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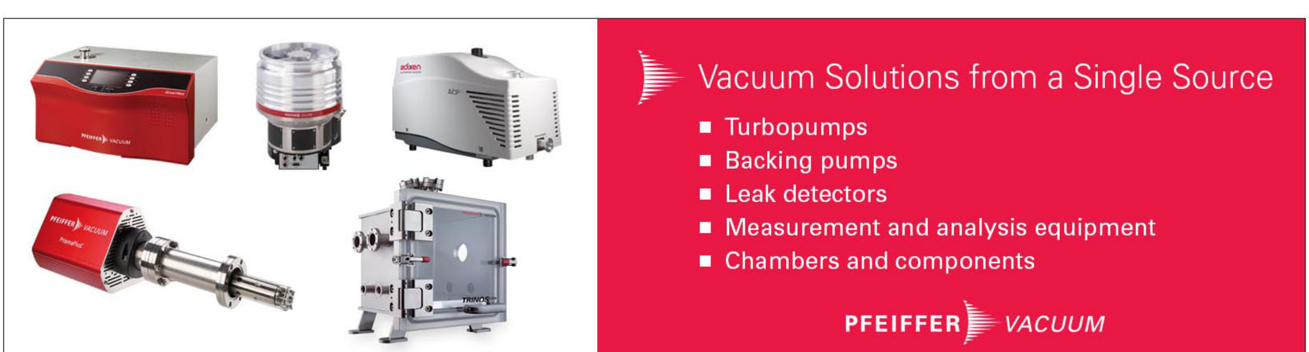


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PHYSICS OF PLASMAS **21**, 082304 (2014)

# Ion acoustic solitons and supersolitons in a magnetized plasma with nonthermal hot electrons and Boltzmann cool electrons

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Arbitrary amplitude, ion acoustic solitons, and supersolitons are studied in a magnetized plasma with two distinct groups of electrons at different temperatures. The plasma consists of a cold ion fluid, cool Boltzmann electrons, and nonthermal energetic hot electrons. Using the Sagdeev pseudo-potential technique, the effect of nonthermal hot electrons on soliton structures with other plasma parameters is studied. Our numerical computation shows that negative potential ion-acoustic solitons and double layers can exist both in the subsonic and supersonic Mach number regimes, unlike the case of an unmagnetized plasma where they can only exist in the supersonic Mach number regime. For the first time, it is reported here that in addition to solitions and double layers, the ion-acoustic supersoliton solutions are also obtained for certain range of parameters in a magnetized three-component plasma model. The results show good agreement with Viking satellite observations of the solitary structures with density depletions in the auroral region of the Earth's magnetosphere. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4891877]

## I. INTRODUCTION

Spacecraft observations[1–3] as well as laboratory experiments,[4,5] and theoretical models[6,7] have provided evidence of the occurrence of abnormal energetic particle presence in the Earth's magnetosphere. Plasmas with thermal equilibrium (Maxwellian) velocity distributions have been studied over several years.[8–17] In recent times, the consistent attempts have been made to study the energetic particles effects through Cairn's nonthermal distribution model,[7,18–27] as well as the Kappa's distribution model.[28–35]

Cairns *et al.*[7] proposed a nonthermal distribution model for the highly energetic electron species in order to explain the observations made by the FREJA and Viking satellites in auroral regions of the Earth's magnetosphere. Cairn's nonthermal distribution is not the only distribution that can be used to model energetic electrons. There are kappa- and Tsallis-distributions which are also nonthermal distributions. Such nonthermal distributions have pronounced energetic particle tails which arise as a consequence of several different acceleration mechanisms,[36] e.g., DC parallel electric fields,[37] field-aligned potential drops in reconnection regions,[38,39] wave-particle interactions due to kinetic Alfven wave turbulence[40] or cyclotron resonance.[41] Furthermore, in space plasmas, Ma and Summers[42] proposed the formation of power-law distributions due to electron acceleration by whistler-mode waves.

The study on oblique propagation of ion-acoustic solitons in a magnetized plasma with nonthermal electrons and warm adiabatic ions has shown the coexistence of rarefactive and compressive solitary waves.[43] Mamun[18] investigated the effects of adiabatic ion temperature and the contribution of a nonthermal distribution of electron species, on arbitrary amplitude ion acoustic solitons, using the pseudopotential technique for a two component unmagnetized plasma. The time evolution of coexisting positive and negative potential solitary waves in nonthermal plasma model revealed that the positive initial disturbance breaks up into a series of solitary waves, whereas the behaviour of negative potential solitary waves appears to be different. The negative potential waves appear to be unstable and produce positive solitary waves at a later time.[44] The effect of superthermal electrons on the dynamics of ion-acoustic solitons has been investigated by Abbasi and Pajouh.[45] In a magnetized, superthermal plasma, the properties of the obliquely propagating ion-acoustic solitary waves are influenced by the presence of excess superthermal electrons.[29] Also, the existence of arbitrary amplitude ion-acoustic solitary waves in an unmagnetized plasma consisting of ions and excess superthermal electrons and an electron beam have been investigated.[46]

