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Nighttime equatorial ionosphere: GPS scintillations and differential carrier phase fluctuations

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Abstract. The presence of scintillation-producing irregularities in the nighttime equatorial ionosphere, in the path of Global Positioning System (GPS) signals received at an equatorial station, causes dual-frequency measurements of the differential carrier phase of GPS L1 and L2 signals to have a contribution from phase scintillations on the two signals. Dual-frequency data for fluctuations in the total electron content (TEC) along the path of GPS signals to the equatorial station Ancon (1.5° dip), sampled at a rate of 1 Hz, are used to separate this contribution from the slower TEC variations. Rapid fluctuations in the differential carrier phase, usually on timescales < 100 s, which result from diffraction, are seen to follow the pattern of intensity scintillations on the L1 signal. Intensity scintillations are also related to the variations in TEC which arise from density fluctuations associated with ionospheric irregularities. An approximate version of the transport-of-intensity equation, based on a phase screen description of the irregularities, suggests that a quantitative measure of intensity scintillations may be provided by the derivative of rate of change of TEC index (DROTI), obtained from the second derivative of TEC. This equation also yields the dependence of the scaling factor between DROTI and S_4 on the Fresnel frequency. Comparison of DROTI computed from relative TEC data to corresponding S_4 indices indicates that there may be lesser uncertainity in a quantitative relation between the two than between the index ROTI, introduced in recent years, and S_4. Power spectral analysis of TEC fluctuations and simultaneous intensity scintillations on L1 signal, recorded at Ancon, does not indicate any simple dependence of the scaling factor between DROTI and S_4 on the spectral characteristics.

1. Introduction

Recognition of the radio wave signals transmitted from Global Positioning System (GPS) satellites as useful tools for probing the Earth's ionosphere has led to a number of such studies in recent years [*Wanninger*, 1993; *Aarons et al.*, 1996; *Kelley et al.*, 1996; *Weber et al.*, 1996; *Musman et al.*, 1997; *Pi et al.*, 1997]. At the frequencies of the GPS signals the electron-density -dependent refractive index of the ionosphere introduces range errors and range rate errors which, to lowest order, are due to the change in optical path length introduced by the total electron content (TEC) of the intervening ionosphere [*Klobuchar*, 1996]. For an undisturbed ionosphere where the electron density does not vary significantly over short ($\leq$ 1 km) scale lengths, this is the only effect the ionosphere has on GPS signals. However, the ionospheres in the equatorial and polar cap/auroral regions often exhibit departures from such a state when electron density variations in them span a large range of scale sizes, extending well into the subkilometer range at the small-scale end of the spectrum [*Kelley*, 1989]. A number of examples of scintillations in the intensity of recorded GPS signals have

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been reported in recent literature [*Weber et al.*, 1996; *Beach*, 1998]. The present study is concerned with the relationships between rapid fluctuations in intensity and differential carrier phase due to scintillations on the GPS signals and slower changes in differential carrier phase due to TEC variations.

The GPS signals provide a unique opportunity to study simultaneously the short-scale-length variations in TEC along the signal path in the presence of ionospheric irregularities and the scintillations associated with these irregularities. In the phase screen description of ionospheric irregularities it is the variation, in the plane of the phase screen, of the TEC along the signal path, which introduces the initial phase perturbation that gives rise to the scintillations. Thus GPS data may be used to clear up some ambiguity that exists regarding the distinction between the initial phase perturbation and phase scintillations which result from further propagation. Whereas the initial phase perturbation may be simply retraced to variations in TEC along the signal path, phase scintillations do not relate to density fluctuations in the same manner. Starting with a simple theoretical description of phase scintillations in section 2, dual-frequency observations of GPS signals summarized in section 3 are used to delineate, in section 4, the contribution of phase scintillations to the measured "TEC fluctuations." A measure of TEC variations which may be associated with scintillations on GPS signals is the rate of change of TEC index (ROTI) introduced by *Pi et al.* [1997]. While on many occasions it is able to track intensity scintillations, as measured by the S_4 index, a quantitative relationship between the two is lacking. In an attempt to bridge this gap, in section 5 a new index designated derivative of rate of change of TEC index (DROTI), which may be derived from 30 s GPS TEC fluctuation data, is introduced on the basis of theoretical considerations. This index may be directly scaled to S_4 using a scaling factor which is determined by the Fresnel frequency. Investigation of the spectral characteristics of intensity scintillations and fluctuations in TEC along the signal path does not shed any new light on the relationship between DROTI and S_4. On the basis of results described in this paper it is concluded in section 6 that the applicability of the phase screen approximation, on which indices such as ROTI and DROTI are based, is an important factor in their use as a measure of scintillation-producing irregularities.

2. Theoretical Description of Phase Scintillations

A theoretical description of phase scintillations based on the phase screen approximation is briefly recounted here in order to define phase scintillations as distinct from the phase variations imposed on a radio wave by electron density irregularities encountered in its path. More detailed accounts of various theories of scintillation may be found in reviews by *Yeh and Liu* [1982] and *Bhattacharyya et al.* [1992] and references therein. In the phase screen approximation the irregularities are assumed to be confined to a relatively thin layer such that the amplitude of the incident radio wave remains unchanged on emerging from the layer and only an irregular phase perturbation is imposed on the radio wave on its passage through the irregularities. Further propagation through free space to the plane of the receiver results in the buildup of spatial variations in amplitude and phase. A relative motion between the density irregularities and the signal path results in fluctuations of amplitude and phase of the signal recorded by a stationary receiver on the ground.

