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No electrostatic supersolitons in two-component plasmas

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The concept of acoustic supersolitons was introduced for a very specific plasma with five constituents, and discussed only for a single set of plasma parameters. Supersolitons are characterized by having subsidiary extrema on the sides of a typical bipolar electric field signature, or by association with a root beyond double layers in the fully nonlinear Sagdeev pseudopotential description. It was subsequently found that supersolitons could exist in several plasma models having three constituent species, rather than four or five. In the present paper, it is proved that standard two-component plasma models cannot generate supersolitons, by recalling and extending results already in the literature, and by establishing the necessary properties of a more recent model. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4881471]

I. INTRODUCTION

When Dubinov and Kolotkov introduced the concept of acoustic supersolitons in their seminal papers,[1,2] it was for a very specific plasma with five constituents, which moreover was only discussed for a single set of plasma parameters. This clearly established that what they coined "super solitary waves" could exist, mathematically speaking, when a plasma model is able to support three consecutive local extrema of the Sagdeev pseudopotential[3] between the undisturbed conditions and an accessible root. This leads to a characteristic supersoliton signature in the electric field, viz., the simple bipolar shape associated with a usual soliton is enriched by the presence of subsidiary extrema on each side of the structure.

In their original paper, Dubinov and Kolotkov[1] also claimed that at least four plasma species were necessary to generate supersolitons, and subsequently discussed a corresponding plasma model.[4] This raised the question that supersolitons might be artefacts of rather complicated plasma compositions. As plasma models with soliton roots beyond a double layer had been encountered before,[5–8] in plasmas with fewer than four constituents, the challenge was to study supersolitons in various plasmas with only three components. These were duly found and their properties investigated in a systematic way,[9–12] in models that were sufficiently different to prove the point that it was not necessary to have at least four species in the plasma composition, as used, e.g., by Maharaj *et al.*[13]

It is to be remarked that in the latter paper a different definition is used of what a supersoliton is, namely the existence of an accessible root beyond the double layer range. The argument is that the soliton velocity (or equivalent Mach number) rather than the electric field profile is a quantity which can be observed, and that the soliton velocity is not necessarily restricted to a rather narrow interval. However, for the discussion in this paper the distinction between the two definitions of what is understood by supersolitons does not play a role, since the occurrence of soliton roots beyond the double layer range is the necessary condition to have them, in both cases.

During those explorations of various plasma compositions, it was felt that in plasmas with only two components supersolitons could not occur, because such plasmas do not have enough adjustable parameters.[9,10] We therefore prove in the present paper that known standard two-component plasma models cannot satisfy the requirements, by recalling and extending results already in the literature, and by establishing the necessary properties of a more recent model.

Fully nonlinear acoustic solitary waves in a plasma involving two polytropic species are investigated in Sec. II A. In Secs. II B–II D, we deal successively with three commonly used plasma models that are characterised by an adiabatic inertial component and a hot species whose velocity distribution exhibits an enhanced non-Maxwellian superthermal "tail," viz., the so-called Cairns nonthermal distribution,[14] the kappa distribution,[15,16] and the q-nonextensive (or Tsallis) distribution.[17,18] These three distributions have all previously formed the basis of numerous investigations of soliton behaviour and characteristics. Our conclusions follow in Sec. III.

II. DISCUSSION OF VARIOUS TWO-COMPONENT PLASMAS

A. Polytropic pressure-density relations

First, we begin with a two-component plasma, where, in a fluid description, the pressure-density relations are polytropic. One may wish to think of a classic electron-proton plasma, but no specific mass ratios or thermal velocity properties are assumed, and for the two species both inertial and pressure effects are included. There is only a requirement that the two species should have distinct and sufficiently separated thermal speeds, as otherwise, acoustic waves are not supported. Using the McKenzie fluid-dynamical interpretation[19]

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of nonlinear acoustic modes, it was proved earlier[20] that at most one compressive soliton solution could be found, the polarity of which is governed by the sign of the cooler of the two species. Here cool is defined in terms of appropriate thermal velocities, and the cooler species is then supersonic (its thermal velocity being smaller than the soliton velocity), the hotter species being necessarily subsonic in this picture.

