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Ion acoustic solitons/double layers in two-ion plasma revisited

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Ion acoustic solitons and double layers are studied in a collisionless plasma consisting of cold heavier ion species, a warm lighter ion species, and hot electrons having Boltzmann distributions by Sagdeev pseudo-potential technique. In contrast to the previous results, no double layers and super-solitons are found when both the heavy and lighter ion species are treated as cold. Only the positive potential solitons are found in this case. When the thermal effects of the lighter ion species are included, in addition to the usual ion-acoustic solitons occurring at $M > 1$ (where the Mach number, M, is defined as the ratio of the speed of the solitary wave and the ion-acoustic speed considering temperature of hot electrons and mass of the heavier ion species), slow ion-acoustic solitons/double layers are found to occur at low Mach number ($M < 1$). The slow ion-acoustic mode is actually a new ion-ion hybrid acoustic mode which disappears when the normalized number density of lighter ion species tends to 1 (i.e., no heavier species). An interesting property of the new slow ion-acoustic mode is that at low number density of the lighter ion species, only negative potential solitons/double layers are found whereas for increasing densities there is a transition first to positive solitons/double layers, and then only positive solitons. The model can be easily applicable to the dusty plasmas having positively charged dust grains by replacing the heavier ion species by the dust mass and doing a simple normalization to take account of the dust charge. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4884791]

I. INTRODUCTION

Ion-acoustic solitary waves in unmagnetized and magnetized plasmas have been studied extensively for nearly 5 decades.[1–3] Observations of solitary waves and double layers (DLs) by S3-3,[4] Viking,[5,6] Freja,[7] etc. motivated the research in ion acoustic solitons/DLs.[8–21] These models considered the auroral plasma consisting of two-electron temperature and multi-ion beams in various combinations and with or without the background magnetic field with an aim to get negative potential ion-acoustic solitons/double layers to explain S3-3, Viking, and Freja observations. Recently, Dubinov and Kolotkov[22,23] have discovered ion acoustic "supersolitons" in five species plasmas. Later on these authors[24] showed that ion-acoustic supersolitons can exist in a plasma containing at least four charged particle species. Whereas the ordinary ion acoustic solitons cannot exist for Mach numbers greater than that of the double layer, the supersolitons exist beyond the DL Mach number, and hence the prefix "super" in their name. In a series of papers, Verheest *et al.*[25–27] have done further analysis and shown that ion acoustic supersolitons can exist even in three species plasmas, for example, plasmas having (1) cold positive and cold negative ions and nonthermal electrons, (2) negative dust and two-temperature Boltzmann and nonthermal positive ions, and (3) two temperature electrons and cold ions. Maharaj *et al.*[28] have studied dust acoustic supersolitons in plasma comprising of cold negative dust, adiabatic positive dust, Boltzmann electrons, and non-thermal ions. Mathematically, supersolitons require that Sagdeev pseudopotential should have 3 finite consecutive roots for the potential ϕ. The supersolitons are possible only when the third root becomes accessible, i.e., when the plasma model is able to support three consecutive local extrema of the Sagdeev pseudopotential (see Figure 1 of Ref. 24 and Figures 2 and 4 of Ref. 27). Whereas the electric potential profile of a supersoliton looks quite similar to that of a regular solitons, its electric field profile is distinctly different from that of a soliton, i.e., it has a subsidiary maximum on each side of the usual bipolar soliton structure (see Figure 3 of Ref. 24 and Figure 4 of Refs. 22 and 27).

Way back in 1972, White *et al.*[29] studied ion acoustic solitons and shocks (double layers) in two positively charged ions and hot electron plasma. They obtained Sagdeev potential profiles (see their Figure 1) which were similar to that of "supersolitons" (see Figure 1 of Ref. 24 and Figure 2 of Refs. 22 and 27). However, they missed calling them as "supersolitons." Here, we revisit the model of White *et al.*[29] and show that for the case of two cold ion species and hot Boltzmann distributed electrons, the Sagdeev potential profiles shown in their Figure 1 are not possible, and that the supersolitons cannot exist in this model. However, we get a new ion-ion hybrid mode, the so called slow ion acoustic mode, when the light ion species have a finite temperature. The model is described in Sec. II. The special cases of both ion species being cold, and the heavy ions being cold and light ion being warm are discussed in Sec. III, and the results are summarized in Sec. IV.

II. MODEL

We consider an infinite, collisionless, and unmagnetized plasma consisting of three species, namely, heavy ions (N_1, T_1), light ions (N_2, T_2), and hot electrons (N_e, T_e), where N_j

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and T_j represent the equilibrium density and temperature of the jth species, and $j = 1$, 2, and e for the heavy ions, light ions, and hot electrons, respectively. To maintain charge neutrality in the equilibrium state, we take $N_e = N_1 + N_2$. For simplicity, we treat the heavy ion species as cold, i.e., we set $T_1 = 0$, and therefore we do not need any equation of state for the heavy ion species. For the light ion species, we assume the adiabatic equation of state, i.e., $P_2 n_2^{-\gamma} = constant$, where the adiabatic index, γ, is defined as $\gamma = (2+N)/N$, with N denoting the degree of freedom. Since we are dealing with a one-dimensional case (i.e., $N = 1$), we consider $\gamma = 3$ here. Dynamics of both heavy and light ions are described by the multi-fluid equations of continuity, momentum, and an energy equation derived from the equation of state for the light ion species having finite temperature T_2 (Ref. 16):

$$\frac{\partial n_j}{\partial t} + \frac{\partial (n_j v_j)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial v_j}{\partial t} + v_j \frac{\partial v_j}{\partial x} + \frac{1}{\mu_j n_j}\frac{\partial P_j}{\partial x} + \frac{Z_j}{\mu_j}\frac{\partial \phi}{\partial x} = 0, \tag{2}$$

$$\frac{\partial P_2}{\partial t} + v_2 \frac{\partial P_2}{\partial x} + 3P_2 \frac{\partial v_2}{\partial x} = 0. \tag{3}$$