For a simple theoretical picture of the spatial variations that would appear in the reception plane a model phase screen is assumed to be located in the $x-y$ plane at $z=0$, while an incident plane wave propagates through it in the z direction to a receiver at a distance z from the screen. It is assumed that there is no variation in electron density along the y axis, and the resultant one-dimensional phase screen imposes a phase perturbation $\phi(x)$ on the incident radio wave, which is determined by the variation $\Delta N_T(x)$ of the TEC along the signal path through the irregularity layer:

$$\phi(x) = -\lambda r_e \Delta N_T(x), \tag{1}$$

where r_e is the classical electron radius ($= 2.8\times10^{-15}$ m). In this situation the complex amplitude $u(x,z)$ of the wave at a point (x,y,z) in the reception plane will be given by the Fresnel diffraction result under a forward scattering assumption [*Ratcliffe*, 1956]:

$$u(x,z) = \sqrt{\frac{i}{\lambda z}} \int_{-\infty}^{\infty} u(x',0) \exp\left[-i\pi(x-x')^2/\lambda z\right] dx', \tag{2}$$

where λ is the wavelength of the incident wave and $u(x,0)$ is the complex amplitude of the wave immediately after it emerges from the phase screen

$$u(x,0) = e^{-i\phi(x)}. \tag{3}$$

The role of Fresnel diffraction in generating scintillations may be better understood by transforming to the Fourier domain. A Fourier transform of (2) yields the relation

$$u_F(q,z) = u_F(q,0)e^{-i\pi\lambda z q^2} \tag{4}$$

between $u_F(q,z)$, the Fourier transform of $u(x,z)$, and $u_F(q,0)$, the Fourier transform of $u(x,0)$. It is seen that if $u(x,0)$ has variations only on scale lengths much larger than the Fresnel scale size $d_F (= \sqrt{2\lambda z})$, $u_F(q,0)$ will be nonvanishing for only $q^2 \ll (2\lambda z)^{-1}$, such that

$$u_F(q,z) \approx u_F(q,0). \tag{5}$$

In this case, just the original phase variation $\phi(x)$, due to the large- scale TEC variations, will be recorded by a receiver as the irregularities drift across the signal path. When shorter-scale-length density variations are also present along with the large-scale-length ones, as in the case of equatorial spread F (ESF) irregularities, the effect of Fresnel diffraction is seen through the exponential term in (4). Thus fluctuations in phase of the radio wave received on the ground cover the whole range of scale lengths present in TEC variations as well as the diffraction effects they cause, while amplitude scintillations arise because of irregularities of scale sizes of the order of the Fresnel dimension or shorter. It is, therefore, expected that the pattern of phase scintillations, when separated from the phase trend in the carrier phase recorded by a ground receiver, should follow that of amplitude scintillations.

3. Dual-Frequency Observations of GPS Signals

Simultaneous observations of both L1 (1.57542 GHz) and L2 (1.2276 GHz) signals transmitted from GPS satellites have provided information on TEC variations which have beeen used in studies of ionospheric irregularities. TEC fluctuations obtained at intervals of 15 s from dual-frequency GPS receivers set up at a single equatorial station were used in the study by *Musman et al.* [1997], while 30 s samples of TEC derived from the International GPS Service for Geodynamics (IGS) network observations constituted the database for results reported by *Aarons et al.* [1997] and *Pi et al.* [1997]. For a GPS L1 signal, if it is assumed that the effect of ionospheric irregularities may be represented by that of an equivalent phase screen located at a distance of 350 km from the ground receiver, the Fresnel scale size d_F is ~360 m. In a situation where the eastward motion of the ionospheric penetration point for the signal path from a particular satellite nearly matches the eastward drift of the ionospheric irregularities themselves, the irregularity drift velocity $\vec{v}$ relative to the signal path may be small enough to yield a Fresnel frequency ν_F $(= v/d_F)$ smaller than 0.1 Hz. Generally, a sampling interval much shorter than 15 s or 30 s used in the earlier studies is required to quantify the contribution of scintillations to the observations.

In a campaign carried out in April 1997 by the Cornell University group, an Allen Osborne ICS-4000Z dual-frequency receiver was deployed at the equatorial station Ancon (11.77°S, 77.15°W, 1.5° dip) to collect data at a higher sampling rate of 1 Hz, specifically for studying ionospheric effects on GPS signals [*Beach*, 1998]. A dual-frequency receiver records the number of cycles of the carrier of each of the GPS frequencies, starting at the time the receiver locks onto the signal. The accuracy of measurements is usually 0.01 cycles or better [*Hofman-Wellenhof et al.*, 1994]. The receiver used in the April 1997 campaign was adapted to log pseudorange TEC and phase TEC directly. The phase measurements do not yield absolute TEC information, but they provide information about variations in TEC along the signal path as the satellite moves, after the receiver locks onto the signals. As will be seen in section 4, the TEC fluctuation data are a combination of TEC variations caused by the presence of ionospheric irregularities in the path of the radio waves as well as short-timescale fluctuations associated with phase scintillations on L1 and L2 signals. Carrier-to-noise ratio records logged by the receiver give a measure of amplitude scintillations on the L1 signal. A sampling interval of 1 s is inadequate for obtaining a complete picture of ionospheric scintillations, for example, through a study of their power spectral features. A single-frequency (L1) GPS scintillation monitor which could record data at a rate of 50 Hz was developed at Cornell University [*Beach*, 1998] and deployed at Ancon alongside the dual-frequency receiver during the April 1997 campaign. In this campaign, TEC fluctuations and scintillations were simultaneously observed on three nights: April 8-9, April 10-11, and April 16-17, on signals transmitted by three satellites, PRN 5, 9, and 26. The tracks followed by the ionospheric penetration points of the signal paths at an altitude of 350 km, around the time of the scintillation event of April