For the sake of readability, we recall some elements of this proof, which is not based on Sagdeev pseudopotential expressions, but relies on determining the number of charge-neutral points, as necessary conditions for solitons to occur, although these are in themselves not sufficient conditions.[20] We start from the integrated form of the momentum equation per species, which in the fluid-dynamic approach is written in terms of velocities rather than densities, since, for a polytropic fluid, $p_j \propto n_j^{\gamma_j} \propto v_j^{-\gamma_j}$. Here p_j, n_j, v_j, and γ_j refer to the pressure, density, fluid velocity, and polytropic index of species j, respectively. For simplicity, we use $j = i$ for the ions and $j = e$ for the electrons, but no special mass or temperature relations are involved. Mass flux conservation yields $n_e v_e = n_0 V = n_i v_i$, where V is the velocity a soliton would have in a laboratory frame, but the treatment has been worked out in a frame co-moving with the soliton.

We thus get an energy expression of the form

$$\mathcal{E}_j \equiv \frac{m_j}{2}\left(v_j^2 - V^2\right) + \frac{m_j c_{tj}^2}{\gamma_j - 1}\left(\frac{V^{\gamma_j-1}}{v_j^{\gamma_j-1}} - 1\right) = -q_j\,\phi, \qquad (1)$$

for $\gamma_j \neq 1$. Here, m_j, q_j, and c_{tj} represent the mass, charge, and thermal velocity per species, and the electrostatic potential is ϕ. We reiterate that the only assumption regarding the thermal velocities is that $c_{ti} < c_{te}$.

Hence, these integrals are such that for a two-component plasma $\mathcal{E}_e + \mathcal{E}_i = 0$ always holds. In a charge-neutral point, one has $n_e = n_i$, and from mass flux conservation also that $v_e = v_i$, denoted as v. Combining this information gives

$$\begin{aligned} f(v) \equiv{}& \frac{m_i + m_e}{2}\left(v^2 - V^2\right) + \frac{m_i c_{ti}^2}{\gamma_i - 1}\left(\frac{V^{\gamma_i-1}}{v^{\gamma_i-1}} - 1\right) \\ &+ \frac{m_e c_{te}^2}{\gamma_e - 1}\left(\frac{V^{\gamma_e-1}}{v^{\gamma_e-1}} - 1\right) = 0. \end{aligned} \qquad (2)$$

It is obvious that $\lim_{v\to 0+} f(v) \to +\infty$, $f(V) = 0$ and $\lim_{v\to+\infty} f(v) \to +\infty$. Since

$$\begin{aligned} \frac{d^2 f(v)}{dv^2} ={}& m_i + m_e + \frac{m_i c_{ti}^2 \gamma_i}{V^2}\left(\frac{V}{v}\right)^{\gamma_i+1} \\ &+ \frac{m_e c_{te}^2 \gamma_e}{V^2}\left(\frac{V}{v}\right)^{\gamma_e+1} > 0, \end{aligned} \qquad (3)$$

$df(v)/dv$ is monotonically increasing from $-\infty$ at $v = 0$ to $+\infty$ for $v \to +\infty$, and can be shown to be positive already for $v = V$. Because of the monotonicity, $df(v)/dv$ has one and only one root, in the range $v < V$, where $f(v)$ has a (negative) minimum. This means that $f(v)$ goes through $v = V$ with a positive slope, and hence has one and only one root outside $v = V$, $\bar{v}$ in $]0, V[$, which gives a charge-neutral point on the side where $v < V$. In terms of the densities, this means that $n > n_0$. In two-component plasmas both species are either compressed or rarefied (here compressed), otherwise no charge neutral points can be generated outside the undisturbed conditions.[19,20] Other conditions are then needed to realize the soliton existence.

We have earlier excluded the cases $\gamma_j = 1$, which represent an isothermal pressure-density assumption, in which case

$$\begin{aligned} \lim_{\gamma_j\to 1} \frac{m_j c_{tj}^2}{\gamma_j - 1}\left(\frac{V^{\gamma_j-1}}{v_j^{\gamma_j-1}} - 1\right) &= m_j c_{tj}^2 \ln\left(\frac{V}{v_j}\right) \\ &= m_j c_{tj}^2 \ln\left(\frac{n_j}{n_0}\right), \end{aligned} \qquad (4)$$

and one may repeat the above arguments in a slightly modified form, to find the same result.[21] Moreover, when inertia is neglected, as might be the case for hot electrons, one recovers the well known Maxwell-Boltzmann distributions.

This proof includes in particular the standard ion-acoustic model, where ion pressure and electron inertia are neglected. Obtaining at most one compressive soliton is a far cry from the three roots needed to have a double layer and a third root beyond that. Hence, the polytropic model rules out having supersolitons, and covers many two-component plasma models in the literature, such as a plasma with adiabatic positive ions and Boltzmann electrons, or familiar notions of cold, isotropic or adiabatic constituents.