Gill *et al.*[47] used the reductive perturbation method to derive the Korteweg-de Vries (KdV) and the (modified) m-KdV equations to investigate ion acoustic solitons and double layers for a plasma consisting of unmagnetized warm positive and negative ions with different masses and charged states, and nonthermal distribution of electron species. Bahamida *et al.*[20] presented a three component plasma model consisting of unmagnetized positively charged ions, nonthermally distributed electrons and Boltzmann positrons, to study the properties of arbitrary amplitude ion acoustic solitons observed by satellites in different regions of the Earth's magnetosphere. Verheest and Hellberg[48] studied the characteristics of compressive and rarefactive ion acoustic solitary waves in a plasma consisting of positive ions and nonthermal distribution of electron species. Rarefactive solitary waves and double layer structures were obtained when the electron nonthermality exceeded a certain minimum. Jung and Hong[30] investigated nonthermal effects on the

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propagation of ion acoustic solitons in generalized Lorentzian electron-ion plasmas. Their analyses were done by obtaining the KdV equation as a function of the spectral index in generalized Lorentzian plasmas. Jilani *et al.*[49] studied the properties of nonlinear ion acoustic solitary waves in an unmagnetized and collisionless pair-ion plasma with a nonthermal distribution of electron population. Using the reductive perturbation technique, they obtained the nonlinear Korteweg-de Vries (KdV) equation for the soliton structures.

The supersolitons were first introduced by Ustinov[50] to explain resonance due to nonlinear excitations in inhomogenous Josephson junctions. Recently, there has been great deal of interest in the study of ion-acoustic supersolitons, which were discovered by Dubinov and Kolotkov[51,52] in five species plasmas. More recently, they have shown that supersolitons of the ion acoustic type can exist in an unmagnetized plasma that contains at least four kinds of charged particle species with inertialess species following the Boltzmann distributions.[53] These new kind of solitons with distorted potential and electric field profiles are known as supersolitons which appear beyond the double layers. The Sagdeev pseudopotential for the supersolitons should have 3 finite consecutive roots for the real potential, and the supersolitons structures can occur only when the third root becomes accessible, i.e., when the plasma model is able to support three consecutive local extrema of the Sagdeev pseudopotential (cf. Figure 1 of Dubinov and Kolotkov,[53] Figures 2 and 4 of Verheest *et al.*[54]). In contrast, the ordinary ion acoustic solitons cannot exist for Mach numbers greater than that of the double layer. Further, in three different theoretical models, Verheest *et al.*[54–56] have shown that ion acoustic supersolitons can exist even in three species plasmas, e.g., (1) plasmas having nonthermal electrons, cold positive and negative ions (2) negative dust and two temperature Boltzmann and nonthermal positive ions, and (3) two temperature electrons and cold ions. Dust-acoustic supersolitons have been studied by Maharaj *et al.*[57] in a four species plasma comprising of cold negative dust, adiabatic positive dust, Boltzmann electrons, and non-thermal ions. Recently, Lakhina *et al.*[58] have shown that supersolitons cannot exist in 3-component plasmas having two types of ion species (heavier and lighter ions) and one type of electron species with Boltzmann distribution. Verheest *et al.*[59] have ruled out the existence of ion acoustic supersolitons in two component plasmas.

In this paper, the effect of a nonthermal hot species of electrons is examined on low frequency ion-acoustic solitary waves in a magnetized plasma. In Sec. II, the formulation of the research problem and the localized solution for the nonlinear waves using the Sagdeev pseudo-potential approach are presented. For the first time, it is shown that in addition to solitons and double layer, the supersoliton structures can also form in the three component magnetized plasmas. Numerical results and discussions are presented in Sec. III, with a summary of our findings presented in Sec. IV.

## II. THEORETICAL MODEL

We consider the propagation of ion acoustic waves in a three-component, collisionless, magnetized plasma consisting of two distinct group of electrons, namely cool electrons following a Boltzmann distribution and hot electrons having nonthermal Cairns type of distribution and cold fluid ions. The ambient magnetic field $B_0\hat{\mathbf{z}}$ is considered to be in the $z$-direction, and waves are propagating obliquely to the ambient magnetic field in the x-z plane. The magnetic field limits particle free-streaming, but in the directions perpendicular to the magnetic field. In our model, the motion of magnetized electrons is nearly one dimensional. Further, the motion of electrons, both cooler and hot, is considered much faster than the phase velocity of the wave; therefore, we are justified to treat them as inertialess species. Being much hotter, the nonthermal electron species have not achieved the Boltzmann distribution yet due to collisional effects. We are interested in time scales much shorter than the thermalization time of the hot electrons. We assume the phase velocity of the wave to be larger than the ion thermal speed; therefore, the assumption of ions being cold is satisfied, and their dynamics are governed by the fluid continuity and momentum equations

$$\frac{\partial N_i}{\partial t} + \nabla.(N_i \mathbf{V_i}) = 0, \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V_i}.\nabla\right)\mathbf{V_i} = -\frac{e\nabla\phi}{m_i} + e\frac{\mathbf{V}_i \times \mathbf{B}_o}{m_i c}, \tag{2}$$

where $N_i$ and $\mathbf{V_i}$ are the ions number density and the fluid velocity, respectively, $m_i$ is the ion mass, e is the magnitude of the electron charge, c is the speed of light in vacuum, t is time, and $\phi$ is the electrostatic potential.