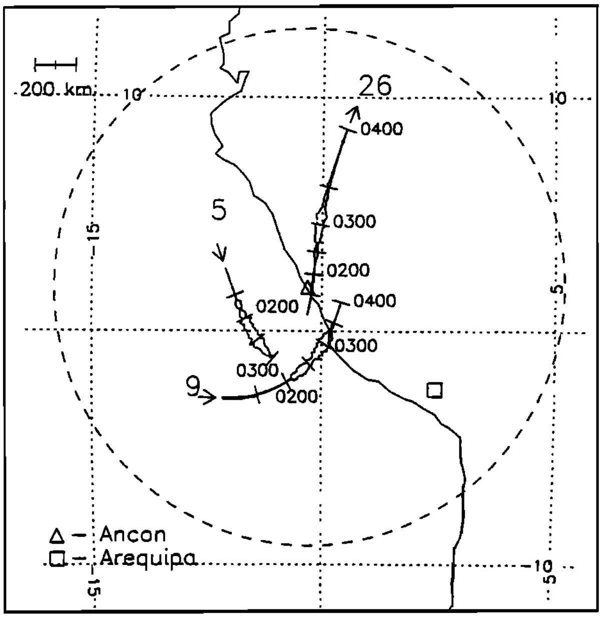


Figure 1. Tracks followed by the ionospheric penetration points, at an altitude of 350 km, of the signal paths from GPS satellites PRN 5, PRN 9, and PRN 26 to Ancon, on April 9, 1997. The time marks are in universal time (UT). Geomagnetic coordinates are given on the grid and the dashed circle represents the 350 km altitude visibility limit at Ancon for an elevation of 10°. Thickness of a trace indicates strength of scintillation activity. [after *Beach and Kintner*, 1999].

9, are shown in Figure 1 [Beach and Kintner, 1999]. The times marked in Figure 1 are in universal time (UT) which is local time plus 5 hours. The tracks followed by the ionospheric penetration points of the signal paths from these satellites, during the other two nights when scintillations were observed, are similar to those shown in Figure 1.

4. Contribution of Phase Scintillations

The respective numbers of cycles, ϕ_1 and ϕ_2, including fractional cycles, of L1 and L2 carriers, recorded by a dual-frequency receiver after it locks onto the signal, yield the relative TEC along the signal path, in TEC units (1 TECU = 10^{16} electrons/m^2), uncertain to an additive constant:

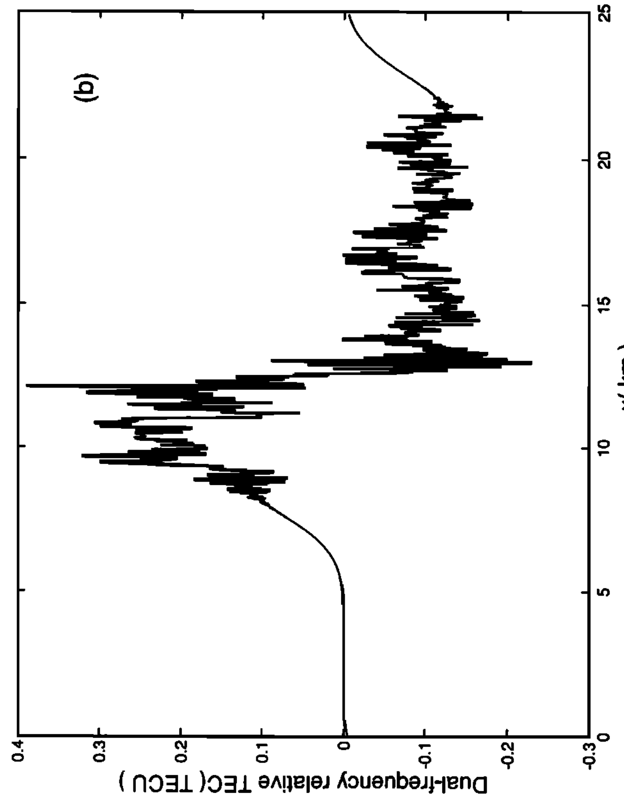
$$N_T = 1.17(\beta_1\phi_1 + \beta_2\phi_2). \tag{6}$$

Here β_1 and β_2 are determined by the frequencies f_1 and f_2, respectively, of the two signals:

$$\beta_1 = \frac{f_2^2}{f_1^2 - f_2^2}; \qquad \beta_2 = \frac{-f_1 f_2}{f_1^2 - f_2^2}. \tag{7}$$

In the presence of electron density irregularities in the ionosphere, there may be fluctuations in the amplitudes and phases of both L1 and L2 signals. The magnitude of the contribution of phase scintillations on L1 and L2 signals to the measured relative TEC may be approximately estimated by using a phase screen model for the irregularities as described in section 2. For a simple geometry, where a signal is perpendicularly incident on a one-dimensional phase screen, the complex amplitude of the signal in the plane of the receiver is given by (2). Results have been derived by *Rino* [1979] for plane waves obliquely incident on a phase screen. In the case of GPS satellites the situation is more complex since the geometry of the signal path vis-à-vis the irregularities keeps changing in the course of a satellite's orbit. In order to simply demonstrate that the magnitude of the contribution of phase scintillations to the relative TEC derived from dual-frequency observations of GPS signals may not be negligible, complications due to changing geometry shall be ignored at present.