B. Nonthermal Cairns distributions

What remains to be discussed are several instances where the pressure is not a polytropic function of the species' density, or when the density is derived from one or the other velocity distribution function in phase space. Three models are currently encountered in the space physics literature, all three designed to account for non- or superthermal wings in the distribution, as compared to the standard Maxwellian: the Cairns nonthermal distribution,[14] the kappa superthermal distribution,[15,16,22] and the more recent q-nonextensive Tsallis distribution.[17,18] We will discuss these in what follows, in that order, and see what they allow in two-component plasmas.

The Cairns distribution,[14] commonly used in theoretical space plasma studies, models a Maxwellian with an enhanced nonthermal tail, which may be characterized at the macroscopic level by a parameter β.[23] It will be recalled that Cairns *et al.*[14] introduced this velocity distribution function to model the excess of superthermal particles often observed in space plasmas, as an *ad hoc* vehicle to explore their effects on nonlinear waves in space.

Cairns *et al.*[14] found rarefactive solitary structures (of the polarity associated with the subsonic species) even in a two-component plasma, besides the ubiquitous compressive modes. There are also compositional and Mach number parameter ranges for which solutions of both polarities can "coexist," with the (implicit) understanding that only one solution can be realized at a time. However, Cairns *et al.* were satisfied with establishing the possibility of having rarefactive (negative polarity) solutions, for fairly large β, but did not give an exhaustive discussion of all possibilities in terms of compositional parameters and Mach numbers (equivalent to soliton velocities).

Our study,[24] based on cold inertial ions and Cairns-distributed electrons confirmed the possibility of having rarefactive solitons at sufficiently high values of β (while retaining a single-humped distribution).[23] The range in soliton speed of the negative polarity solitons was limited by a rarefactive double layer. To broaden the discussion, we will now assume that the ions are warm rather than cold. The ion density follows from the stationary form (in a frame co-moving with the solitary structure) of the normalized fluid equations of continuity and momentum,

$$\frac{d}{dx}(n_i v_i) = 0, \tag{5}$$

$$v_i \frac{dv_i}{dx} + \sigma n_i \frac{dn_i}{dx} + \frac{d\varphi}{dx} = 0, \tag{6}$$

as

$$n_i = \frac{1}{2\sqrt{\sigma}}\left[\sqrt{(M+\sqrt{\sigma})^2 - 2\varphi} - \sqrt{(M-\sqrt{\sigma})^2 - 2\varphi}\right]. \tag{7}$$

The restriction to adiabatic ions ($p_i \propto n_i^3$) has been introduced for analytical tractability. The notation is standard and normalized as usual, $M = V/V_a$ is a Mach number, in terms of an acoustic reference velocity; $\sigma = c_{ti}^2/V_a^2$ measures the ion pressure effects and φ is the normalized electrostatic potential. Further details of the normalization are not relevant for the discussion given here. In (7), we have followed the way of writing introduced by Ghosh *et al.*,[25,26] as being easier to deal with. The ions are superacoustic, so that $\sigma < M^2$.

Besides the warm ions, the model also includes nonthermal Cairns electrons, with normalized density

$$n_e = (1 - \beta\varphi + \beta\varphi^2)\exp[\varphi]. \tag{8}$$

These densities are inserted in Poisson's equation (not given), the integration of which with respect to φ yields a typical energy-like conserved integral

$$\frac{1}{2}\left(\frac{d\varphi}{dx}\right)^2 + S(\varphi) = 0. \tag{9}$$

This can be analyzed as in classical mechanics,[3] in terms of the Sagdeev pseudopotential

$$S(\varphi, M) = \frac{\left[(M-\sqrt{\sigma})^2 - 2\varphi\right]^{3/2} - \left[(M+\sqrt{\sigma})^2 - 2\varphi\right]^{3/2}}{6\sqrt{\sigma}} + \frac{3M^2+\sigma}{3} + 1 + 3\beta - (1 + 3\beta - 3\beta\varphi + \beta\varphi^2)\exp[\varphi]. \tag{10}$$