Here, $j = 1$ and 2, $\mu_j = m_j/m_1$, and Z_j is the positive electronic charge of the jth ion species. We shall consider both ion species to be singly charged, i.e., $Z_j = 1$. We have normalized all densities with unperturbed electron density, $N_e = N_1 + N_2$, velocities with the ion acoustic speed of the heavy ions, $C_a = (T_e/m_1)^{1/2}$, time with the inverse of ion plasma frequency, $\omega_{pi} = (4\pi N_e e^2/m_1)^{1/2}$, the lengths with hot electron Debye length, $\lambda_{de} = (T_e/4\pi N_e e^2)^{1/2}$, the electrostatic potential ϕ with T_e/e, and the thermal pressure P_2 with $N_e T_e$.

We transform the above set of equations to a stationary frame moving with velocity V, the phase velocity of the wave, i.e., $\xi = (x - Mt)$, where $M = V/C_a$ is the Mach number with respect to the ion acoustic speed, considering temperature of hot electrons and mass of the heavier ion species. In such a reference frame, all variables, e.g., densities and pressure tend to their undisturbed values and potential ϕ tends to zero at $\xi \to -\infty$. Then, from Eqs. (1)–(3), we can get the following expressions for the cold heavy ions and warm light ions,[16,18] respectively:

$$n_1 = \frac{N_1}{(1 - 2\phi/M^2)^{1/2}} \tag{4}$$

$$n_2 = \frac{N_2}{2\sqrt{3T_2/\mu_2}}\Big\{[(M + \sqrt{3T_2/\mu_2})^2 - 2\phi/\mu_2]^{1/2} - [(M - \sqrt{3T_2/\mu_2})^2 - 2\phi/\mu_2]^{1/2}\Big\}. \tag{5}$$

Here, the densities n_1 and n_2 are the normalized densities. Since the hot electrons are treated as Boltzmann distributed, their normalized number density in presence of ion-acoustic wave is given by

$$n_e = \exp(\phi). \tag{6}$$

The basic set of equations is closed by the Poisson's equation

$$\frac{\partial^2 \phi}{\partial \xi^2} = n_e - n_1 - n_2. \tag{7}$$

On substituting n_1, n_2, and n_e, in the transformed Poisson's (7), multiplying it with $d\phi/d\xi$ and integrating with the boundary conditions that $\phi = 0$, $d\phi/d\xi = 0$ at $\xi \to \pm\infty$, we get the energy integral

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi, M) = 0, \tag{8}$$

where the pseudopotential $V(\phi, M)$ is given by

$$V(\phi,M) = [1 - \exp\phi] + N_1 M^2\Big[1 - (1 - 2\phi/M^2)^{1/2}\Big] + \frac{N_2}{6\sqrt{3T_2/\mu_2}}\Big[(M + \sqrt{3T_2/\mu_2})^3 - \Big\{(M + \sqrt{3T_2/\mu_2})^2 - 2\phi/\mu_2\Big\}^{3/2} - (M - \sqrt{3T_2/\mu_2})^3 + \Big\{(M - \sqrt{3T_2/\mu_2})^2 - 2\phi/\mu_2\Big\}^{3/2}\Big]. \tag{9}$$

For soliton solutions, the Sagdeev potential $V(\phi, M)$ must satisfy the following conditions:

$$\begin{aligned} &(i)\ V(\phi,M) = 0,\ dV(\phi,M)/d\phi = 0, \text{ and} \\ &\quad d^2V(\phi,M)/d\phi^2 < 0 \text{ at } \phi = 0, \\ &(ii)\ V(\phi,M) = 0 \text{ at } \phi = \phi_0, \text{ and} \\ &(iii)\ V(\phi,M) < 0 \text{ for } 0 < |\phi| < |\phi_0|. \end{aligned} \tag{10}$$

The double layer solutions can exist provided one more condition, in addition to the solitons conditions given by Eq. (10), is satisfied, namely,

$$(iv)\ dV(\phi,M)/d\phi = 0 \text{ at } \phi = \phi_0. \tag{11}$$

The supersolitons can exist when there is an accessible root of Sagdeev potential beyond the double layer. This demands that

$$(v) V(\phi,M) = 0 \text{ for } M > M_{DL} \text{ and } \phi > \phi_{DL}. \tag{12}$$

From Eq. (9), it is seen that the Sagdeev pseudopotential, $V(\phi, M)$ and its first derivative with respect to ϕ vanish at $\phi = 0$.