The model phase screen is based in part on in situ measurements of plasma density in the equatorial F region, by a retarding potential analyzer on board the Atmospheric Explorer E (AE-E) satellite [*Basu et al.*, 1980]. Segments of such data have been used by *Beach* [1998] to construct equivalent phase screens for studying intensity scintillations on a GPS L1 signal. In the present study, a segment of AE-E measurements made at ~22.13 magnetic local time (MLT), within a region of depleted electron density in the equatorial ionosphere, has been used to construct an equivalent phase screen at an altitude of 350 km above the plane of the receiver, by assuming that this type of density variation exists over an altitude range extending from 340 to 360 km. The variations in TEC along the signal path, $\Delta N_T(x)$, which impose phase perturbations on the incident radio wave, are shown in Figure 2a. Artificial smooth transition zones have been appended at both ends for the computation of Fourier transforms without significant edge effects, in the calculation of the complex amplitude of the radio wave when it reaches the plane of the receiver (z=350 km). In this plane the phases ϕ_1 and ϕ_2 of signals L1 and L2, respectively, which are superpositions of the basic phase variations imposed on the signals by the TEC variations $\Delta N_T(x)$ as given by (1) and phase



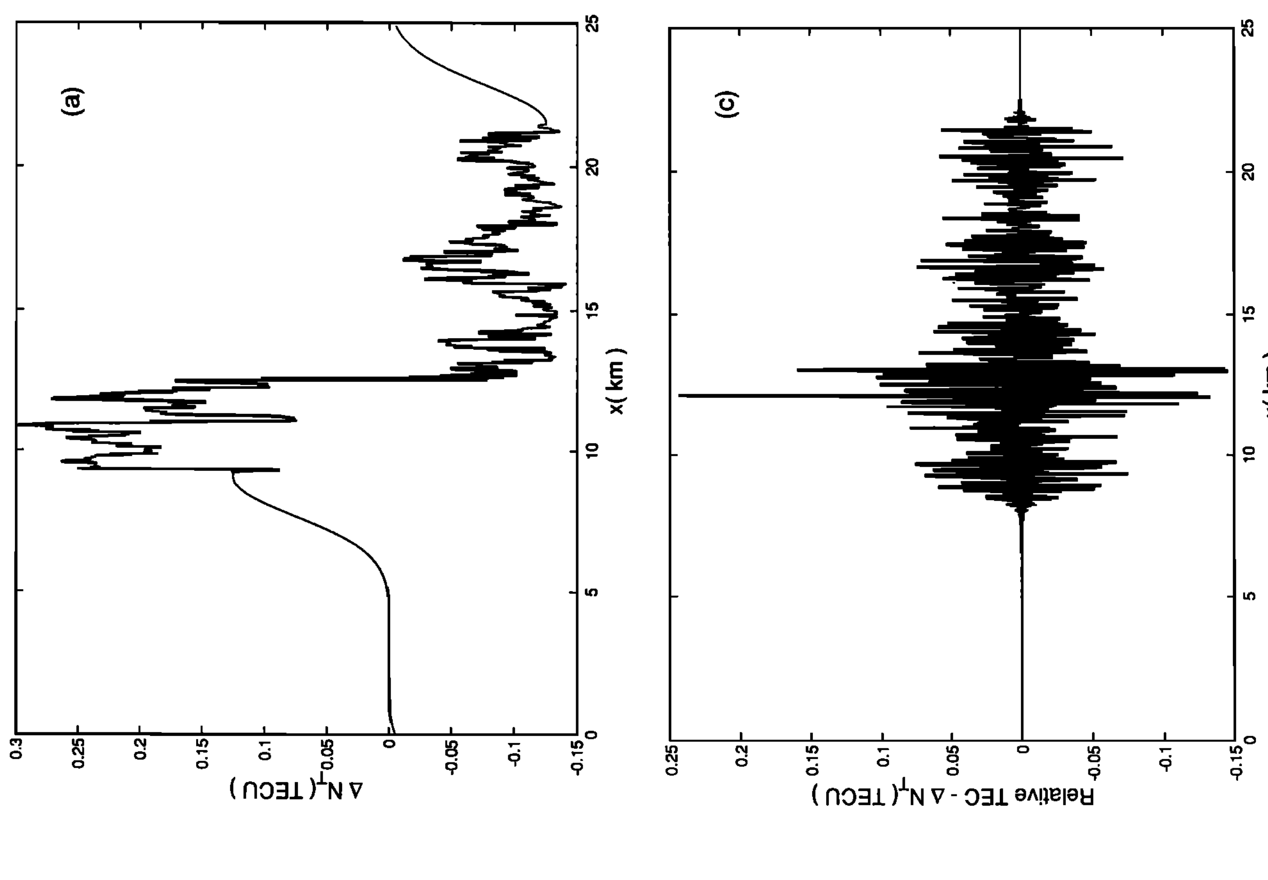


Figure 2. (a) Variation, ΔN_T, along a transverse (x) direction, in TEC along the signal path, for a model irregularity layer of 20 km thickness. (b) Variation in relative TEC derived from dual-frequency observations of L1 and L2 signals after propagation through the model irregularity layer. (c) Contribution of phase scintillations on L1 and L2 signals to Figure 2b.

