \mathbf{E}_{\perp})/H_0$ and μ_0 is the magnetic permeability of a free space. We apply to (1) a Fourier transform by x and t with parameters k and ω. Fourier harmonics of $\mathbf{E}(z,x,t)$, $\mathbf{H}(z,x,t)$, and $\mathbf{J}(z,x,t)$ are denoted as $\mathbf{e}(z,k,\omega)$, $\mathbf{h}(z,k,\omega)$, and $\mathbf{j}(z,k,\omega)$, respectively. Further, for brevity, we omit the arguments k and ω. For the disturbances propagating along the meridian ($k = k_x$), the equation (1) has the form

$$\left(\nabla_{\parallel}^2 + k_P^2\right) \frac{e_x}{\sin I} - i k_H^2 e_y = 0 \qquad (2)$$

$$i k_H^2 \frac{e_x}{\sin I} + \left(\nabla^2 + k_P^2\right) e_y = -i\omega\mu_o j_0 \qquad (3)$$

Here we introduce the following notations: $k_P^2 = i\omega\mu_o\sigma_P$ $k_H^2 = \omega\mu_o\sigma_H$, and $k_C^2 = \omega\mu_o\sigma_C$, where σ_C is the Cowling conductivity of the ionosphere, given by $\sigma_C = \sigma_P + \sigma_H^2/\sigma_P$. The operators in (2) and (3) are $\nabla_{\parallel} = \nabla_{\parallel}(k) = \cos I \cdot ik + \sin I \cdot d_z$ and $\nabla^2 = d_z^2 - k^2$. The electric and magnetic components of disturbances are related by the following relationships:

$$h_x = -\frac{1}{i\omega\mu_0} d_z e_y \qquad h_y = \frac{1}{i\omega\mu_0} \nabla_{\parallel} \frac{e_x}{\sin I}$$
$$e_z = -\cot I e_x \qquad h_z = \frac{k}{\omega\mu_0} e_y \qquad (4)$$

In deriving the expression for e_z, we assumed that the field-aligned component $e_{\parallel} = e_x \cos I + e_z \sin I = 0$. From (3) and (4), the electric components of a disturbance can be derived through the vertical magnetic component, h_z, as follows:

$$e_y = \frac{\omega\mu_0}{k} h_z \qquad e_x = -\frac{\sin I}{ik\sigma_H}(\nabla^2 + k_P^2) h_z - \frac{\sin I}{\sigma_H} j_0 \qquad (5)$$

Substituting the above expressions into (2), we reduce the basic system of equations to the following

$$\left(\nabla_{\parallel}^2 + k_P^2\right)\left(\nabla^2 + k_P^2\right) h_z - k_H^4 h_z = -ik\left(\nabla_{\parallel}^2 + k_P^2\right) j_0 \qquad (6)$$

Now we define the boundary conditions for this equation. To simplify the calculations, we neglect the atmospheric conductivity, the influence of the Earth, and the emission of MHD waves into the magnetosphere. Then, outside the ionospheric layer ($z < 0$ and $z > l$), the following equations hold:

$$\nabla^2 \mathbf{H} = 0 \qquad \nabla^2 \mathbf{E} = 0 \qquad (7)$$

It follows from (7), taking into account of the finite value of h_z at $|z| \to \infty$ and the continuity of h_z, $d_z h_z$ at $z = 0, l$, that the following conditions must be satisfied:

$$d_z h_z(0) = |k| h_z(0) \qquad d_z h_z(l) = -|k| h_z(l) \qquad (8)$$

The requirement that the normal component of current vanishes at the boundaries of a conductive layer, $j_z(z = 0, l) = 0$, results in $h_y(z = 0, l) = 0$. Because of (4), the latter relationships are equivalent to $\nabla_{\|} e_x(z = 0, l) = 0$. Replacing e_x with h_z, we come to the desired boundary condition

$$\nabla_{\|} \left(\nabla^2 + k_P^2\right) h_z(z = 0, l) = -ik \nabla_{\|} j_0(z = 0, l) \qquad (9)$$