By choice of integration constants and invoking overall charge neutrality in the undisturbed conditions, one sees that $S(0, M) = S'(0, M) = 0$, with a prime denoting a derivative with respect to φ. The required curvature to make the origin an unstable maximum, needed to give solitary wave solutions, is $S''(0, M) \leq 0$. At the same time, the global acoustic Mach number M_s is a solution of $S''(0, M) = 0$, yielding in this case

$$M_s^2 = \sigma + \frac{1}{1-\beta}. \tag{11}$$

There is another interesting property, found empirically earlier for a number of plasma models,[6,24,27–29] but proved in general only recently,[30] in that $S'''(0, M_s) = 0$ gives the value of the compositional parameters where the soliton polarity changes sign. Here, we have that

$$S'''(0, M_s) = 2 - 6\beta + 3\beta^2 + 4\sigma(1-\beta)^3 = 0. \tag{12}$$

For different values of the effective temperature ratio, σ, one thus finds β_c, critical values of the Cairns parameter, β, at which the soliton polarity is reversed. It is seen that polarity reversals are possible inside the permissible range $0 \leq \beta < 4/7$ (where the Cairns distribution is properly defined),[23] going from $\beta_c = 0.423$ at $\sigma = 0$ to $\beta_c = 0.5$ at $\sigma = 0.5$. We note that the latter value of σ would already be too large to leave a sizeable window in Mach number space for solitons to occur.[31] Thus, unlike the polytropic fluid case, Cairns electrons support both positive (compressive) and negative (rarefactive) solitons, and we shall consider the two polarities separately.

Negative polarity roots of the pseudopotential can only occur in pairs, owing to the required convexity at the origin and the asymptotic behavior for large negative φ. Plotting (10) for combinations of β and σ such that negative double layers are found clearly shows that there are no roots beyond these, so that these double layers are indeed the limit of a range of rarefactive solitons.

On the other hand, on the positive polarity side, there is only the possibility of having at most one (compressive) root as in the polytropic pressure-density model. This has been checked on the relevant Sagdeev pseudopotential graphs for a whole range of parameter combinations. The compressive soliton range is limited by encountering a sonic point,[19,20] or by infinite ion compression in the case of cold ions. Hence, it follows that Cairns electrons, too, do not support the existence of supersolitons in a two-component plasma.

C. Superthermal kappa distributions

Another way of modeling particle distributions with a high energy tail which can deviate significantly from a Maxwellian is by a kappa or generalized Lorentzian distribution function[15,16,22] that appears to be more appropriate than a thermal Maxwellian distribution in a wide range of plasma situations. Both space and laboratory plasma environments may have such an excess superthermal electron population due to velocity space diffusion, leading to an inverse power-law distribution at a velocity much higher than the electron thermal speed. The kappa distribution has thus been widely adopted to model observed distributions.[32,33]

Because of this interest, Saini *et al.*[34] gave a thorough discussion of what replacing a Maxwellian by a kappa distribution for the electrons entailed for ion-acoustic solitary modes in a two-component plasma. In the limit $\kappa \to \infty$ the kappa distribution reduces to a Maxwellian, much as the Cairns distribution does for $\beta = 0$. Their model included cold ions, but we will again assume warm adiabatic ions with density (7) (as in Sec. II B), and kappa-distributed electrons, with density[34,35]

$$n_e = \left(1 - \frac{\varphi}{\kappa - 3/2}\right)^{-\kappa+1/2}. \quad (13)$$

The Sagdeev pseudopotential for this plasma composition is

$$S(\varphi, M) = \frac{[(M-\sqrt{\sigma})^2 - 2\varphi]^{3/2} - [(M+\sqrt{\sigma})^2 - 2\varphi]^{3/2}}{6\sqrt{\sigma}} + \frac{3M^2+\sigma}{3} + \left[1 - \left(1 - \frac{\varphi}{\kappa - 3/2}\right)^{-\kappa+3/2}\right]. \quad (14)$$

Here, we have used a similar normalization to that above. The global acoustic Mach number M_s is a solution of $S''(0, M) = 0$, here defined through

$$M_s^2 = \sigma + \frac{2\kappa - 3}{2\kappa - 1}. \quad (15)$$

For changes of polarity, one would need to find a root κ of

$$S'''(0, M_s) = \frac{4(2\kappa-1)(\kappa-1)}{(2\kappa-3)^2} + 4\sigma\,\frac{(2\kappa-1)^3}{(2\kappa-3)^3}. \quad (16)$$

Given the standard restriction $\kappa > 3/2$, one sees that $S'''(0, M_s)$ is always positive for the kappa distribution model. Between parentheses, at the time when Saini *et al.* wrote their paper,[34] the properties of the Sagdeev pseudopotential at the acoustic speed were not fully realized, in particular the influence of $S'''(0, M_s)$ on the soliton polarity.[30]

Hence, in contrast to the Cairns model, which, at sufficiently high β (but not exceeding the limits of acceptability of the model),[23] allowed solitons and double layers of the reverse polarity (rarefactive) to exist, the kappa model was unable to generate the latter, and only the well known compressive soliton was obtained.[34] In this sense, the behavior of the kappa model in a two-component plasma is not vastly different from that of the Maxwellian or polytropic descriptions, at least as far as the soliton polarity is concerned. There are, of course, considerable changes in the quantitative details of the nonlinear modes. In summary, then, kappa-distributed hot species, too, do not allow supersolitons in a two-component plasma.