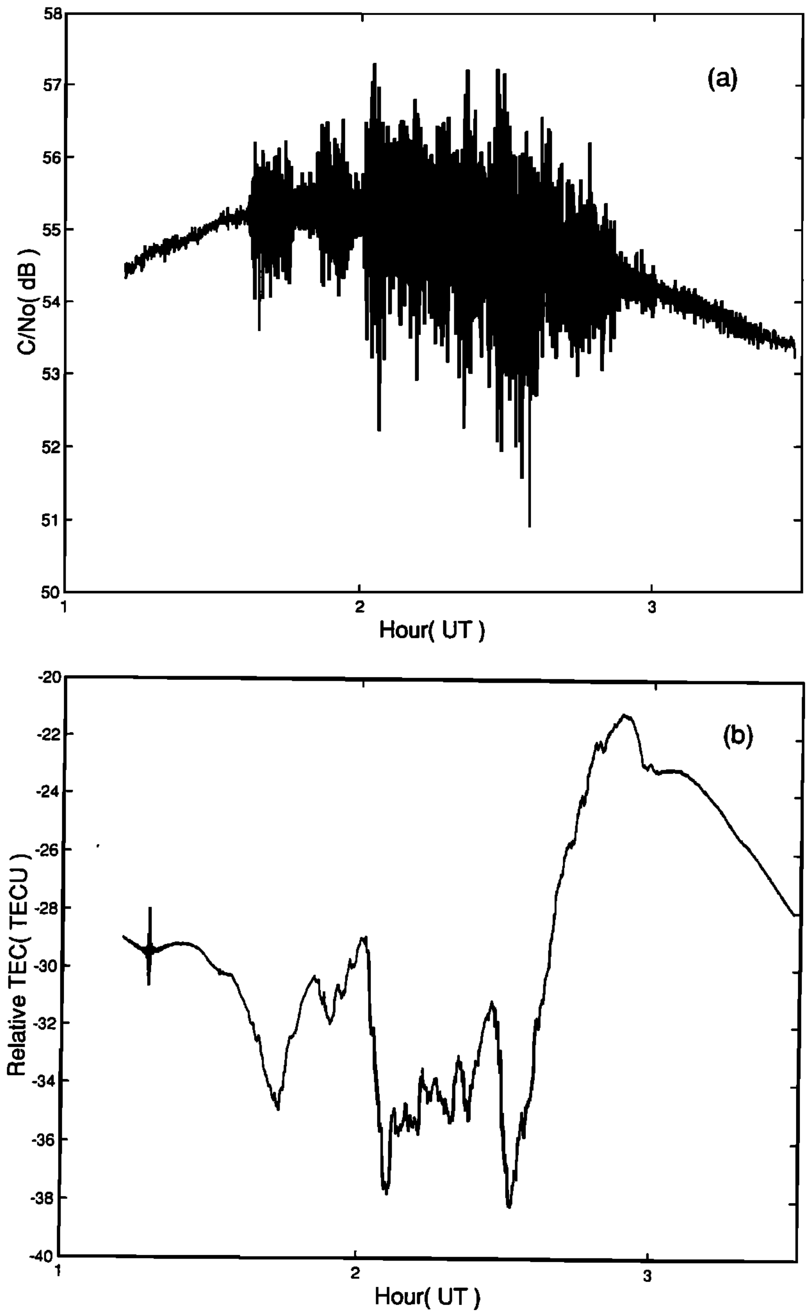


Figure 3. (a) Intensity scintillations on the L1 signal transmitted from PRN 5 to Ancon, and (b) simultaneous variations in TEC along the same signal path, measured using a dual-frequency receiver on April 9, 1997.

scintillations, are calculated using (2). Combination of ϕ_1 and ϕ_2 in accordance with (6) now yields the relative TEC accompanied by the effects of scintillations on L1 and L2 as seen in Figure 2b. In this model calculation it is possible to derive the contribution of phase scintillations exactly by subtracting $\Delta N_T(x)$ shown in Figure 2a from the relative TEC shown in Figure 2b. This contribution, displayed in Figure 2c, is found to follow the pattern of intensity scintillations on either of the signals.

An example of intensity scintillations on the L1 signal transmitted from PRN 5 to Ancon on April 9, 1997, is seen in a plot, in Figure 3a, of the carrier-to-noise ratio (C/No) recorded by a dual-frequency receiver. Simultaneous variations in the relative TEC along the same signal path, measured by the dual-frequency receiver, are displayed in Figure 3b. The elevation angle of the signal path was in the range 40°-44° for the duration of this scintillation event. Estimation of the contribution of phase scintillations to the differential carrier phase data derived from dual-frequency observations requires the separation of this contribution and the slower TEC variations by the application of an appropriate filter [*Fremouw et al.*, 1978; *Bhattacharyya and Rastogi*, 1986]. A low-pass, six-pole Butterworth filter, with a 3 dB cutoff of 0.03 Hz, was applied to the intensity data to isolate the trend in the data which arises as a result of satellite motion. Choice of a cutoff frequency is dictated by the requirement that the computed S_4 index, which is the standard deviation of the normalized detrended intensity, is not significantly altered by excluding the trend and is hence determined by the Fresnel frequency, which depends on the slant distance along the signal path from the receiver to the irregularity layer as well as the transverse drift speed of the irregularities relative to the signal path. The cutoff frequency is critical for defining phase scintillations since the measured phase fluctuations encompass all the scales present in the initial phase perturbation imposed on the radio wave by the irregular variations in ionospheric electron density, as well as phase scintillations. The TEC fluctuation data are detrended with the same cutoff frequency to estimate the contribution, $\Delta\phi$, of phase scintillations. Values of $\sigma_{\Delta\phi}$, the standard deviation of detrended "TEC fluctuations" or the differential carrier phase fluctuations, are computed along with the corresponding S_4 values at 1 min intervals. For the data shown in Figure 3, values of these indices plotted in Figure 4a follow nearly the same pattern during the scintillation event.