The two distinct group of electrons are: cool electrons in thermal equilibrium with a Boltzmann distribution, i.e,

$$N_c = N_{c0}\exp\left(\frac{e\phi}{T_c}\right), \tag{3}$$

while the extra-energetic hot electrons deviate from Maxwellian behavior, as a result of high electron temperatures attributed to the solar radiation. We adopt for our population of hot electrons the nonthermal distribution function given by Cairns *et al.*[7]

$$f_h(v) = \frac{N_{h0}}{(3\alpha+1)\sqrt{2\pi v_h^2}}\left(1 + \frac{\alpha v^4}{v_h^4}\right)\exp\left(-\frac{v^2}{2v_h^2}\right), \tag{4}$$

where $N_{h0}$ is the equilibrium density, $v_h = \sqrt{T_h/m_e}$ is the thermal speed of the hot electrons, and $\alpha$ is the nonthermal parameter. The nonthermal electron distribution in the presence of the ion acoustic wave field can be found by replacing $v^2/v_h^2$ by $v^2/v_h^2 - 2e\phi/T_h$ in Eq. (4), which on integration over velocity space gives the following expression for the electron density:[7]

$$N_h = N_{h0}\left[1 - \beta\left(\frac{e\phi}{T_h}\right) + \beta\left(\frac{e\phi}{T_h}\right)^2\right]\exp\left(\frac{e\phi}{T_h}\right), \tag{5}$$

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where $\beta = \frac{4\alpha}{1+3\alpha}$. It is noted that $\alpha = 0$ corresponds to the Boltzmann distribution of electrons.

To investigate the existence of arbitrary amplitude nonlinear waves in such a plasma, we normalize the variables as follows: densities $N_i$, $N_c$, $N_h$ by the total ion equilibrium density $N_{i0} = N_{c0} + N_{h0}$, distance $x$ by the effective ion Larmor radius, $\rho_i = c_s/\mathbf{\Omega}$; $c_s = \sqrt{T_{eff}/m_i}$, time t by the inverse of ion gyro-frequency $\mathbf{\Omega}^{-1}$, where $\mathbf{\Omega} = eB_0/m_i c$, and potential $\phi$ by $T_{eff}/e$. Here, the temperature ratio $\tau = T_c/T_h$, cool electron density ratio $f = N_{c0}/N_0$, where $N_{j0} = (j = c, h, i)$ are the equilibrium densities, and effective temperature $T_{eff} = T_c/(f + (1-f)\tau)$. We further define $\alpha_c = T_{eff}/T_c$, $\alpha_h = T_{eff}/T_h$, and $\psi = e\phi/T_{eff}$.

In order to solve Eqs. (1)–(3) and (5), we follow the method adopted by Lee and Kan[60] (refer to their eq. (5)) and Reddy *et al.*[61,62] Thus, the above mentioned equations can be transformed into stationery frame by using the transformation as $\xi = (\varphi x + \gamma z - Mt)/M$, (where $M = V/c_s$ is the Mach number and $\varphi = \sin\theta$, $\gamma = \cos\theta$, and $\theta$ is the angle of propagation with respect to ambient magnetic field) to express as

$$n_c = f e^{\alpha_c \psi}, \tag{6}$$

$$n_h = (1-f)(1 - \beta(\alpha_h\psi) + \beta(\alpha_h\psi)^2) e^{\alpha_h\psi}, \tag{7}$$

$$\frac{d}{d\xi}(L_v n_i) = 0, \tag{8}$$

$$L_v \frac{dv_x}{d\xi} = -\varphi \frac{d\psi}{d\xi} + M v_y, \tag{9}$$

$$L_v \frac{dv_y}{d\xi} = -M v_x, \tag{10}$$

$$L_v \frac{dv_z}{d\xi} = -\gamma \frac{d\psi}{d\xi}, \tag{11}$$

where $L_v = -M + \varphi v_x + \gamma v_z$. Our system of equation is closed with the quasi-neutrality condition

$$n_i = f e^{\alpha_c\psi} + (1-f)(1 - \beta(\alpha_h\psi) + \beta(\alpha_h\psi)^2) e^{\alpha_h\psi}. \tag{12}$$