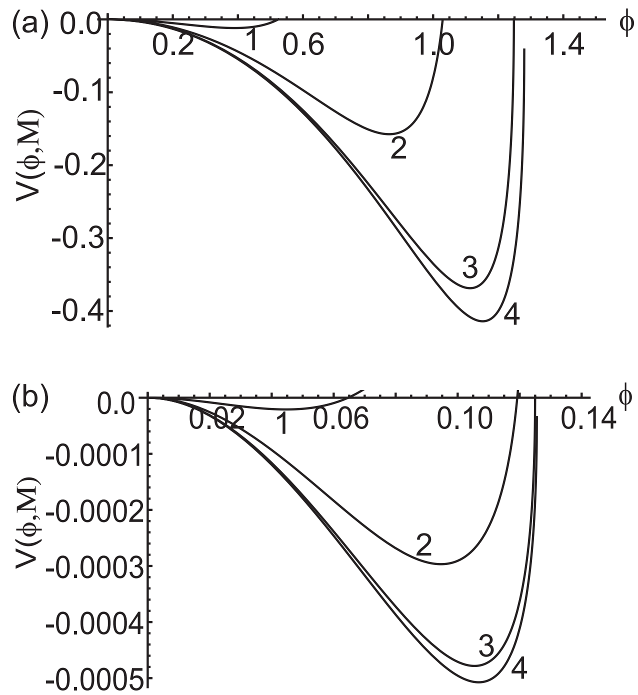

FIG. 1. Shows Sagdeev potential profiles, $V(\phi, M)$ versus ϕ, for various values of the Mach number M. Panel (a) is for a cold single heavy ion plasma, i.e., $N_2 = 0$ and for $M = 1.20$, 1.45, 1.58, and 1.6 for the curves 1, 2, 3, and 4, respectively. The ion-acoustic solitons do not exist for $M \geq 1.6$. Panel (b) is for two cold ion plasma with $\mu_2 = 1/5$ and normalized lighter ion number density $N_2 = 0.01$, and for $M = 1.05$, 1.1, 1.175, and 1.2 for the curves 1, 2, 3, and 4, respectively. The ion-acoustic solitons do not exist for $M \geq 1.2$.

III. SPECIAL CASES

In this section, we shall now discuss the solitary wave solutions of Eq. (8) for two special cases.

A. Both ion species are cold

In this case, as both $T_1 = 0$ and $T_2 = 0$, Eq. (3) becomes redundant. The Sagdeev pseudopotential is simplified to

$$V(\phi, M) = [1 - \exp\phi] + N_1 M^2\left[1 - (1 - 2\phi/M^2)^{1/2}\right] + \mu_2 N_2 M^2\left[1 - \{1 - 2\phi/(\mu_2 M^2)\}^{1/2}\right]. \tag{13}$$

Now, the condition $\mathrm{d}^2V(\phi, M)/\mathrm{d}\phi^2 < 0$ at $\phi = 0$ is satisfied provided the Mach number, M, exceeds a critical value $M_0 = (N_1 + N_2/\mu_2)^{1/2}$. It is interesting to note that for $N_2 = 0$, we have $M_0 = 1$ as it should be for a single ion species plasma.

In Figure 1, we show Sagdeev pseudo potential profiles, $V(\phi, M)$ versus ϕ, for various values of the Mach number M. We consider lighter ion to heavy ion mass ratio, $\mu_2 = m_2/m_1 = 1/5$ (same as the case considered by White *et al.*[29] in their Figure 1), and the plots are shown for two values of lighter ion number density of $N_2 = 0.0$ (panel (a)) and 0.01 (panel (b)). On comparing the curve 1 of panel (a) ($N_2 = 0$ and $M = 1.2$) with panel (b) of White *et al.*,[29] we note that both have similar shapes, i.e., only one extremum (minimum in this case). This corresponds to ion acoustic soliton in an usual plasma consisting of electrons and single type of ions. The comparison of panel (b) which is for two ion component plasma with $N_2 = 0.01$ with panel (a) of White *et al.*[29] tells an entirely different story. In our case, the Sagdeev potential profiles have a single minimum for all values of M thereby predicting only solitons. Whereas for White *et al.*,[29] the profiles have three extrema of supersolitons. It is clear from panels (a) and (b), that there is an upper limit on M beyond which ion acoustic solitons cannot exist (cf curves 4).

We have explored Sagdeev potential profiles over a range of N_2 values varying from 0.0 to 1.0 in two ion component plasmas with $\mu_2 = 1/5$ and 1/16. The latter case corresponds to oxygen-proton plasma encountered frequently on the auroral field lines of the Earth's magnetosphere. In all cases, the Sagdeev pseudopotential profiles have a single minimum corresponding to ion-acoustic solitons.

In Figure 2, we have shown the variation of critical, M_0, and maximum, M_{max}, Mach numbers, and ϕ_{max}, the maximum value of ϕ with different values of N_2 for $\mu_2 = 1/5$ and 1/16. The behavior of M_0, M_{max}, and ϕ_{max} is found to be similar in both cases. It is interesting to note that introduction of a small percentage of the second lighter ion species gives rise to a sharp decrease in M_{max} and ϕ_{max}, further increase in N_2 leads to increase in their magnitudes. Further, as N_2 tends to 1, the maximum potential tends to $\phi_{max} = 1.25$, the same value as for $N_2 = 0$, i.e., as expected for a single ion plasma.

In Figure 3, we show the profiles of electrostatic potential, ϕ, and electric field, E, for the case of $N_2 = 0$ (single heavy ion plasma) and for $N_2 = 0.01$ (two ion component plasma with $\mu_2 = 1/5$). These are the typical profiles for the ion acoustic solitons. It is clearly seen that magnitudes of both potential (panel (a)) and electric field (panel (b)) are reduced significantly by the presence of a mere 1% of the lighter ion number density (cf curves 1 and 2).