3. Vertical Distribution of Disturbances in the Ionosphere

Within a conductive layer ($0 < z < l$), a general solution of (6) can be searched for in the form

$$h_z(z) = \sum_{n=1}^{4} C_n \exp(\lambda_n z) + h_*(z) \qquad (10)$$

where C_n ($n = 1-4$) are arbitrary constants and $h_*(z)$ is some partial solution of (6). The parameters λ_n are found from the characteristic equation

$$\Delta(k, \lambda) = (k_{\|}^2 - k_P^2)(q^2 - k_P^2) - k_H^4 = 0 \qquad (11)$$

Here $k_{\|} = k \cos I - i\lambda \sin I$ is the field-aligned projection of the wave vector, and $q^2 = k^2 - \lambda^2$ is the square of the wave vector. Equation (11) is actually an algebraic equation of the fourth order in λ, and its solution has a cumbersome form. So, we will look for an approximate solution, using value of the inclination angle I as a small parameter. When $I \to 0$, two roots (λ_1, λ_2) become infinite, whereas two other roots (λ_3, λ_4) are finite. In the first approximation by $|I| \ll 1$, we obtain

$$\lambda_{1,2} = (-ik \mp ik_P) I^{-1} + O(I) \qquad (12)$$

$$\lambda_{3,4} = \pm \left[k^2 - k_P^2 - k_H^4/(k^2 - k_P^2)\right]^{1/2} + O(I) \qquad (13)$$

Equation (12) can be rewritten in the form $k_P^2 = k_{\|}^2$, which testifies that the (λ_1, λ_2) modes correspond to Alfven waves of the ideal magnetohydrodynamics. From (13) it can be shown that the (λ_3, λ_4) modes in a plasma with $\sigma_H = 0$ are isotropic and correspond to compressional (magnetosonic) waves. In a plasma with anisotropic conductivity ($\sigma_H \neq 0$), compressional modes were called gyrotropic modes by *Sorokin and Fedorovich* [1982].

When the Pedersen conductivity is negligible, i.e., $\sigma_P \to 0$, the formula (12) is not valid and roots λ_1, λ_2 differ between each other only in a second approximation:

$$\lambda_{1,2} = -ikI^{-1} \pm ik_H^2 k^{-1} + O(I) \qquad (14)$$

Further, we assume that the Pedersen conductivity is always finite.

Substituting general solution (10) in the boundary conditions (8) and (9), we find a solution of the boundary problem considered. The obtained solution satisfies boundary conditions at upper and lower boundaries of the conductive layer. After simple but cumbersome calculations (see details in appendix A), we obtain the following expression for the vertical magnetic component $h_z(z, k, \omega)$ of a disturbance in the ionosphere

$$h_z = j_0 \left\{ \frac{Q(1 - e^{\lambda_2 l})}{(k + k_P)^2} e^{\lambda_1 z} + \frac{Q(1 - e^{\lambda_1 l})}{(k - k_P)^2} e^{\lambda_2 z} + \frac{4ik}{\Delta_S \lambda_3^2} \left[\lambda_3 \cosh\frac{\lambda_3 l}{2} + |k| \sinh\frac{\lambda_3 l}{2}\right] \times \left[\lambda_3 \sinh\frac{\lambda_3 l}{2} - 2|k| \sinh\frac{\lambda_3 z}{2} \sinh\frac{\lambda_3 (z - l)}{2}\right] \right\} \qquad (15)$$

The terms Δ_S and Q are determined in appendix A (equations (A4) and (A5), respectively). Other components of the disturbance at the lower boundary of the conductive layer can be found from (4) and (5). For instance, the horizontal component, as results from $h_x(0) = -(ik)^{-1} d_z h_z(0)$ and (8), is given by

$$h_x(k, \omega) \equiv h_x(0, k, \omega) = i(|k|/k) h_z(0, k, \omega) \qquad (16)$$

In the extreme case of $\sigma_H \to 0$, when $C_1 = C_2$ and $\lambda_3 = (k^2 - k_P^2)^{1/2}$, the approximate solution obtained (15) reduces to the exact solution for the Pedersen layer with arbitrary I. In the long-wavelength limit, i.e., when $|k| l \to 0$, the approximate solution given by (15) coincides with the exact results [*Surkov*, 1990a, 1992] obtained for the equatorial ionosphere ($I = 0$).