The index ROT, introduced in earlier papers [*Wanninger*, 1993; *Doherty et al.*, 1994; *Aaron et al.*, 1996; *Pi et al.*, 1997] for measuring GPS differential phase fluctuations, is computed from the time rate of change of TEC, obtained at 30 s intervals from dual-frequency measurements on GPS signals. In order to identify irregularity structures, *Pi et al.* [1997] introduced ROTI based on the standard deviation of ROT, computed at 5 min intervals. *Beach and Kintner* [1999] have compared ROTI with S_4 using the three nights of observed scintillations on GPS signals recorded at Ancon during the April 1997 campaign. These case studies demonstrate that ROTI was adequate for tracking qualitatively the irregularities which produced intensity scintillations on the L1 signal received at Ancon on April 9, 1997, but failed to do so consistently on the other two nights. Figure 4b shows that for data from PRN 26 on April 17, 1997, $\sigma_{\Delta\phi}$, as defined in the present paper, follows the pattern of increase of the corresponding values of S_4. The reason for this can be found in the decrease in elevation angle of the signal path for this satellite from ~40° to 15° during the period covered in Figure 4b. Data pertaining to elevation angles less than ~15° have not been considered as there might be contamination from multipath propagation. During the period under consideration, motion of PRN 26 is nearly along the magnetic meridian, along which the irregularities tend to be aligned, such that the length of the part of the signal path from PRN 26 to Ancon, which lies within a field-aligned irregularity, is large for low elevation angles. This would result in the development of significant amplitude fluctuations while the waves are traversing the layer of irregularities. The density gradients identified using ROTI would be effective in tracking the amplitude scintillations provided the irregularity layer was sufficiently thin to be adequately described by a phase screen approximation, which fails when there are significant amplitude fluctuations within the irregularity layer. Phase scintillations, on the other hand, track the intensity scintillations through a thick layer of irregularities as well.

5. Relative TEC-Based Index for Scintillation Effects

In this section, the possibility of deriving a measure for density gradients from the relative TEC data, which would allow a quantitative comparison with the S_4 index, is explored. Once again, as in the case of ROTI, this index will be based on a phase screen description of the irregularities encountered by the signal. The intensity and phase of a radio wave, after it emerges from a phase screen and propagates to the receiver through free space, satisfy the so-called transport-of-intensity equation (TIE) [*Teague*, 1983]. For a one-dimensional phase screen an approximate version of the TIE relates the intensity I measured