Therefore, from the above results, we can conclude that in two ion component plasma, when both heavy and light ion species are cold, only ion-acoustic solitons are possible, and that neither ion acoustic double layer nor supersolitons can exist in such a system.

B. Cold heavy ions and warm light ions

In this case, from Eq. (9), the soliton condition $\mathrm{d}^2V(\phi, M)/\mathrm{d}\phi^2 < 0$ at $\phi = 0$ demands that $M > M_0$, where the critical Mach number, M_0, satisfies the following equation:

$$\frac{N_1}{M_0^2} + \frac{N_2}{\mu_2(M_0^2 - 3T_2/\mu_2)} - 1 = 0. \tag{14}$$

The above equation has two possible positive roots, namely, an ion acoustic mode modified by the light ion temperature

$$M_0^2 \approx \left[N_1 + \frac{N_2}{\mu_2} + \frac{3T_2}{\mu_2}\right] \tag{15}$$

and a new slow ion acoustic mode[16]

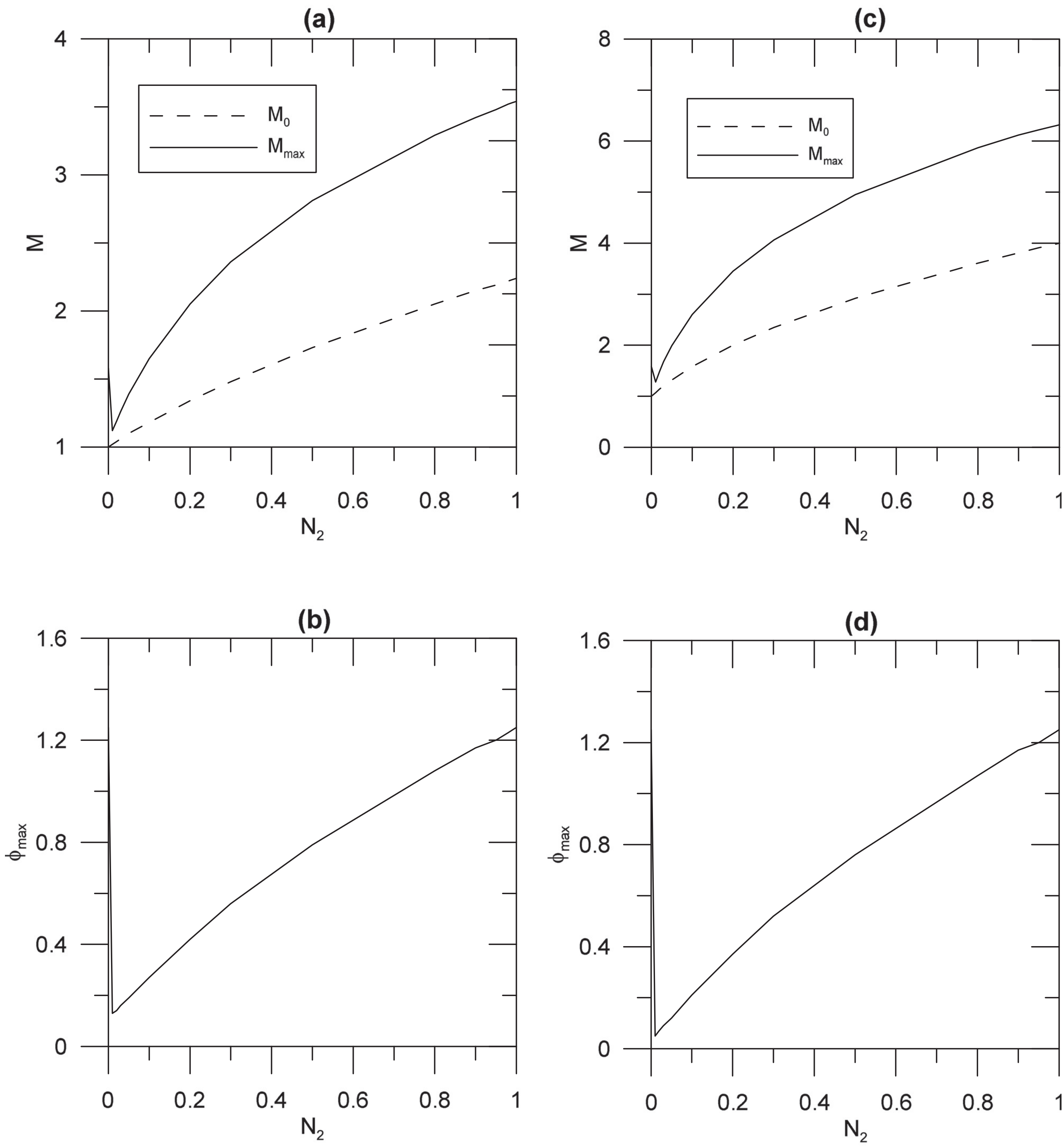

FIG. 2. Shows variation of critical Mach number M_0 (dashed curves, - - -), maximum Mach number M_{max} (solid curves), and maximum potential ϕ_{max} (solid curves), with different values of N_2 in a two cold ion species plasmas. Panels (a) and (b) are for case $\mu_2 = 1/5$, and panels (c) and (d) are for $\mu_2 = 1/16$.

$$M_0^2 \approx \frac{3T_2}{\mu_2} \frac{N_1}{\left[N_1 + \frac{N_2}{\mu_2} + \frac{3T_2}{\mu_2}\right]}. \tag{16}$$

The slow ion acoustic mode[16] is actually an ion-ion hybrid mode that requires essentially a two ion component plasma with warm lighter ions. As one can verify from Eq. (16), this mode vanishes when either $T_2 = 0$ or $N_1 = 0$. The slow ion acoustic modes (given by Eq. (16)) occur at smaller critical Mach numbers as compared to the usual ion acoustic mode (given by Eq. (15)).