4. Spatial Horizontal Distribution of the Disturbances

The inverse Fourier transform of (15) gives the spatial distribution of a disturbance, $\overline{h}_z(z, x, \omega)$, excited by monochromatic fluctuations of the equatorial electrojet, $\overline{J}_0(z, x, \omega)$:

$$\overline{h}_z(z, x, \omega) = \int_{-\infty}^{\infty} h_z(z, k, \omega) \exp(ikx) dk \qquad (17)$$

In order to calculate the integral (17), let us consider the function $h_z(z, k, \omega)$ in the plane of complex wave numbers k. First, in all the above formulas, we replace $|k|$ with $\sqrt{k^2}$. Then, we make cuts from $+0i$ to $+\infty i$ and from $-0i$ to $-\infty i$, so the branches of the square root in plane k are as follows: for Re $k > 0$, one gets $\sqrt{k^2} = k$, and for Re $k < 0$, one gets $\sqrt{k^2} = -k$. For $x > 0$ the integral along the real axis in (17) is equivalent to the integral over the borders of the cut and to the sum of residuals at the poles of $h_z(z, k, \omega)$:

$$\overline{h}_z(z, x, \omega) = \int_{+0i}^{+\infty i} \Delta h_z(z, k, \omega) \exp(ikx) dk + 2\pi i \sum_n \text{Res}\, h_z(z, k_n, \omega) \exp(ik_n x) \qquad (18)$$

where $\Delta h_z(z, k, \omega) = h_+(z, k, \omega) - h_-(z, k, \omega)$ is the difference between values of $h_z(z, k, \omega)$ at the borders of the cut, $h_+(z, k, \omega) = h_z(z, k + 0, \omega)$, and $h_-(z, k, \omega) = h_z(z, k - 0, \omega)$. We obtain a similar formula for $\overline{h}_x(z,x,\omega)$, in which, according to (16), $\Delta h_x(z,k,\omega) = i[h_+(z,k,\omega) + h_-(z,k,\omega)]$. The poles of functions $h_z(z,k,\omega)$ and $h_x(z,k,\omega)$ are determined by the equation $\Delta_S(k) = 0$, which has the form

$$\tanh \lambda_3 l = -\frac{2\lambda_3\sqrt{k^2}}{\lambda_3^2 + k^2} \tag{19}$$

The roots k_n of dispersion equation (19) determine the wave numbers of eigen modes with a discrete spectrum. The spectrum $\{k_n\}$ is composed of two series: $\{k_{Sm}\}$ $(m = 0,1,..)$ and $\{k_{Pn}\}$ $(n = 0,1,..)$. The first series consists of a finite number of points; whereas the second series consists of an infinite number of points with the accumulation point $k = k_P$. For the case $I = 0$, this spectrum $\{k_{Pn}\}$ was considered by *Surkov* [1990b]. However, as calculations show, the input into a total disturbance from the residuals in points k_{P0}, k_{P1}, k_{P2},... is small and can be neglected.

5. Thin Ionosphere Approximation

Let us consider the case of large-scale disturbances, when the ionosphere can be treated as a thin sheet. This can be done when the width l of a conductive layer of the ionosphere is less than the skin-length and spatial scale of a disturbance, i.e., $\delta_{P,H} = (2/\omega\mu_0\sigma_{P,H})^{1/2} \gg l$ and $|k|l \ll 1$ [*Alperovich and Fedorov,* 1992]. The above expressions can be simplified in this case. Strictly, two intervals of possible inclination angles I exist.

1. The ionosphere is an optically thin layer only for the gyrotropic compressional mode, i.e., $|\lambda_n|l < 1$ $(n = 3,4)$. At the same time, the ionosphere cannot be considered as a thin layer for an Alfven mode, i.e., $|\lambda_n|l \geq 1$ $(n = 1,2)$. This situation exists when

$$|I| \leq |k|l \ll 1 \qquad |I| \leq |k_P|l \ll 1 \qquad k_C l \ll 1 \tag{20}$$

In this case, the scale of Alfven disturbances across the ionosphere is comparable to the geometric thickness of the ionosphere.

2. The ionosphere is an optically thin layer for all modes, i.e., $|\lambda_n|l \ll 1$ $(n = 1-4)$. This condition imposes the following restrictions on parameters:

$$|k|l \ll |I| \ll 1, \qquad |k_P|l \ll |I| \ll 1, \qquad k_C l \ll 1. \tag{21}$$

Under the conditions (21), neglecting $O(l^2)$, we obtain from (15)

$$\begin{aligned} h_z(k,\omega) &= j_0 l \frac{ik}{(\lambda_3^2 + k^2)l + 2\sqrt{k^2}} \times \{1 - I^2 T(k)\} \\ T(k) &= \frac{k^2 k_H^4 (k^2 + 3k_P^2)}{k_P^2 (k_P^2 - k^2)^3} \end{aligned} \tag{22}$$

where $h_z(k,\omega) \equiv h_z(z = 0,k,\omega)$. For the function $\Delta h_z(k,\omega)$ in (18), we obtain

$$\Delta h_z = -j_0 l \frac{4ik^2}{(\lambda_3^2 + k^2)^2 l^2 - 4k^2} \times \{1 - I^2 T(k)\} \tag{23}$$

The small terms $\propto I^2$ in parentheses in (22) and (23) are related to the input of Alfven-type disturbances.