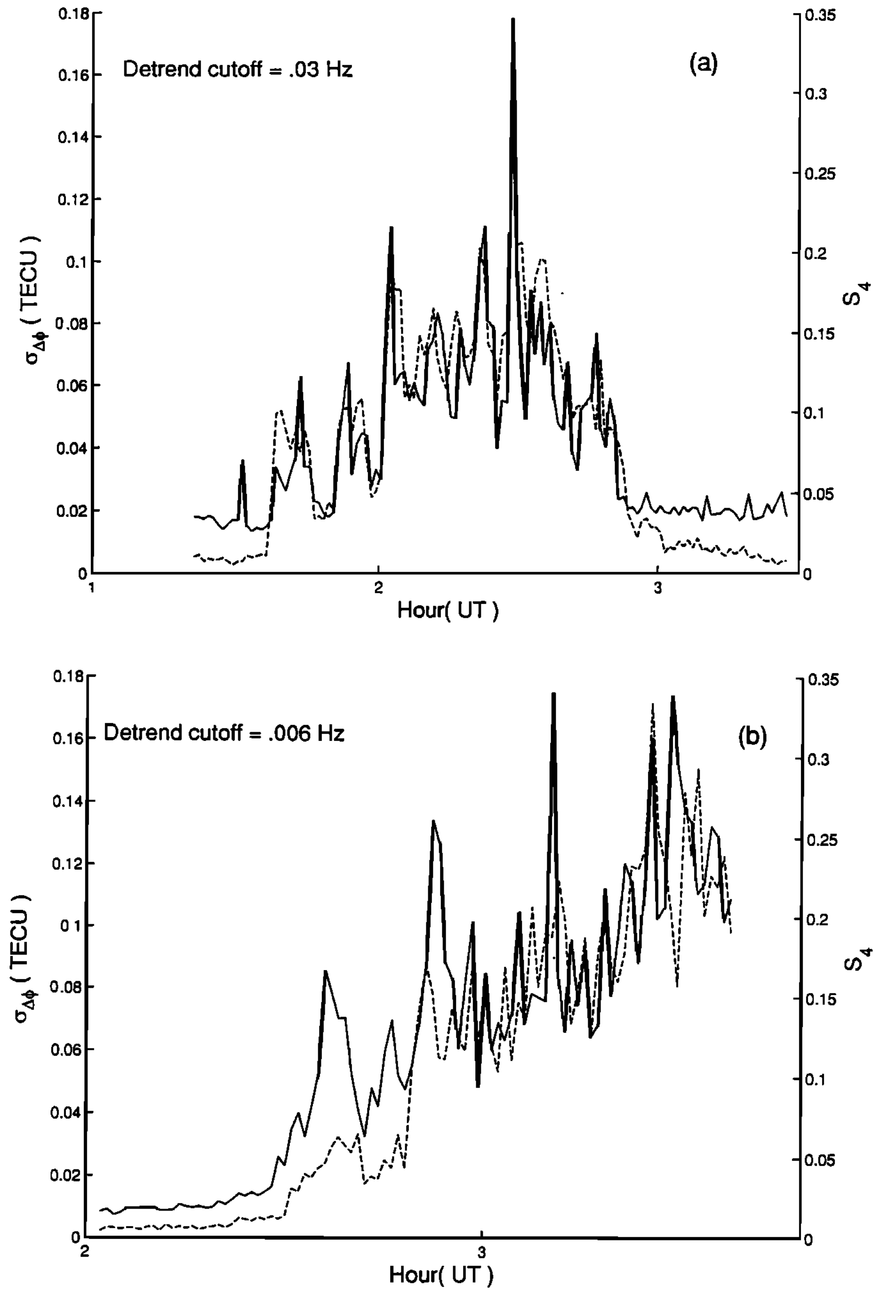


Figure 4. The $\sigma_{\Delta\phi}$ (solid line) and S_4 (dashed line), computed from 1 Hz, dual-frequency, detrended TEC fluctuation data and intensity scintillation data on L1, respectively, recorded at Ancon for (a) PRN 5 on April 9, 1997, and (b) PRN 26 on April 17, 1997.

by a receiver to the phase variations $\phi(x)$ that are present when the wave just emerges from the screen [*Bhattacharyya*, 1999]:

$$\frac{d^2\phi}{dx^2} = \frac{2\pi}{\lambda z}\left[1 - \frac{I(x,z)}{I_0}\right], \tag{8}$$

where z is the distance of the receiver from the phase screen along the signal path which is assumed to be perpendicular to the phase screen and I_0 is the uniform intensity of the incident wave before it encounters the irregularities. Phase variations $\phi(x)$ imposed by the screen are related to variations $\Delta N_T(x)$ in TEC along the signal path according to (1). As noted earlier, spatial variations along the x direction are converted to temporal variations at the site of the receiver because of relative motion of the irregularities with respect to the signal path. Thus, representing the x component of this motion by v and ignoring the changes in v with time over relatively short periods during the course of the scintillation event, (8) is expressed as

$$\frac{d^2}{dt^2}\Delta N_T = -\frac{2\pi}{\lambda r_e}\frac{v^2}{\lambda z}\left[1 - \frac{I(t,z)}{I_0}\right]. \quad (9)$$

This equation suggests that an index based on the standard deviation of $d^2\Delta N_T/dt^2$ rather than $d\Delta N_T/dt$ may be a more natural choice as a measure of scintillation activity. In fact, the S_4 index is the standard deviation of the normalized intensity I/I_0 which appears on the right-hand side of (9). This argument forms the basis for the introduction of a new index DROTI, which may be computed from GPS data for the relative TEC variation in the same manner as ROTI, except that the second time derivative of the variation in TEC would now be used. One advantage of using DROTI is that a quantitative comparison with the S_4 index would now be possible, if the unknown factor v^2/z is estimated from the Fresnel frequency $\nu_F = v/\sqrt{(2\lambda z)}$, where the power spectral density of intensity scintillations, caused by ESF irregularities, displays a broad maximum. For signals transmitted from GPS satellites the factor v^2/z may change considerably with satellite motion during a scintillation event lasting an hour or longer.

In order to simulate the IGS data which have been used for earlier computations of ROTI [*Pi et al.*, 1997], for every interval of 30 s, the last ten samples of the relative TEC data, which are derived from dual- frequency measurements of the differential carrier phase at the rate of 1 Hz, are averaged to obtain one value for the simulated data. The second derivative, $d^2\Delta N_T/dt^2$, is computed from this 30 s data, and its standard deviation calculated for a 5 min interval yields DROTI for the interval. The corresponding intensity scintillation data for the L1 signal are used to compute the S_4 index for the same 5 min intervals, and ν_F obtained from a power spectrum of the intensity data yields an estimate for $v^2/\lambda z$ during the time interval selected for computation of the spectrum. According to (9) the scaling between DROTI and the corresponding S_4 index should be DROTI (TECU min^{-2}) $= 8.5 \times 10^3 \nu_F^2 S_4$ when ν_F is in Hz. However, the data analyzed for April 9, 1997, indicate a DROTI which is smaller by a factor of ~5, yielding a scale factor of $\approx 1.6 \times 10^3 \nu_F^2$ between DROTI (TECU min^{-2}) and S_4. Values of S_4 indices are derived by scaling DROTI, calculated from the relative TEC variation data for the signal paths from PRN 5 to Ancon, on the nights of April 9 and 17, 1997, with the above factor. Another set of S_4 indices are derived from ROTI, which are obtained from the same relative TEC variation data, using an arbitrary scaling between ROTI and S_4: ROTI (TECU min^{-1}) $= 2.5 S_4$. The S_4 indices computed from intensity scintillations on the L1 signal from PRN 5 to Ancon for the same two nights are compared with the respective S_4 indices derived from ROTI and DROTI in Figures 5a and 5b. Elevation angle for the signal paths varied between 40° and 44° and 36° and 43°, respectively, during the two scintillation events. Figure 5 shows that the Fresnel frequency dependence of the scaling factor between DROTI and S_4 is able to accomodate to some extent the different drifts of the irregularities relative to the signal path on different nights.