D. *q*-Nonextensive Tsallis distributions

This brings us to the third of the commonly used nonthermal distributions with an enhanced non-Maxwellian "tail," the q-nonextensive distributions introduced by Tsallis.[17] Whereas the Cairns and kappa distributions are essentially empirical models, the Tsallis distribution, based on non-extensive statistical mechanics,[17] has the attraction that it may provide a rigorous theoretical foundation, although it appears that the approach is still somewhat controversial. However, we note that the Tsallis approach has also been linked to the kappa distribution.[36]

The family of Tsallis distributions, with a parameter q, has recently been much used in soliton theory, based on earlier linear wave studies,[18] but not always with the required care, as will be seen below. We therefore propose to follow the methodology of Verheest and Hellberg for the Cairns distribution[24] and of Saini *et al.*[34] for the kappa distribution, as far as the determination of the properties of large amplitude nonlinear waves is concerned.

The plasma is composed of adiabatic positive ions and Tsallis-distributed electrons, the latter with density[37]

$$n_e = [1 + (q-1)\varphi]^{(q+1)/(2q-2)}. \quad (17)$$

This electron density has been obtained by integration of the Tsallis distribution in phase space over all velocities, the details of which are given elsewhere.[18,37,38] However, the restrictions on q that arise due to the convergence of some of the integrals need to be kept in mind, because they are not visible when one inspects expressions like (17). The phase space distribution is unnormalizable for $q \le -1$, just for the number density, but the energy integral diverges for $q \le 1/3$.[18,37,38] This restriction is equivalent to the well-known limit $\kappa > 3/2$ that is always included by users of the kappa distribution.[16] This means that the superthermal range has to be restricted to $1/3 < q < 1$, as for $q = 1$ the Tsallis distribution reduces to a Maxwellian. Having a finite energy is necessary if one wants to introduce concepts like pressure and/or temperature in the fluid description of Tsallis species. Unfortunately, the restriction to $1/3 < q < 1$ is often not adhered to in numerical applications,[37] and invalidates some of the conclusions in the literature, as we will point out below.

For $q > 1$, the phase space velocities are restricted to a finite interval, and so this part of the Tsallis distribution cannot be used for the modeling of superthermal wings, normally needed for space plasma applications. Nonetheless, we shall show below that the argument applicable to the range $1/3 < q < 1$, applies for $q > 1$, too.

Using, again, a similar normalization, the Sagdeev pseudopotential for this problem is

$$S(\varphi, M) = \frac{[(M-\sqrt{\sigma})^2 - 2\varphi]^{3/2} - [(M+\sqrt{\sigma})^2 - 2\varphi]^{3/2}}{6\sqrt{\sigma}} + \frac{3M^2+\sigma}{3} + \frac{2}{3q-1}\left\{1 - [1 + (q-1)\varphi]^{(3q-1)/(2q-2)}\right\}. \quad (18)$$

Here the quantities of note are, from $S''(0, M_s) = 0$ that

$$M_s^2 = \sigma + \frac{2}{q+1}, \quad (19)$$

and

$$S'''(0, M_s) = q(q+1) + \frac{1}{2}\,\sigma(q+1)^2. \qquad (20)$$

It is seen that for $1/3 < q < 1$ this is always positive and so changes in polarity cannot occur.

Clearly, the above constraint that prevents the occurrence of negative polarity solitons for $1/3 < q < 1$, applies equally well for $q > 1$.

Only a few papers in the literature deal with large amplitude ion-acoustic solitons in two-component plasmas having Tsallis electrons, of which we discuss two representative ones. The paper by Tribeche *et al.*[39] works with cold ions ($\sigma = 0$), and their Sagdeev pseudopotential is a special case of ours. They indicate that changes of polarity occur at $q = 0$, and thus seemingly rarefactive solitons can exist for $q < 0$. Alas, the range $q \leq 1/3$ is excluded for reasons of energy convergence at the phase space level, so that there can be no rarefactive solitons.

The other paper, by Roy *et al.*,[40] includes ion thermal effects and would correspond to our model, except that the pressure term in their ion equation of motion [Eq. (2) of Ref. 40] is written for isothermal pressures ($p_i \propto n_i$). Upon integration that should give a logarithmic term, which renders the inversion of the equation to extract the density as function of φ very complicated. To the contrary, their density expression [Eq. (8) of Ref. 40] is clearly written for adiabatic pressures ($p_i \propto n_i^3$), so there is a disconnect in their algebra! Of their two figures, the first is for $q > 1$, the range which cannot serve to model superthermal wings, whereas the second is invalid as q is too close to 0, outside the allowed range. In any case, they only discuss positive solitons, and each time find only one root, the compressive ion-acoustic mode.