The quasi-neutrality condition (12) is satisfied for low-frequency waves such as ion-acoustic waves,[29] where all perturbed quantities goes back to their equilibrium values at boundary conditions $\xi \to \pm\infty$. Thus, we solve Eqs. (8), (9), and (11) with the boundary conditions $n_i \to 1$, $\psi \to 0$, and $d\psi/d\xi \to 0$ at $\xi \to \pm\infty$, and eliminating $v_x$, $v_y$, and $v_z$, we obtain

$$\frac{d^2 P(\psi)}{d\xi^2} = M^2(n_i - 1) - n_i \gamma^2 Q(\psi), \tag{13}$$

where

$$P(\psi) = \left(\psi + \frac{M^2}{2n_i^2}\right)$$

$$Q(\psi) = \frac{f}{\alpha_c}(e^{\alpha_c\psi} - 1) + \frac{(1-f)}{\alpha_h}\left\{e^{\alpha_h\psi}\left(1 + 3\beta - 3\beta\alpha_h\psi + \beta\alpha_h^2\psi^2\right) - (1 + 3\beta)\right\}.$$

Then, we multiply both sides of Eq. (13) by $2\frac{dP}{d\xi}$ and integrate with the prescribed boundary conditions; we obtain

$$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2 + V(\psi, M) = 0. \tag{14}$$

Equation (14) is regarded as an "Energy Integral" of an oscillating pseudo-particle of unit mass, with the velocity $d\psi/d\xi$ at time $\xi$ and the position $\psi$ in a potential $V(\psi, M)$.

The Sagdeev pseudo potential corresponds to

$$V(\psi, M) = -\frac{\left[-\dfrac{M^4}{2n_i^2(1-n_i)^2} - M^2(1-\gamma^2)\psi + M^2 Q(\psi) - \dfrac{\gamma^2 Q^2(\psi)}{2} - \dfrac{\gamma^2 M^2 Q(\psi)}{n_i}\right]}{\left[1 - \dfrac{M^2}{n_i^3}\left(f\alpha_c e^{\alpha_c\psi} + (1-f)\alpha_h\left(1 - \beta + \beta\alpha_h\psi + \beta\alpha_h^2\psi^2\right)e^{\alpha_h\psi}\right)\right]^2}. \tag{15}$$

## III. NUMERICAL RESULTS

In the Boltzmann limit for the hot electrons, $\alpha = \beta = 0$, the Sagdeev potential given by Eq. (15) above reduces to equation (19) of Rufai *et al.*[16] Soliton solutions exist when the following conditions are satisfied: $V(\psi, M) = 0$, $dV(\psi, M)/d(\psi) = 0$, $d^2V(\psi, M)/d(\psi)^2 < 0$ at $\psi = 0$; $V(\psi, M) = 0$ at $\psi = \psi_m$, and $V(\psi, M) < 0$ for $0 < |\psi| < |\psi_m|$, where $\psi_m$ is the maximum amplitude of the solitons. It must be noted that the additional requirement for double layer solution is $dV(\psi, M)/\mathrm{d}\psi = 0$ at $\psi = \psi_m$.

From the above conditions for a soliton, we have from Eq. (15)

$$\frac{d^2V(\psi, M)}{d\psi^2}\Big|_{\psi=0} = \frac{M^2 - M_o^2}{M^2 - M_1^2} < 0, \tag{16}$$

where

$$M_o^2 = \frac{\gamma^2}{f\alpha_c + (1-f)\alpha_h - (1-f)\beta\alpha_h} \tag{17}$$

is the critical Mach number, and the upper limit is

$$M_1^2 = \frac{1}{f\alpha_c + (1-f)\alpha_h - (1-f)\beta\alpha_h}. \tag{18}$$

Since, $\gamma^2 = \cos^2\theta < 1$, this implies that $M_o < M_1$. Further, if $M > M_1 \Rightarrow M > M_0$ which means that $M^2 - M_o^2 > 0$ and $M^2 - M_1^2 > 0$, consequently (16) is not satisfied.

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TABLE I. Properties of ion-acoustic solitons, such as Mach number range $M$, Soliton Velocity, $V$ (km/s) Electric Field $E$ (mV/m), Soliton Width $W$ (m), and Pulse Duration, $\tau^* = W/V$ (ms), for various values of the nonthermal contribution ($\alpha$) with $\theta = 35°$, Cool electron density $f = 0.1$, Electron temperature ratio, $T_c/T_h = 0.04$.

| $\alpha$ | $M_o < M < M_1$ | $V$ (km/s) | $E$ (mV/m) | $W$ (m) | $\tau^* = W/V$ (ms) |
|---|---|---|---|---|---|
| 0.0 | 0.821–0.999 | 21.26–25.87 | 0.04–23.52 | 978.64–226.20 | 46.03–8.74 |
| 0.01 | 0.825–1.005 | 21.37–26.03 | 0.03–21.87 | 1050.4–212.69 | 49.15–8.17 |
| 0.05 | 0.8402–1.023 | 21.76–26.50 | 0.024–17.68 | 1094.6–192.92 | 50.3–7.28 |
| 0.1 | 0.856–1.043 | 22.17–27.01 | 0.017–15.31 | 1186.12–183.04 | 53.5–6.78 |
| 0.15 | 0.869–1.059 | 22.51–27.43 | 0.01–13.83 | 1409.20–177.84 | 62.61–6.48 |
| 0.2 | 0.88–1.073 | 22.79–27.79 | 0.004–12.94 | 1778.40–173.16 | 78.03–6.23 |

Similarly, if $M < M_0 \Rightarrow M < M_1$ from which $M^2 - M_o^2 < 0$ and $M^2 - M_1^2 < 0$, once again (16) is not satisfied.