Figures 4 and 5 show Sagdeev potential, $V(\phi, M)$, profiles for slow ion-acoustic (panel (a)) and regular ion-acoustic (panel (b)) modes for various values of Mach numbers, M, for an oxygen-hydrogen ion plasma with $\mu_2 = 1/16$, $T_2 = 0.01$, and for hydrogen ion density $N_2 = 0.2$ and 0.8, respectively.

Figure 4(a) shows clearly that the slow ion-acoustic solitons have negative potentials for $N_2 = 0.2$. Their potential ϕ_0

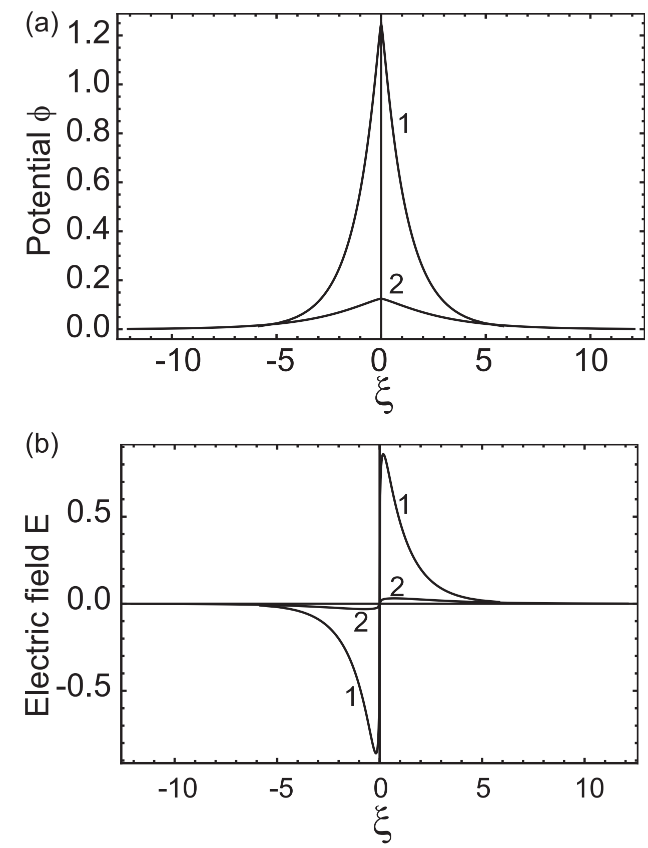

FIG. 3. Shows the profiles of potential ϕ (panel (a)) and electric field E (panel (b)). The curves labelled 1 are for a single heavy ion plasma, i.e., $N_2=0$ case, and curves labelled 2 are for the two component plasma with lighter ion number density, $N_2=0.01$ and lighter to heavy ion mass ratio, $\mu_2=1/5$.

increases with the increase of the Mach number (curves 1 and 2), and they end up with a DL at the upper limit of the Mach number, $M_{max}=0.30369$ (curve 3). For M greater than DL Mach number, slow ion acoustic solitons or supersolitons do not exist (curve 4).

Figure 4(b) shows that for the same plasma parameters as in Figure 4(a), but for higher values of the Mach numbers ($M=2.25$, 2.5, 2.537, and 2.55 for the curves 1, 2, 3, and 4, respectively), we have usual positive potential ion-acoustic solitons. Their potential increases with M (curves 1, 2, and 3) till the upper limit M_{max} is reached. Beyond M_{max}, ion-acoustic solitons do not exist.

Figures 5(a) and 5(b) show the variation of $V(\phi,M)$ versus ϕ for the same plasma parameters as in Figures 4(a) and 4(b) except that for $N_2=0.8$. Here, both the slow ion-acoustic (Figure 5(a)) and the ion-acoustic (Figure 5(b)) solitons have positive potentials. For both slow ion-acoustic and ion-acoustic case the double layers do not exist, so the question of supersoliton's existence does not arise. For both types of solitons, ϕ_0 increases with M till upper limit M_{max} beyond which solitons do not exist (curves 1, 2, 3, and 4). In Figure 6, we have shown the variation of critical, M_0, and maximum, M_{max}, Mach numbers and ϕ_{max} with N_2, for the slow ion acoustic (Panels (a) and (b)) and ion acoustic (Panels (c) and (d)) modes for $\mu_2=1/16$ and $T_2=0.01$. The circles on ϕ_{max} curve show the existence of DLs. From Panel (a), it is clear that as N_2 increases the difference between M_{max} and M_0 for slow ion acoustic case initially decreases,