In case 1, the ionospheric E layer can be replaced by a thin film for a gyrotropic mode only. However, the input of Alfven-type disturbances into the component h_z is negligible, $\sim I^2$. The relevant expressions coincide with (22) and (23), where the second terms in brackets are omitted. Therefore, in both cases, the same formulas (22) and (23) can be used formally with omitted inputs of Alfven-type disturbances.

As a result, the dispersion equation (19) for the thin ionosphere is simplified and takes the form

$$(\lambda_3^2 + k^2)l + 2\sqrt{k^2} = 0 \tag{24}$$

In the approximation of a thin ionosphere from the whole infinite set of horizontal wave numbers, only four constituents with propagation constants $\pm k_S$ and $\pm k_{P0}$ remain in the spectrum.

Upon reduction to the model of a thin ionosphere, the height-integrated conductivities $\Sigma_{P,H} = \sigma_{P,H} l$ remain finite for $l \to 0$, while the parameter $|k|l$ tends to 0. Thus the solution $k(\omega)$ of dispersion equation (24) can be reduced to the form

$$k_S = i\omega/2V_C \tag{25}$$

Here the velocity $V_C = (\mu_o \Sigma_C)^{-1}$ has been introduced, which is determined by a characteristic combination of height-integrated conductivities $\Sigma_C = \Sigma_P + \Sigma_H^2/\Sigma_P$. The dispersion equation (25) predicts that the eigenmode is spatially aperiodic with the damping scale $\delta_S = (\mathrm{Im} k_S)^{-1} = 2/(\omega\mu_0\Sigma_C)$. Formally, (25) coincides with the dispersion equation for a surface Zennek's wave above a conductive half-space in the theory of radio-wave propagation. We call a disturbance described by the dispersion relationship (25) a surface gyrotropic mode. Herewith we keep the same notation as given by *Sorokin and Fedorovich* [1982] because the described mode belongs to the same type of compressional MHD waves, whose properties are modified by the plasma gyrotropy due to presense of Hall conductivity. This point will be discussed in greater detail in section 6. The scaling of the terms neglected in (24) for $|k| \simeq |k_S| \simeq \omega\mu_0\Sigma_C$ shows that (25) is valid under the condition $k_S^2\delta_P^2 \ll 1$, which holds in the ULF range.

Another root of (24) is given by the following:

$$k_{P0} = k_P + \frac{k_H^4 l}{4k_P^2} \tag{26}$$

The expression (26) can be used only for small inclination angles, such as $|I| \ll (k_P l/2)^{1/2}$, since (19) is not valid while $k \to k_P$. Calculations show that the residual at the pole (26) provides an expression which is small as l^2. So the relevant terms are not accounted for in the final solution.

Within the thin ionosphere approximation, the explicit formula for the spatial distribution of magnetic fields produced by the linear external current can be obtained. These expressions can be used when the distance from an electrojet is bigger than its transverse scale a. The procedure of estimating the intergals (17) and (18) is described in appendix B. The integral expressions for $\overline{h}_z$ and $\overline{h}_x$ with the use of known table integrals are reduced to the following

$$\overline{h_z} = -\frac{I_o}{4\pi x}\left[2 - se^{-s}\overline{Ei}(s) + se^{s}Ei(-s) + i\pi se^{-s}\right]$$
$$\overline{h_x} = -i\frac{I_o}{4\pi x}\left[-se^{-s}\overline{Ei}(s) - se^{s}Ei(-s) + i\pi se^{-s}\right] \quad (27)$$

Here $s = x/\delta_S = (\omega X_C/2c)x$ denotes dimensionless distance from the electrojet, $X_C = \mu_0 c\Sigma_C = 120\pi\Sigma_C$ is the dimensionless Cowling conductance of the ionosphere, $Ei(s)$ and $\overline{Ei}(s)$ are ordinary and supplementary integral exponential functions, correspondingly.