Therefore, the condition (16) is satisfied only if

$$M_o < M < M_1. \tag{19}$$

Since, $f\alpha_c + (1-f)\,\alpha_h = 1$, Eq. (19) becomes

$$\frac{\gamma}{\sqrt{1-(1-f)\beta\alpha_h}} < M < \frac{1}{\sqrt{1-(1-f)\beta\alpha_h}}, \tag{20}$$

which in the case of Boltzmann hot electrons ($\beta = 0$) goes back to Rufai *et al.*[16]

The nonlinear ion acoustic solitary waves propagating along the external magnetic field are investigated numerically for a plasma in which the dominant species are the energetic hot electrons. The typical parameters considered for the numerical evaluation are: density ratio, $f = N_{c0}/N_0$, temperature ratio $\tau = T_c/T_h$, Mach number $M$, nonthermal contributions $\alpha$, and wave propagating direction $\gamma = \cos\theta$, where $\theta$ is the propagating angle.

Table I shows the unnormalized values of the soliton velocity $V$, electric field $E$, soliton width $W$, and pulse duration $\tau^* = W/V$ for various values of $\alpha$ and the Mach number range $M$, respectively. It is seen from Table I that for increasing $\alpha$, the minimum Mach number $M_o$, the soliton velocity, width, and pulse duration tend to increase with $\alpha$, but the electric field decrease. Also, at the maximum Mach number range $M_1$, only the soliton velocity increases with $M_1$ and $\alpha$, but the maximum electric field, width and pulse duration decrease.

We plotted the Sagdeev potential $V(\psi, M)$ with normalized potential $\psi$ (i.e., amplitude and depth) for the above mentioned parameters for various values of plasma parameters. Figure 1 shows the Mach number ranges that support the existence of finite amplitude ion-acoustic solitons and double layers. The curves show the maximum and minimum Mach number values, starting at $f = N_{c0}/N_0 = 0.05$ to 1.0. The numerical values correspond to the maximum and minimum Mach number range in the analytical expression (20).

Figure 2, shows the variation of Sagdeev potential $V(\psi, M)$ versus the normalized electrostatic potential ($\psi$) for different values of Mach number $M$. The other fixed plasma parameters are: cool electron number density, $f = N_{c0}/N_0 = 0.1$, cool to hot electron temperature ratio, $\tau = 0.04$, angle of propagation, $\theta = 15°$ and nonthermal electron contribution $\alpha = 0.01$. The negative potential ion-acoustic soliton amplitude $\psi$ increases with increase in Mach number $M$. Here, the soliton structures can exist for subsonic and supersonic Mach number regime, whereas for cold ions and two Boltzmann electrons plasma the soliton solutions are possible only for $M < 1$.[16] The reason for this is that the presence of nonthermal electrons changes the dispersion characteristics of the ion acoustic mode. As a result of this, both the phase velocity of the mode (i.e., $M_0$) and the upper limit on Mach number,

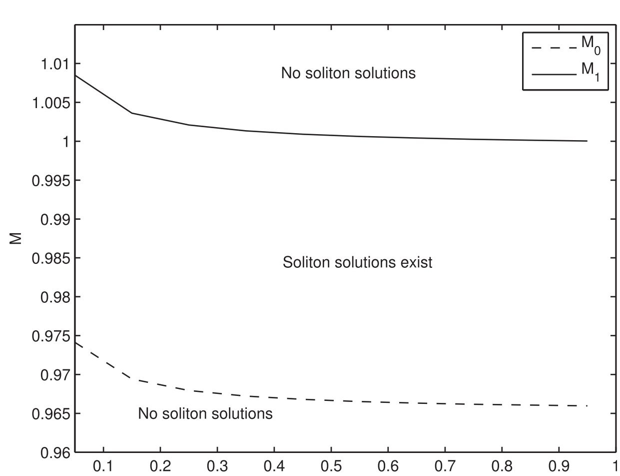


FIG. 1. Critical Mach number, $M_0$, and maximum Mach number, $M_1$, of ion-acoustic solitons shown as a function of the normalized cool electron number density $f = N_{c0}/N_0$, for $\tau = 0.04$, $\alpha = 0.01$, $\theta = 15°$.