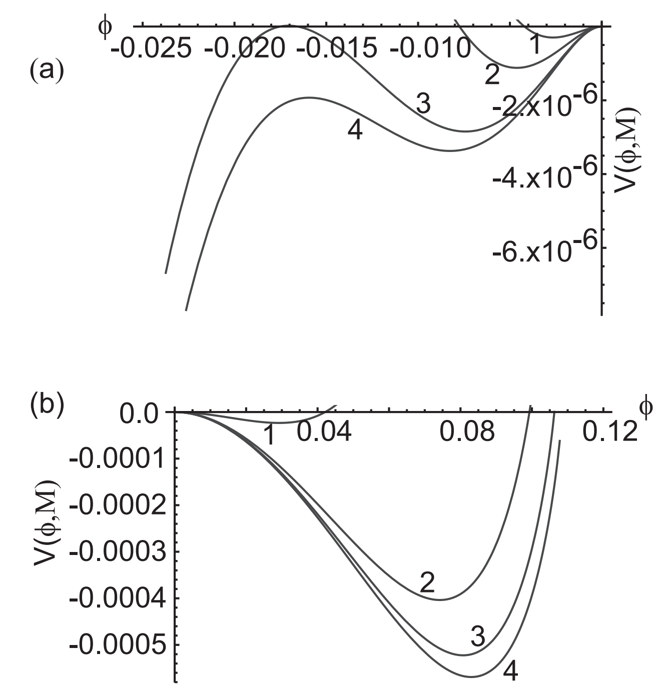

FIG. 4. Shows plots of Sagdeev pseudopotential $V(\phi,M)$ versus the potential ϕ for $N_2=0.2$, $\mu_2=1/16$, and $T_2=0.01$. Panel (a) shows slow ion-acoustic solitons for the Mach number $M=0.3$, 0.302, 0.30369 (DL), and 0.304 for the curves 1, 2, 3, and 4, respectively. The curve 3 corresponds to the double layer. Panel (b) shows ion-acoustic solitons for the Mach number $M=2.25$, 2.5, 2.537, and 2.55 for the curves 1, 2, 3, and 4, respectively.

then start increasing and then again decreases as N_2 approaches 1. Panel (b) shows that as N_2 increases initially negative potential slow ion acoustic solitons ending with

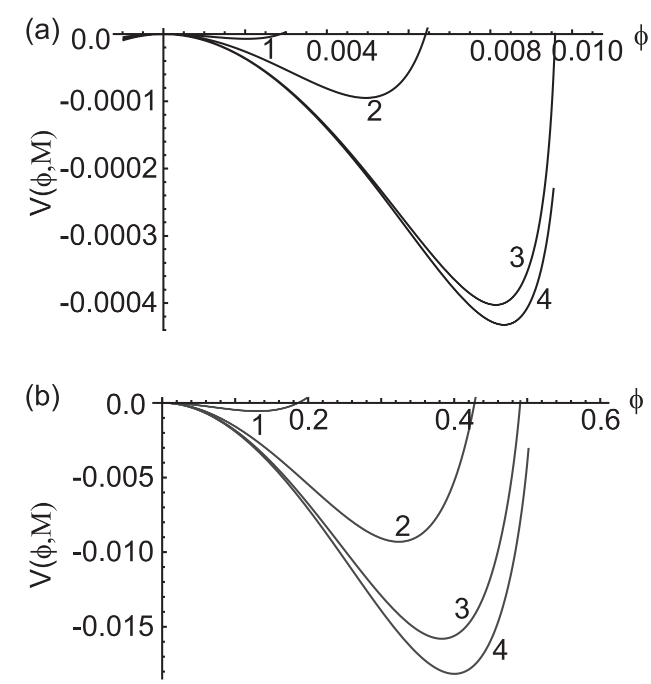

FIG. 5. Shows plots of Sagdeev pseudopotential $V(\phi,M)$ versus the potential ϕ for the same plasma parameters as in Figure 4 except that $N_2=0.8$. Panel (a) shows slow ion-acoustic solitons for the Mach number $M=0.1$, 0.12, 0.13894, and 0.14 for the curves 1, 2, 3, and 4, respectively. Panel (b) shows ion-acoustic solitons for the Mach number $M=4.0$, 4.5, 4.655, and 4.7 for the curves 1, 2, 3, and 4, respectively.