At large distances from a source ($s \gg 1$), the relations given by (27), are reduced to

$$\overline{h}_z = \frac{I_0(\omega)\delta_S^2}{\pi x^3}\left[1 - i\frac{\pi}{4}s^3\exp(-s)\right]$$
$$\overline{h}_x = i\frac{I_0(\omega)\delta_S}{2\pi x^2}\left[1 - i\frac{\pi}{2}s^2\exp(-s)\right] \quad (28)$$

The condition $s \gg 1$ is valid for distances $x \gg \delta_S(km) \simeq 10^5 T(s) X_C^{-1}$. For disturbances with period $T \simeq 10^2$ s in the dayside ionosphere with parameter $X_C \simeq 10^4$, the critical distance $\delta_S \simeq 10^3$ km. The first term in parentheses in (28) corresponds to the quasi-static field of the conductivity and external currents in the quadruple approximation. The terms in (28), which correspond to exponentially damping surface disturbances, provide phase variations of magnetic disturbance away from the electrojet. For example, at $s = 3$, the input of this term in the formula for h_z in (28) reaches a maximum value of about 1.

At small distances ($s \ll 1$) from a source, we find, from (27),

$$\overline{h_z} = -\frac{I_o(\omega)}{2\pi x}\left(1 + i\frac{\pi}{2}s\right)$$
$$\overline{h_x} = i\frac{I_o(\omega)}{2\pi\delta_S}\left(\ln s + C - i\frac{\pi}{2}\right) \quad (29)$$

The first term in $\overline{h_z}$ describes the quasi-static magnetic field of an infinite linear current.

It can be seen from (29) that at small distances ($x \ll \delta_S$) a spectrum of magnetic disturbances $\overline{h}_z(x,\omega)$ coincides with the electrojet fluctuations spectrum $I_0(\omega)$. At large distances, the higher frequencies in a spectrum damp faster, which follows from (28).

6. Discussion and Conclusion

We now summarize the results obtained above and elsewhere and describe the general properties of the surface gyrotropic mode. The study of the general dispersion equation (19) reveals a wide spectrum of decaying MHD disturbances. In the ULF frequency range, when the near-equatorial ionosphere can be modeled as a thin sheet, two branches remain in the spectrum of ionospheric MHD disturbances. The mode of the first branch has a damping spatial scale, $\sim \delta_S = 2/(\omega\mu_0\Sigma_C)$, whereas the second mode has a damping spatial scale about the ionospheric skin depth, $\sim \delta_P = (2/\omega\mu_0\sigma_P)^{1/2}$. Besides a higher damping rate, the excitation effectiveness of the second branch in the near-equatorial ionosphere is low. In the case of a nondissipative plasma layer ($\Sigma_P \to 0$), this spectrum transforms into weakly damped gyrotropic waves [*Sorokin and Fedorovich,* 1982; *Sorokin,* 1987; *Sorokin and Yashenko,* 1988]. However, in a realistic ionosphere, the propagation regime becomes a diffusion-like one (aperiodic decrease of amplitude) [*Mazur,* 1988]). Our consideration shows that the wave regime of propagation of MHD disturbances along the E-layer cannot be realized even in the near-equatorial regions, where $I \to 0$. However, despite its decaying character, the surface gyrotropic mode can be detected experimentally thanks to a phase shift of disturbances recorded at different distances away from the source. This phase shift may be formally attributed to some apparent anti-equatorward phase velocity.

Now we give some additional simple estimates describing the basic properties of a surface gyrotropic mode. For brevity, we consider the situation when there is no coupling between the disturbance in the ionosphere and at the ground, i.e., $kH \gg 1$ (H is the height of the ionosphere). Allowance for the influence of the Earth's surface will modify the dispersion equation, but it will not markedly change the character of wave propagation.

We introduce the following characteristic velocities, determined by height-integrated ionospheric conductivities: $V_{P,H,C} = (\mu_o\Sigma_{P,H,C})^{-1}$. Long-range propagation of a gyrotropic wave is possible only along the ionospheric film with a small Pedersen conductivity: $\Sigma_P/\Sigma_H \ll (kl)^{1/2}$. The propagation velocity in this case is related to V_H: $\omega/k = (2kl)^{1/2}V_H\cos I$.

However, in the real ionosphere, where commonly $\Sigma_P \simeq \Sigma_H$, the properties of a gyrotropic mode change drastically. The wave regime of propagation changes into a diffusive-like one. A compressional large-scale gyrotropic mode does not "feel" the inclination of the geomagnetic field, and the meridional propagation of this mode at all latitudes can be described by the same relations. In the dayside ionosphere, when $\Sigma_P \gg \Sigma_A = 1/\mu_0 C_A$, the apparent phase "velocity" $|\omega/k| = V_S = 2V_C$, whereas the decay length $\delta_S = 2V_C/\omega$ [*Mazur,* 1988]. In the nighttime ionosphere, where $\Sigma_P \ll \Sigma_A$, the apparent propagation velocity $|\omega/k| = 2V_P$ [*Borisov,* 1988].