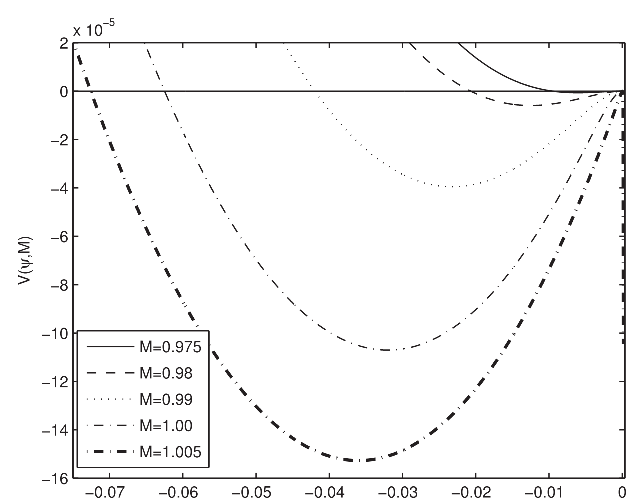


FIG. 2. Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential $\psi$, for $\tau = 0.04$, $f = 0.1$, $\alpha = 0.01$, $\theta = 15°$, and varying different values of $M$.

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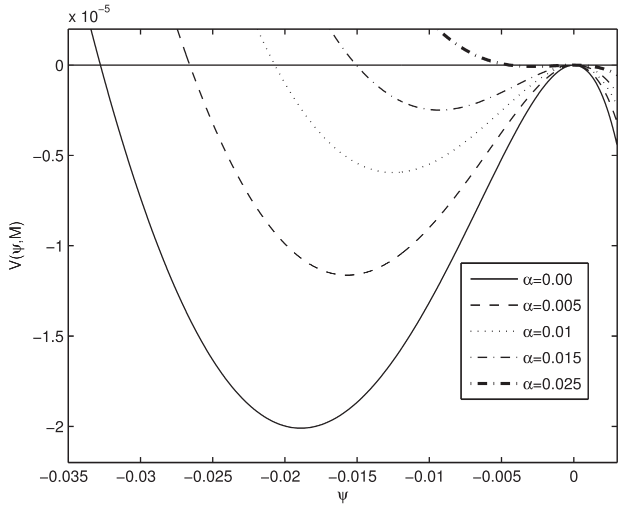


FIG. 3. Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential $\psi$, for $\tau = 0.04, f = 0.1, \theta = 15°, M = 0.98$ with varying nonthermal parameter $\alpha$.

$M_1$, are increased by equal amount (see Eqs. (17) and (18)). Thus, depending on the nonthermality of the electron distribution, i.e., $\beta$ value, $M_1$ can exceed 1, and the solitons can exist in supersonic regime. Further, it is interesting to note that the lower and upper limit on Mach numbers arrived at solving numerically Eq. (15) for the existence of the soliton matches with analytically obtained limit from Eq. (20).

In Figure 3, the curve shows the variation of Sagdeev potential $V(\psi, M)$ with real potential $\psi$ for different values of nonthermal electron contribution $\alpha$ for other fixed parameters namely, cool electron number density, $f = 0.1$, cool to hot electron temperature ratio, $\tau = 0.04$, angle of propagation, $\theta = 15°$, and Mach number, $M = 0.98$. The soliton amplitude decreases with increase in $\alpha$. This may be due to the increase of the critical Mach number, $M_0$, with the increase of $\alpha$. It is interesting to note that for $0 \leq \alpha \leq \infty$, the value of $\beta$ is limited in the range $0 \leq \beta \leq 4/3$. It has been pointed out elsewhere[63] that for $\alpha > 0.25$, i.e., $\beta > 4/7$ the nonthermal distribution starts to acquire ring component and particles become more energetic and might become beam unstable.

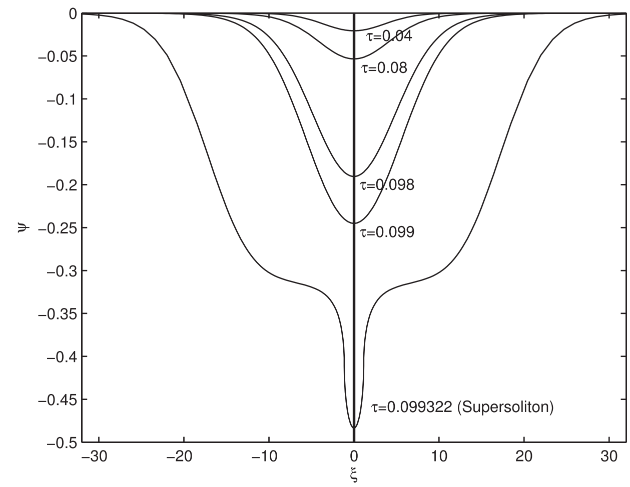


FIG. 5. The solitons and supersoliton potential $\psi$ versus $\xi$ for Figure 4 parameters.

However, in our case for the chosen parameters mentioned above, soliton solutions are not found for $\alpha > 0.025$.