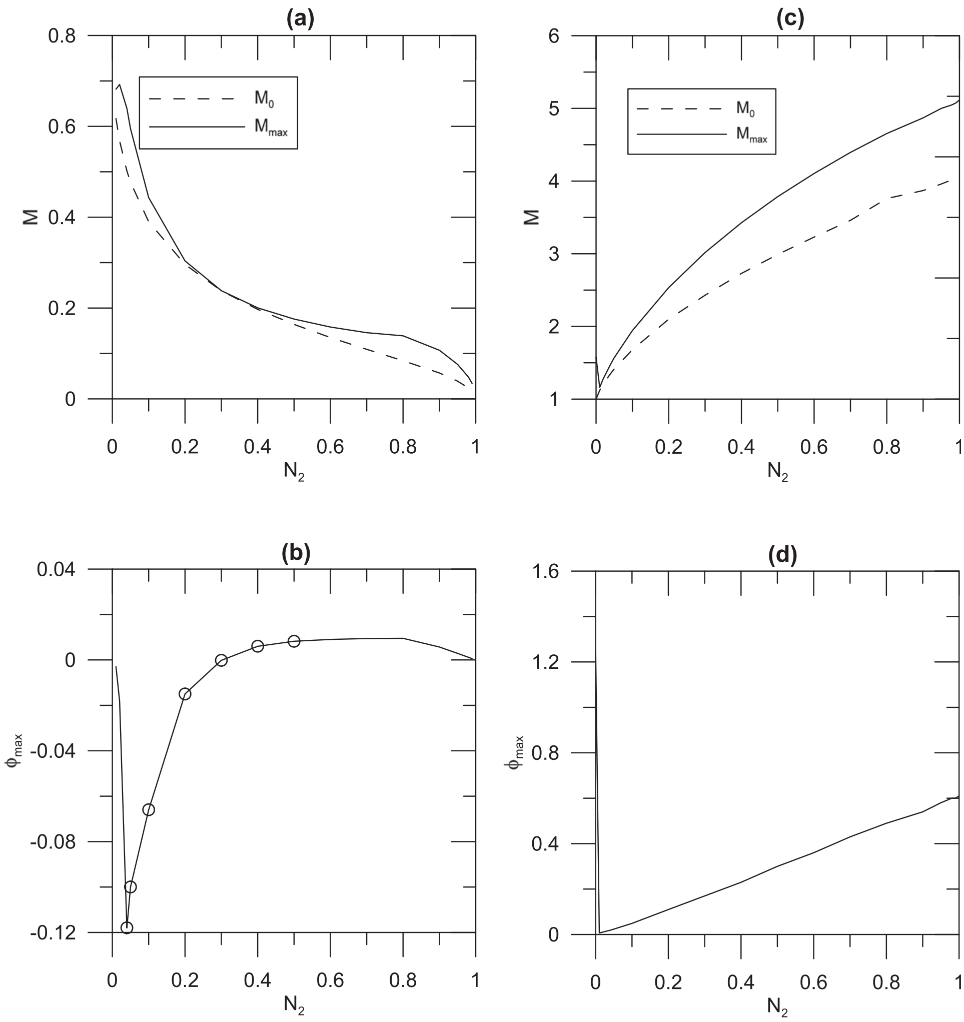

FIG. 6. Shows variation of critical Mach number M_0 (dashed curves, - - -), maximum Mach number M_{max} (solid curves), and maximum potential ϕ_{max} (solid curves), with different values of N_2 in a two component cold heavy ions and warm light ions plasmas with $\mu_2 = 1/16$, and $T_2 = 0.01$. The circles on ϕ_{max} curve show the existence of DLs. Panels (a) and (b) are for slow ion acoustic case, and Panels (c) and (d) are for the ion acoustic case.

double layers are found, further increase in N_2 results in transition to positive DLs (near $N_2 = 0.4$) and then to the existence of solitons only. The slow ion acoustic mode disappears as N_2 approaches 1. From Panels (c) and (d), it is clear that behavior of range of $M_{max} - M_0$ and ϕ_{max} for the ion acoustic mode is similar to Panels (c) and (d) of Figure 2.

Figure 7 shows the variation of M_0, M_{max}, and ϕ_{max} with N_2 for the case of $T_2 = 0.05$. The format is similar to that of Figure 6. From panel (a), it is seen that for slow ion-acoustic case the difference between M_{max} and M_0 is very small till $N_2 = 0.3$, with further increases in N_2, the difference increase and then decreases as N_2 approaches 1. Panel (b) shows a similar behavior of ϕ_{max} with N_2 as for the case of $T_2 = 0.01$ (panel (b) of Figure 6) except that now transition from negative to positive DLs occurs at lower values of $N_2 = 0.2$ and that the amplitudes are higher than that of $T_2 = 0.01$ case. Panels (c) and (d), show that for the ion acoustic case, the behavior of M_{max}, M_0, and ϕ_{max} with variations in N_2 is similar to the case of $T_2 = 0.01$ (panels (c) and (d) of Figure 6), except that the magnitude of ϕ_{max} is much reduced.

IV. DISCUSSION

We have shown that a plasma model comprising of two cold, heavy and light, ion species and hot electrons with Boltzman distribution, can support only positive potential ion acoustic solitons. It cannot support either double layers or supersolitons as implied by White *et al.*[29] model. The reason, it appears to us, is the wrong normalization of the parameter $\psi = 2e\phi/m_1V^2 = 2\phi/M^2$ (in our notation) and treating ψ and M as independent parameters. We, on the other hand, treat ϕ and M as independent parameters, as indeed they are! Our results support the conclusion of Dubinov and Kolotkov[24] that at least four plasma species are required for the existence of ion-acoustic supersolitons when the inertialess species follow Boltzmann distribution.

Do our results contradict the results of Verheest *et al.*[25] who found supersolitons in three-species plasmas? Our answer is not necessarily so. This is because of the fact that our model is different from their model. First, Verheest *et al.*[25] consider two cold ion species, one positively charged and the second negatively charged, whereas we consider both ion species as positively charged. Second, they consider electrons (inertialess species) having nonthermal distributions where as we consider electrons having Boltzmann distribution. Therefore, the dependence of the electron density on ϕ is quite different in their and our models. The profile of Sagdeev pseudo-potential is sensitively controlled by the polarity of the ions species as well as electron density dependence on potential, ϕ (see our Eq. (13) and Eq. (5) of Verheest *et al.*[25]). In our case, it turns out that Sagdeev pseudo-potential has a single minimum, thus giving regular