The common skin effect in the conductive ionosphere provides the phase shifts corresponding to the apparent velocity $|\omega/k| = V_d = (2\omega/\mu_0\sigma_P)^{1/2}$ [*Rostoker,* 1965]. The surface gyrotropic mode can transport magnetic disturbances to a greater distance and at higher velocity than can the ordinary skin effect:

$$\delta_P/\delta_S = (l/\delta_P)(\sigma_C/\sigma_P) \ll 1$$
$$V_d/V_S = (l/\delta_P)(\sigma_C/\sigma_P) \ll 1 \quad (30)$$

The geomagnetic disturbances excited by the equatorial electrojet should be observed mainly in the H and Z components. For the typical dayside ionosphere conductances $\Sigma_H \simeq 1.5\Sigma_P$, $\Sigma_P \simeq 24-5$ S, the effective Cowling conductance is $\Sigma_C \simeq 80-16$ S. These values ensure the ionospheric propagation of disturbances in the Pc3-4 range ($\omega \simeq 0.1$ s^{-1}) with the apparent velocities $V_S = 2/(\mu_0\Sigma_C) \simeq 20-100$ km/s and effective damping scales $\delta_S = V_S/\omega \simeq 2\cdot10^2-10^3$ km. At a distance $x \simeq \delta_S$, a phase shift is $\sim \pi/2$.

It should be stressed that the considered generation mechanism is absolutely nonsensitive to a particular origin of the equatorial electrojet fluctuations. They might be caused by a turbulence of neutral gas, fluctuations of large-scale ionospheric electric field, variations of ionospheric conductivities, incidence of ULF compressional waves from the magnetosphere, etc. The geomagnetic signals observed by *Rao* [1995] at near-equatorial latitudes (about 5^o-10^o) may in principle be induced by ULF fluctuations of the equatorial electrojet, which are transported by gyrotropic modes along the ionosphere. ULF disturbances with anti-equatorward propagation, observed by *Yumoto et al.* [1990] further away from the dip equator at latitudes $\sim 20^o-30^o$, cannot be generated by the equatorial electrojet. They are probably caused by specific features of the phase structure of magnetospheric MHD waves in the region of resonant transformation.

The problem of MHD signal propagation along the ionosphere goes beyond electrojet-related disturbances. It is of key importance to understand the physics of "fast" ionospheric disturbances with velocities above the acoustic ones, excited by a powerful explosive impact on the ionosphere (see review by *Pokhotelov et al.* [1995]). The acoustic impact of earthquakes and thunderstorms (being relatively rare) can cause sufficiently strong effects at ionospheric levels.

The idea of ionospheric MHD propagation has been applied to interpret the propagation effects of Pi2 pulsations excited by the auroral electrojet [*Rostoker*, 1965; *Webster et al.*, 1989]. It should also be mentioned that large-scale ULF magnetic disturbances induced by variations of the equatorial electrojet may exert an influence on the dynamics of trapped particles in the inner radiation zone [*Kuznetsov and Kudela*, 1995].

The model developed may be considered as a physical justification of the idea of *Saito* [1983] about the existence at near-equatorial latitudes of a nonmagnetospheric source of ULF disturbances. In this paper, we have discussed the available experimental indications of a possible role of the equatorial electrojet as a source of ULF disturbances. Only recently have dense meridional profiles with ULF magnetic stations near the geomagnetic equator been installed [*Yumoto et al.*, 1996; *Tachihara et al.*, 1996]. Maybe a special analysis of new data would prove more reliably the existence of electrojet-generated MHD disturbances, in particular, the gyrotropic surface mode.

Appendix A: Solution of the Boundary Problem (6), (8), and (9)

We determine the coefficients C_n in (10), if an external current j_0 does not depend on z. In this case, a partial solution can be chosen as

$$h_* = \frac{ik(k^2-k_P^2)j_0}{\Delta(k,0)} = \frac{ik}{\lambda_3^2}j_0 \qquad (A1)$$