Figure 4 shows the variation of Sagdeev potential $V(\psi, M)$ with the normalized potential $\psi$ for different values of the cool to hot electron temperature ratio, $\tau$ for Mach number $M = 0.98$, and other fixed parameters are the same as in Figure 2. The curves show that the ion-acoustic soliton amplitude increases with the increase in the cool to hot electron temperature ratio. It is interesting to note that at $\tau = 0.099322$ a supersoliton structure appear. The corresponding potential profiles of the solitons and supersoliton are plotted in Figure 5 which clearly shows that the distorted electrostatic potential profile for the supersolitons are distinctly different from the regular solitons.

Figure 6 shows the variation of Sagdeev potential $V(\psi, M)$ versus the normalized electrostatic potential $\psi$ for cool electron density ratio $f$ for cool to hot electron temperature $\tau = 0.04$ and other fixed parameters of Figure 4. As the cool electron density $f$ increases, the soliton amplitude $\psi$ increases, and soliton solutions do not exist beyond $f = 0.364$ which is greater than 0.35 the limit for the two Boltzmann

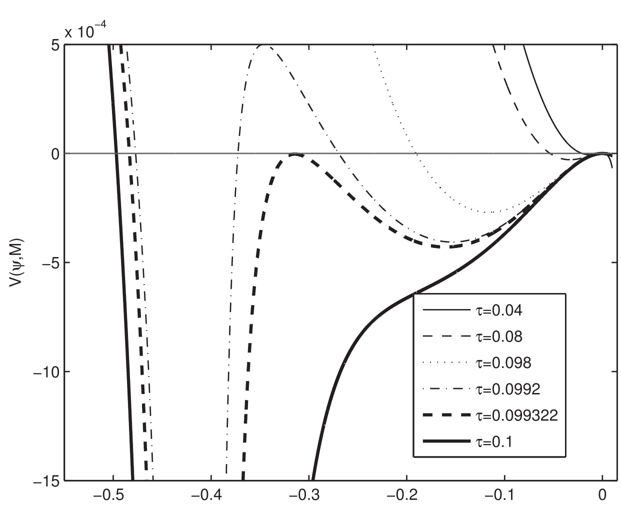


FIG. 4. Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential $\psi$, for $f = 0.1, \alpha = 0.01, \theta = 15°, M = 0.98$ for different values of $\tau$.

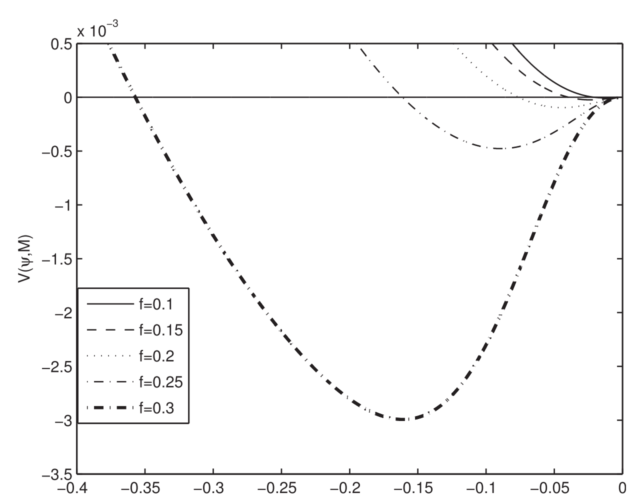


FIG. 6. Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential $\psi$, for $f = 0.1, \alpha = 0.01, \theta = 15°, M = 0.98$ for different values of $f$.

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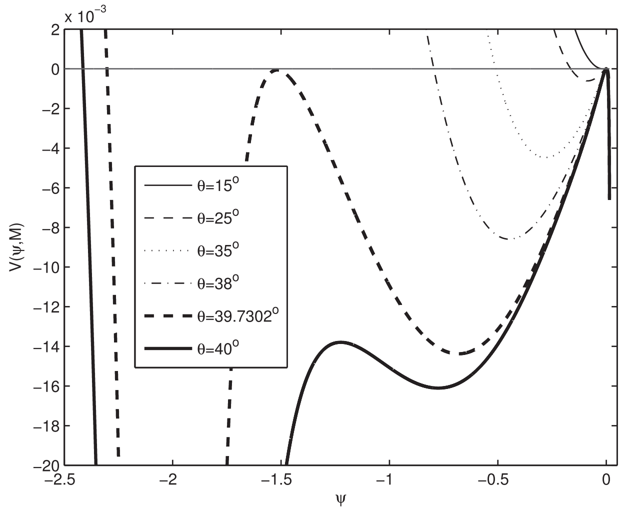


FIG. 7. Sagdeev potential, $V(\psi, M)$ vs normalized electrostatic potential $\psi$, for $f = 0.1$, $\alpha = 0.01$, $\tau = 0.04$, $M = 0.98$ for different values of $\theta$.