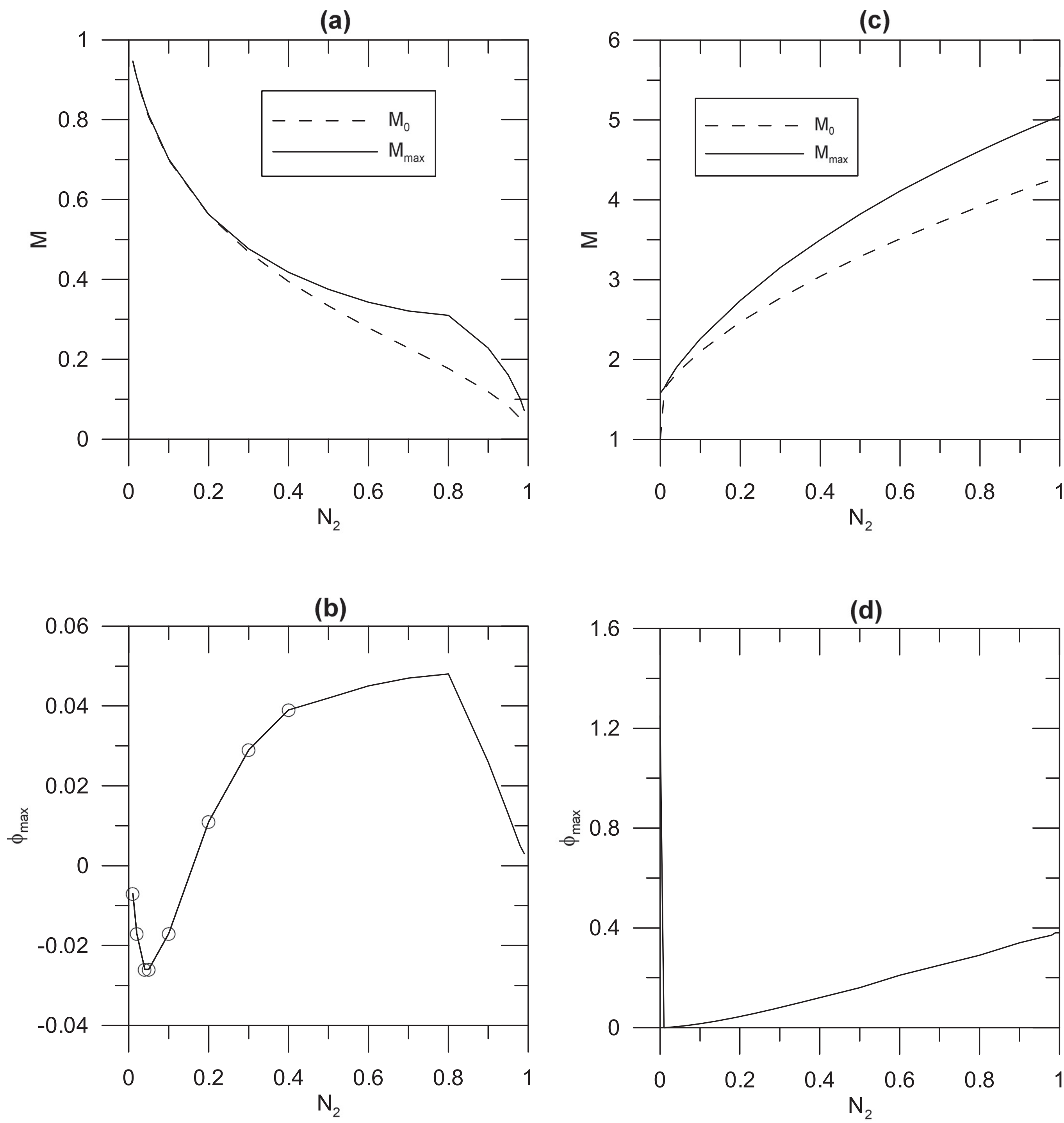

FIG. 7. Shows variation of critical Mach number M_0 (dashed curves, - - -), maximum Mach number M_{max} (solid curves), and maximum potential ϕ_{max} (solid curves), with different values of N_2 in the same format and for the same parameters as that of Figure 6, except that for $T_2 = 0.05$.

ion-acoustic solitons. For the case of Verheest *et al.*,[25] the Sagdeev pseudo-potential has three extrema in some parametric regime leading to the existence of supersolitons.

When the light ion species has a finite temperature, then, in addition to the regular ion acoustic mode, a new slow acoustic ion-ion hybrid mode comes into existences at lower Mach numbers. The slow ion acoustic mode supports negative potential solitons and DLs at low light ion densities, and positive potential solitons and DLs at large light ion densities. The transition density is dependent on plasma parameters, but is usually at $N_2 = 0.2$ to 0.4. The increase in light ion temperature T_2 leads to decrease (increase) in the Mach number regime as well as ϕ_{max} for the negative (positive) solitons/DLs of slow ion acoustic mode. On the other hand, the regular (fast) ion acoustic mode in two ion plasma, irrespective of light ion temperature, supports only positive potential solitons. For the regular ion acoustic mode, there is sharp decrease in ϕ_{max} (also in M) as light ion species is introduced. Then as the relative number density N_2 is increased, both ϕ_{max} and M range increase. The increase in T_2 results in narrowed Mach number regime and reduced ϕ_{max} for the regular ion acoustic solitons. Our model does not support the existence of supersolitons even when the light ion species have finite temperature. The model can be easily applicable to the dusty plasmas having positively charged dust grains by replacing the heavier ion species by the dust mass and doing a simple normalization to take account of the dust charge.

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