Having derived the necessary information, we will plot the existence domains in admissible Mach numbers, in terms of q, for some typical values of σ. The lower bound is clearly given by M_s, whereas the upper bound follows from considering the limits imposed on φ by the reality of the ion density. From (7) we infer that this is given by $\varphi_\ell = \frac{1}{2}(M - \sqrt{\sigma})^2$, reached before $\frac{1}{2}(M + \sqrt{\sigma})^2$. Since a soliton root should be encountered before φ_ℓ, hence $S(\varphi_\ell, M) > 0$, we will use the standard practice of determining the relevant M from $S(\varphi_\ell, M) = 0$. This is illustrated in Fig. 1.

Once we know where to choose appropriate parameter values, we can plot the Sagdeev pseudopotentials, and illustrate them for some typical choices of q and σ. In Fig. 2, the values for σ are the same as used in Fig. 1, with the same curve styles, but panel (a) is for $q = 0.5$, panel (b) for $q = 0.8$ and panel (c) for $q = 1.5$. For negative φ, all curves in Fig. 2 ultimately tend to $-\infty$, whereas on the positive side they have been limited for graphical clarity. The value of

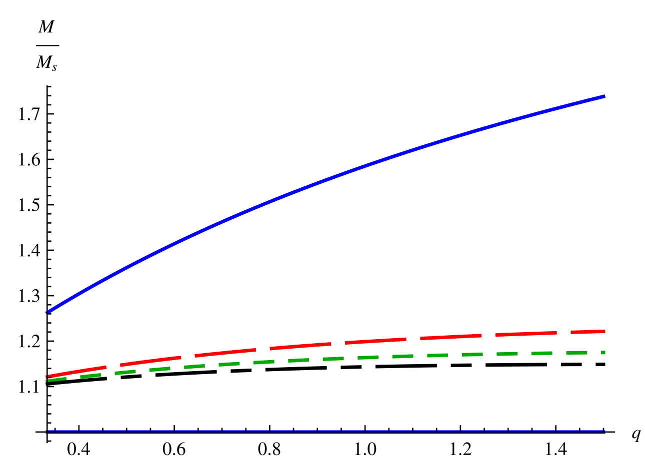

FIG. 1. Existence diagram in $\{q, M/M_s\}$ space. The lower limit for M is at the acoustic speed, or $M/M_s = 1$, whereas the upper limit is given by the respective sonic points, for $\sigma = 0$ (blue solid curve), $\sigma = 0.1$ (red dashed curve), $\sigma = 0.2$ (green dotted curve) and $\sigma = 0.3$ (black dotted-dashed curve).