distribution electron and cold ion plasma.[16] Figure 7 shows the Sagdeev potential $V(\psi, M)$ with real electrostatic potential $\psi$ for different angles of propagation, $\theta$, (where $\cos\theta = \gamma$ the wave obliqueness) for $f = 0.1$ and other fixed parameters are the same as in Figure 5. The curves show that as the angle of propagation $\theta$ increases, the negative potential ion-acoustic soliton amplitude as well as depth of Sagdeev potential increases. It is interesting to observe that at $\theta = 39.7302°$ supersoliton appears. The corresponding solitons and supersoliton potential profiles are plotted in Figure 8. It must be pointed out that for the same set of plasma parameters for two Boltzmann electron and cold ions case,[16] only double layer structure appears at a lower angle of propagation, $\theta = 38.0425°$. The width and electrostatic potential amplitude of the supersoliton appear to be much larger than the normal solitons ($\theta = 38°$) as depicted in Figure 8. The magnetic field does not appear explicitly in the transformed set of equations (8)–(11) due to the normalization adopted here. However, the effect can be seen through the angle of propagation. The amplitude of the supersoliton in this case is larger than the one obtained in Figure 5.

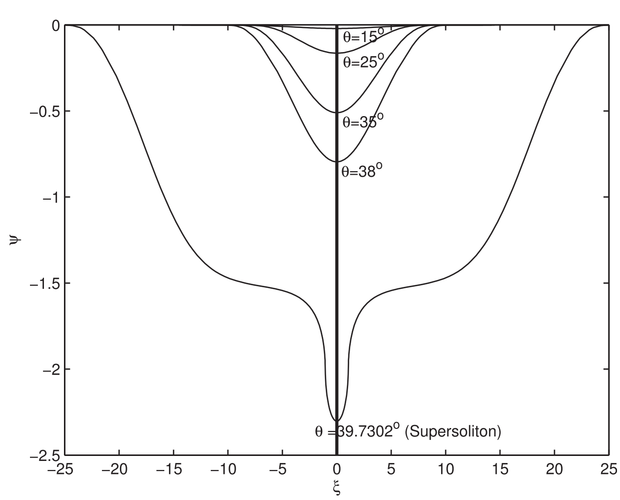


FIG. 8. The solitons and supersoliton potential $\psi$ versus $\xi$ for figure 7 parameters.

## IV. DISCUSSIONS AND CONCLUSIONS

We have studied the nonlinear propagation of arbitrary amplitude ion-acoustic soliton and double layer in a magnetized auroral plasma consisting of cold ions, Boltzmann distributed electron and nonthermal distribution of hot electron species using Sagdeev pseudo-potential approach. We adopted the Cairns proposed nonthermal distribution model[7] for the extra-energetic hot electron species. The most interesting result of the analysis is that the model supports the existence of ion acoustic supersolitons. These are the first reported theoretical results on ion-acoustic supersolitons in a magnetized three-component plasma consisting of Boltzmann electrons, hot non-thermal electrons and fluid ions. Furthermore, it is found that the inclusion of nonthermal hot electron extends the Mach number domain to the supersonic region. The model supports negative potential ion-acoustic soliton, double layers, and supersolitons, and they are found to exist in both subsonic and supersonic Mach number regime. On the other hand, for the case of unmagnetized plasma,[24] these negative potential nonlinear structures can appear only for Mach number range greater than 1 (i.e., supersonic Mach number region). Also, in the absence of nonthermal electrons, the nonlinear structures exist in the subsonic Mach number regime.[16] In the oblique propagation region, we notice that the waves amplitude grows higher than the case of the two Boltzmann distribution electrons plasma, due to the present of the extra-energetic hot electron. The nonthermal distribution model of the extra-energetic hot electrons are of common feature of space plasmas, the present study is applied to examine the low frequency nonlinear fluctuations in the mid-altitude region of the Earth's magnetosphere. The following parameters are taken from the Viking observations,[64] namely, $n_c = 0.2\,\mathrm{cm}^{-3}$, $n_h = 1.8\,\mathrm{cm}^{-3}$, $T_c = 1\,\mathrm{eV}$, $T_h = 26\,\mathrm{eV}$ which gives $T_{eff} \approx 7\,\mathrm{eV}$. The maximum electric field associated with the solitons for $M = 0.98$, $\theta = 25°$, $\alpha = 0.01$ is about 17.2 mV/m and soliton width, pulse duration, and speed comes out to be $\approx$ 208 m, 8.2 ms and 25.4 km/s, respectively, are within the range of values of these parameters as observed by Viking. Thus, the predictions of our model are in good agreement with the observations of solitary waves and double layer observed on the auroral field lines. Further, for the same parameters as mentioned earlier and at $\theta = 39.7302°$, the electric field amplitude of the supersolitons, width, pulse duration, and soliton speed are found to be 21 mV/m, 770 m, 30.3 ms and 25.4 km/s, respectively. It must be emphasized here that the supersoliton amplitude is much higher in the case of $\theta = 39.7302°$ (cf. Figure 8) than for $\tau = 0.099322$ (cf. Figure 5).

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