Here the relationship $\Delta(k,0) = (k^2-k_P^2)\lambda_3^2$ has been used. Substituting a general solution (10) with a chosen h_* into boundary conditions (8) and (9), the following system of linear equations for the determination of C_n is obtained

$$\sum_{n=1}^{4}(\lambda_n - |k|)C_n = |k|h_*$$
$$\sum_{n=1}^{4}(\lambda_n + |k|)C_n\exp(\lambda_n l) = -|k|h_* \qquad (A2)$$

$$\sum_{n=1}^{4}\alpha_nC_n = R \qquad \sum_{n=1}^{4}\alpha_nC_n\exp(\lambda_n l) = R \qquad (A3)$$

where $\alpha_n = (\lambda_n + ik/I)(\lambda_n^2 - k^2 + k_P^2)$ and

$$R = -k^2\frac{k_H^4 j_0}{I\Delta(k,0)}$$

The subsequent analysis of (A2) and (A3) with allowance for (12), (13) and for the estimates $\alpha_{1,2} = O(I^{-3})$ and $\alpha_{3,4} = O(I^{-1})$ shows that C_1, $C_2 = O(I^2)$ and C_3, $C_4 = O(1)$. Hence, in (A2), the terms $C_1$and C_2 can be omitted. We then obtain

$$C_3 = -\frac{ik|k|}{\Delta_S}\frac{j_0}{\lambda_3^2}\left[\lambda_3 + |k| + (\lambda_3 - |k|)e^{-\lambda_3 l}\right] + o(I)$$
$$(A4)$$
$$C_4 = -\frac{ik|k|}{\Delta_S}\frac{j_0}{\lambda_3^2}\left[\lambda_3 - |k| + (\lambda_3 + |k|)e^{\lambda_3 l}\right] + o(I)$$

where $\Delta_S = 2\left[(\lambda_3^2 + k^2)\sinh(\lambda_3 l) + 2\lambda_3|k|\cosh(\lambda_3 l)\right]$. Substituting (A4) into (A3), we obtain a system of equations for the determination of coefficients C_1 and C_2. The solutions of this system are given by

$$C_1 = Q\frac{1-\exp(\lambda_2 l)}{(k+k_P)^2}j_0 \quad C_2 = Q\frac{1-\exp(\lambda_1 l)}{(k-k_P)^2}j_0 \qquad (A5)$$

where

$$Q = -i\frac{2I^2k^2k_H^4[\lambda_3\sinh(\lambda_3 l) + |k|\cosh(\lambda_3 l) - |k|]}{\Delta_A\Delta_S\lambda_3k_P(k^2-k_P^2)}$$
$$\Delta_A = \exp(\lambda_2 l) - \exp(\lambda_1 l)$$

Appendix B: Calculation of Fourier Transform (17)

We extract the singularities of an integrand function (18) at $k = k_S$ for $h_+(k)$ and at $k = -k_S$ for $h_-(k)$. First, we find the residuals $h_+(k)$ and $h_-(k)$ in the points $k = k_S$ and $k = -k_S$, respectively, and then we present Δh_z in the form

$$\Delta h_z(k,\omega) = j_0(k_S,\omega)\cdot l\, C_{-1}\frac{2k^2}{k_S(k^2-k_S^2)} + R_z$$
$$\Delta h_x(k,\omega) = j_0(k_S,\omega)\cdot l\, C_{-1}\frac{2ik}{k^2-k_S^2} + R_x$$

where

$$C_{-1} = \left. \frac{\Delta_S(k)}{\partial_k \Delta_S(k)} \frac{h_z(k,\omega)}{j_0(k,\omega) l} \right|_{k=k_S}$$

and $h_x(k,\omega) \equiv h_x(z=-0,k,\omega)$. Herewith, $j_0(k,\omega)$ is supposed to be a regular function of k in the upper half-space. Numerical estimates show that the approximation errors $R_z(k,\omega)$ and $R_x(k,\omega)$ are small for typical ionospheric parameters and can be omitted below. Taking into account inequalities $|k_S l| \ll 1$ and $|k_S|^2 \ll |k_P|^2$, we get $C^{-1} \simeq 0.5 i k_S$. Then

$$\begin{aligned} \Delta h_z(k,\omega) &= j_0 l \frac{ik^2}{k^2 - k_S^2} \\ \Delta h_x(k,\omega) &= -j_0 l \frac{k k_S}{k^2 - k_S^2} \end{aligned} \tag{B1}$$

Now $\overline{h}_z(x,\omega)$ and $\overline{h}_x(x,\omega)$ can be presented in the form

$$\begin{aligned} \overline{h}_z &\simeq il \int_{+i0}^{+i\infty} \frac{j_0(k)k^2}{k^2 - k_S^2} \exp(ikx) dk \\ \overline{h}_x &\simeq -l \int_{+i0}^{+i\infty} \frac{j_0(k) k k_S}{k^2 - k_S^2} \exp(ikx) dk \end{aligned} \tag{B2}$$

Let us suppose that the external current distribution is modeled by the linear current, i.e., function $\overline{J}_0(x,\omega) l = I_0(\omega)\delta(x)$, where $I_0(\omega)$ is the spectral density of the external current fluctuations at frequency ω, while $j_0(k,\omega) \cdot l = (2\pi)^{-1} I_0(\omega)$. Integrals (B2) should be taken along the positive imaginary half-axis, going around singular points from the right. As a result, the expression (27) is obtained.