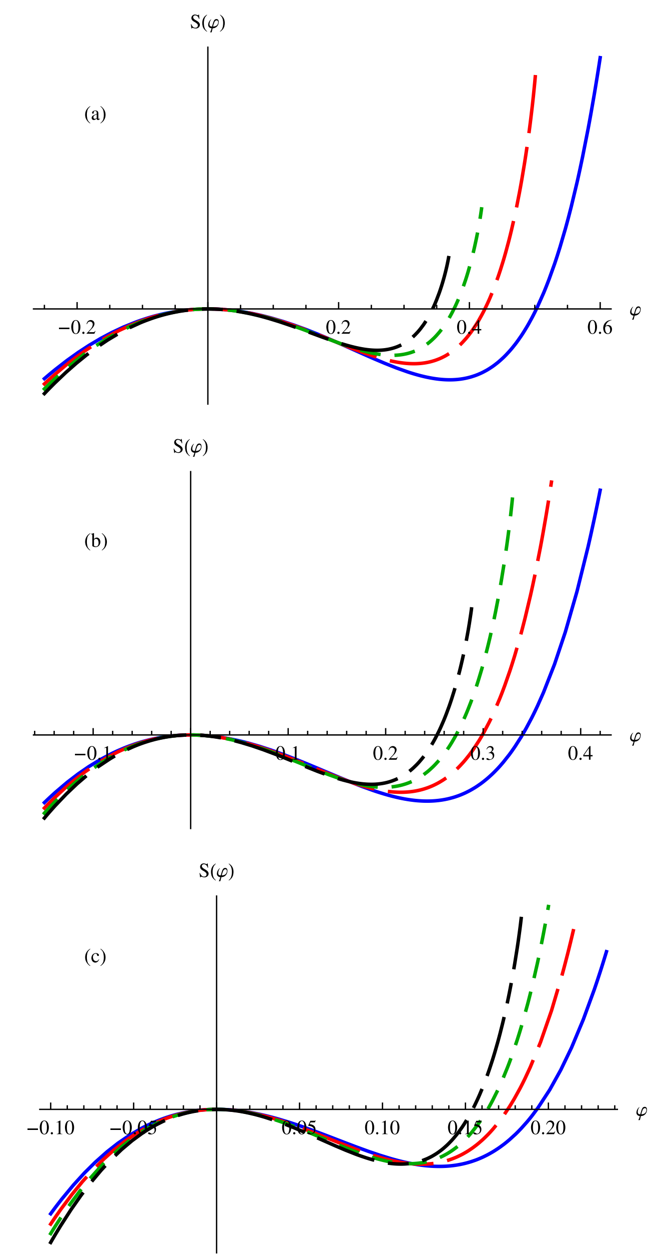

FIG. 2. Graphs of typical Sagdeev pseudopotential curves, for $M/M_s = 1.1$, $\sigma = 0$ (blue solid curve), $\sigma = 0.1$ (red dashed curve), $\sigma = 0.2$ (green dotted curve), and $\sigma = 0.3$ (black dotted-dashed curve). The corresponding values for q are (a) 0.5, (b) 0.8, and (c) 1.5.

$M/M_s = 1.1$ has been chosen as it yields solitons for all the values of σ considered. It is seen that increasing q at a fixed value of M/M_s decreases the soliton amplitudes, and that there are no negative solitons at all.

Although not our main focus, we have briefly considered the case $q > 1$. As we have seen above, polarity changes cannot take place in this range. For completeness, we include in Fig. 2, panel (c), typical Sagdeev pseudopotentials for the case $q = 1.5$ as an illustration of the pseudopotential behaviour in this range.

Again, as expected, Fig. 2 clearly shows that no supersolitons can form, when the electrons are Tsallis-distributed. It is seen that the effective temperature ratio parameter σ has a definite quantitative influence on the soliton amplitudes, but, importantly, it cannot affect the positive or negative character of the solutions. This is why in many applications the ions are treated as cold, for mathematical simplicity.[7,23,24,30]

III. CONCLUSIONS

We recognise that it is not possible to provide a null result for all possible two-component plasma models. Hence, we have investigated a number of standard two-component models that are commonly used in the broader space physics literature. In particular, these include all the well-known examples in which the inertialess species has a velocity distribution that models the presence of an enhanced non-Maxwellian superthermal "tail."

We have shown that supersolitons cannot exist in current two-component plasmas, by recalling results from the literature where the pressure-density relations are polytropic, even when full inertia and pressure effects have been retained for both species. Only compressive solitons have been found, the range of which is limited by ion density limits, not by double layers.

This approach also covers Boltzmann distributions for the hot species, and an extension of those to kappa or Tsallis nonextensive distributions has not been able to generate qualitative differences, or changes in soliton polarity. The cases in the literature which discuss polarity changes for Tsallis-distributed electrons at $q=0$ are mistaken, in that a correct delimitation of the superthermal range limits q to $1/3 < q < 1$ by requiring that the energy in phase space be normalizable. This is equivalent to the well-known lower limit for kappa, viz. the requirement $\kappa > 3/2$, and it should be used as universally for the Tsallis model as is its analogue for the kappa distribution.

The only exception to this general trend is when the electrons have nonthermal Cairns distributions. Here a polarity change is possible for sufficiently large β, but the negative soliton range is limited by double layers, without roots beyond those. It is one of the enduring mysteries why the Cairns distributions should be so different in this respect from kappa or Tsallis distributions, as all include the Boltzmann case as a limit. It appears that a first step in solving that mystery may have been taken in a recent paper by Cairns,[41] who considered another aspect of the different behaviour of waves in kappa- and in Cairns-distributed plasmas.