We note that asymptotic formulas for the $h_z(x)$ produced by the electrojet can be derived directly from (17) and (22) for an arbitrary localized source. At small distances, $x \ll |k_S|^{-1}$, the relevant integral contains the fast oscillating function $\exp(ikx)$, and the main input is provided by a domain, where $|k| \ll |k_S|$. This results in (28), which gives dependence $\propto x^{-3}$. At small distances, $(a,l) \ll x \ll |k_S|^{-1}$, the main input in the integral is given by large wave numbers, $k \gg |k_S|$, whereas the term $(\lambda_3^2 + k^2) l$ in the denominator of (22) can be neglected. Finally, integration by parts results in the leading term in (29), with a decrease rate $\propto x^{-1}$.

Appendix C: Peculiarities of Gyrotropic Surface Waves Near the Equator and at Middle Latitudes

We would like to note some fine difference between the properties of the gyrotropic surface mode at middle latitudes and near the equator. At middle latitudes, when $\delta_{P,H} \gg l$, the condition (21) holds. Then the ionosphere can be treated as a thin sheet in which the electric field weakly changes along altitude. We neglect the emission of Alfven waves into the magnetosphere and suppose that at the upper and bottom boundaries of the conductive layer the condition $h_y = 0$ holds. Hence the jump of the magnetic field is given by $\{h_y\} = h_y(z=l) - h_y(z=0) = 0$. Then after integration of (2) and (3) over the ionosphere, with the approximation $\mathbf{e} \simeq$ const in the conductive layer, we get the relationships for the jump of the horizontal magnetic components:

$$\begin{aligned} \{h_y\} &= -\Sigma_P \frac{e_x}{\sin I} + \Sigma_H e_y = 0, \\ \{h_x\} &= \Sigma_H \frac{e_x}{\sin I} + \Sigma_P e_y + \int j_0 dz = 0 \end{aligned} \tag{C1}$$

After straightforward calculations from (C1), taking (5) and (16) into account, one can obtain a dispersion equation (25) for a surface mode. This equation holds for finite inclinations I (while (27) holds), and it includes a Cowling-like combination of height-integrated conductivities $\Sigma_C = (\Sigma_P^2 + \Sigma_H^2)/\Sigma_P$.

In the near-equatorial region under the conditions (20), the emission of Alfven waves, or leakage of field-aligned current from the ionosphere into the magnetosphere, can be neglected. However, this can be done not because of the great difference between wave conductances of the ionosphere and the magnetosphere, as at middle latitudes, but because of the nearly horizontal orientation of the geomagnetic field. In contrast to middle latitudes, the horizontal components of the electric field cannot be considered height-independent. Instead, we have assumed that the vertical electric current is vanishing. Hence, instead of integral relationships (C1), we come to the local relationship

$$-\sigma_P \frac{e_x}{\sin I} + \sigma_H e_y = 0 \tag{C2}$$

Substituting (C2) into (3), we obtain an equation for a compressional surface mode, which already does not contain the rapidly varying along altitude Alfven disturbances. Because the ionosphere is a thin film for a compressional gyrotropic mode, a dispersion equation for this mode can be easily obtained. This equation is identical to (25), but, instead of Σ_C, it contains the actual height-integrated Cowling conductivity $\int \sigma_C dz$.

Up to now, the Pedersen and Hall conductivities have been assumed, for simplicity, to be constant within the conductive layer. In that case, the dispersion equations for an ionospheric surface mode are formally the same at any I. However, in a realistic, vertically inhomogeneous ionosphere, different equations would be obtained for middle and near-equatorial latitudes. At midlatitudes, owing to the equipotential character of field lines, the features of vertical distribution of ionospheric conductivity do not change the dispersion relation for a surface wave. However, in the near-equatorial ionosphere, the situation is entirely different. The electric field of the wave may vary with altitude, which may cause some effect when the Hall and Pedersen layers are shifted between each other.

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