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[1] A. E. Dubinov and D. Yu. Kolotkov, IEEE Trans. Plasma Sci. **40**, 1429 (2012).
[2] A. E. Dubinov and D. Yu. Kolotkov, High Energy Chem. **46**, 349 (2012).
[3] R. Z. Sagdeev, *Reviews of Plasma Physics*, edited by M. A. Leontovich (Consultants Bureau, New York, 1966), Vol. 4, p. 23.
[4] A. E. Dubinov and D. Yu. Kolotkov, Plasma Phys. Rep. **38**, 909 (2012).
[5] F. Verheest, Phys. Plasmas **16**, 013704 (2009).
[6] T. K. Baluku, M. A. Hellberg, and F. Verheest, Europhys. Lett. **91**, 15001 (2010).
[7] F. Verheest, Phys. Plasmas **18**, 083701 (2011).
[8] A. Das, A. Bandyopadhyay, and K. P. Das, J. Plasma Phys. **78**, 149 (2012).
[9] F. Verheest, M. A. Hellberg, and I. Kourakis, Phys. Plasmas **20**, 012302 (2013).
[10] F. Verheest, M. A. Hellberg, and I. Kourakis, Phys. Rev. E **87**, 043107 (2013).
[11] F. Verheest, M. A. Hellberg, and I. Kourakis, Phys. Plasmas **20**, 082309 (2013).
[12] M. A. Hellberg, T. K. Baluku, F. Verheest, and I. Kourakis, J. Plasma Phys. **79**, 1039 (2013).
[13] S. K. Maharaj, R. Bharuthram, S. V. Singh, and G. S. Lakhina, Phys. Plasmas **20**, 083705 (2013).
[14] R. A. Cairns, A. A. Mamun, R. Bingham, R. Boström, R. O. Dendy, C. M. C. Nairn, and P. K. Shukla, Geophys. Res. Lett. **22**, 2709, doi:10.1029/95GL02781 (1995).
[15] V. M. Vasyliunas, J. Geophys. Res. **73**, 2839, doi:10.1029/JA073i009p02839 (1968).
[16] M. A. Hellberg, R. L. Mace, T. K. Baluku, I. Kourakis, and N. S. Saini, Phys. Plasmas **16**, 094701 (2009).
[17] C. Tsallis, J. Stat. Phys. **52**, 479 (1988).
[18] J. A. S. Lima, R. Silva, Jr., and J. Santos, Phys. Rev. E **61**, 3260 (2000).
[19] J. F. McKenzie, Phys. Plasmas **9**, 800 (2002).
[20] F. Verheest, T. Cattaert, G. S. Lakhina, and S. V. Singh, J. Plasma Phys. **70**, 237 (2004).
[21] T. Cattaert, F. Verheest, and M. A. Hellberg, Phys. Plasmas **12**, 042901 (2005).
[22] D. Summers and R. M. Thorne, Phys. Fluids B **3**, 1835 (1991).
[23] F. Verheest and S. R. Pillay, Phys. Plasmas **15**, 013703 (2008).
[24] F. Verheest and M. A. Hellberg, Phys. Plasmas **17**, 102312 (2010).
[25] S. S. Ghosh, K. K. Ghosh and A. N. Sekar Iyengar, Phys. Plasmas **3**, 3939 (1996).
[26] S. S. Ghosh and A. N. Sekar Iyengar, Phys. Plasmas **4**, 3204 (1997).
[27] T. K. Baluku, M. A. Hellberg, I. Kourakis, and N. S. Saini, Phys. Plasmas **17**, 053702 (2010).
[28] T. K. Baluku and M. A. Hellberg, Plasma Phys. Controlled Fusion **53**, 095007 (2011).
[29] T. K. Baluku and M. A. Hellberg, Phys. Plasmas **19**, 012106 (2012).
[30] F. Verheest, M. A. Hellberg, and T. K. Baluku, Phys. Plasmas **19**, 032305 (2012).
[31] F. Verheest, M. A. Hellberg, and G. S. Lakhina, Astrophys. Space Sci. Trans. **3**, 15 (2007).
[32] V. Pierrard and M. Lazar, Sol. Phys. **267**, 153 (2010).
[33] M. Hapgood, C. Perry, J. Davies, and M. Denton, Planet. Space Sci. **59**, 618 (2011).

[34]N. S. Saini, I. Kourakis, and M. A. Hellberg, Phys. Plasmas **16**, 062903 (2009).
[35]T. K. Baluku and M. A. Hellberg, Phys. Plasmas **15**, 123705 (2008).
[36]G. Livadiotis and D. J. McComas, Space Sci. Rev. **175**, 183 (2013).
[37]F. Verheest, J. Plasma Phys. **79**, 1031 (2013).
[38]E. Saberian and A. Esfandyari-Kalejahi, Phys. Rev. E **87**, 053112 (2013).
[39]M. Tribeche, L. Djebami, and R. Amour, Phys. Plasmas **17**, 042114 (2010).
[40]K. Roy, T. Saha, and P. Chatterjee, Phys. Plasmas **19**, 104502 (2012).
[41]R. A. Cairns, J. Plasma Phys. **79**, 1035 (2013).