Depending on the relative drift speed between the irregularities and the signal path, a sampling interval of 30 s implies that the scale lengths associated with fluctuations in TEC which contribute to ROTI or DROTI may be much larger than those which contribute the most to intensity scintillations.To test the conjecture that a shorter sampling interval may yield a better agreement between S_4 derived from intensity scintillation data and that derived from DROTI, 10 s averages of the 1 Hz relative TEC data are used to compute DROTI at 5 min intervals. The scaling factor between DROTI (TECU min^{-2}) and S_4 is now closer to the theoretical one, being $\approx 3.4 \times 10^3 \nu_F^2$ for the data of April 9 and $\approx 9.4 \times 10^3 \nu_F^2$ for the data of April 11 and 17. S_4 indices computed from intensity scintillations on L1 and S_4 indices derived using the above scale factors with DROTI are compared for PRN 5, PRN 9, and PRN 26 on April 9 and PRN 5 on April 17 in Figures 6a, 6b, 6c, and 6d, respectively. The agreement is closest for PRN 5 on all 3 days, since there is least variation in the elevation angle for this satellite during the scintillation events. For PRN 26 the S_4 index tends to diverge increasingly from the value predicted by DROTI as its elevation angle decreases. The closer agreement with theory when the sampling interval is reduced

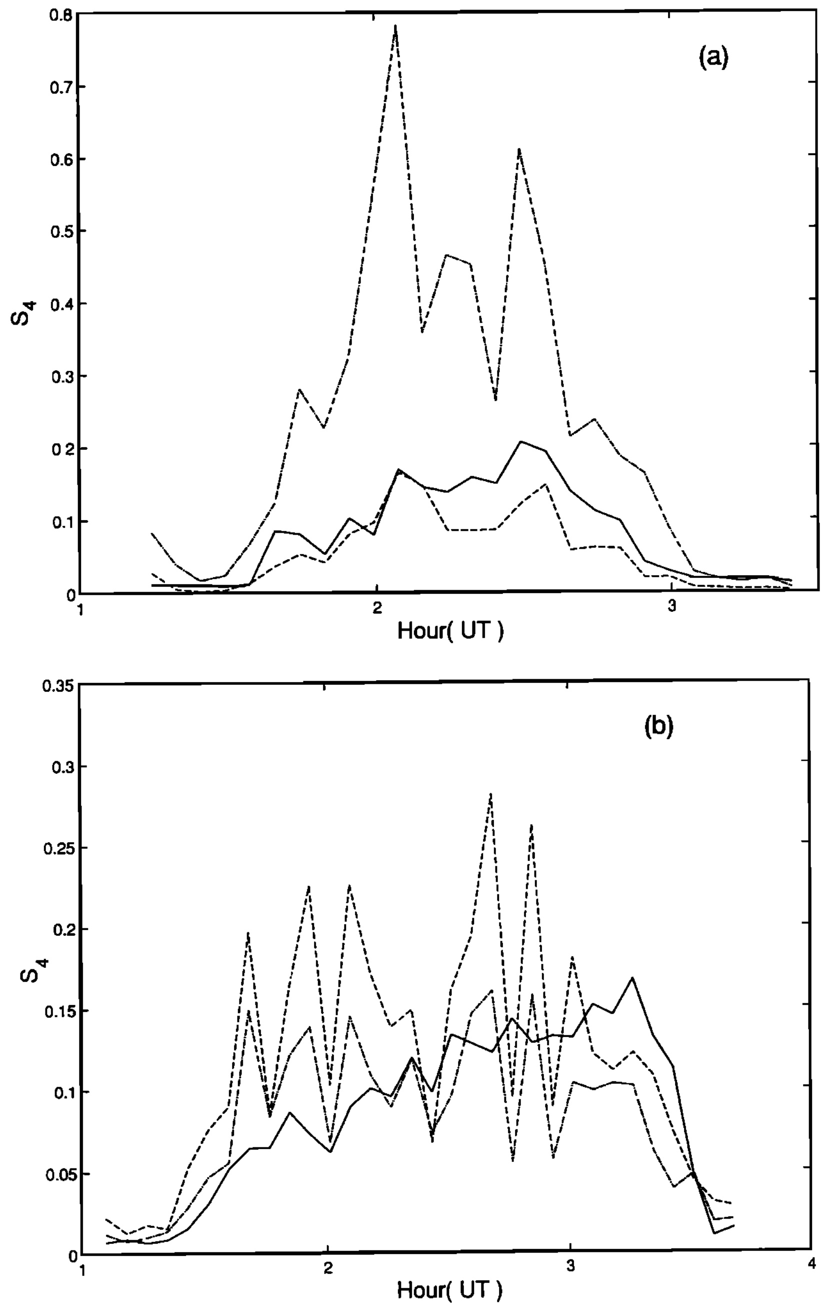


Figure 5. S_4 indices: computed from intensity scintillations on L1 (solid line), derived from DROTI ($= 1.6 \times 10^3 \nu_F^2 S_4$) (dashed line) with DROTI in units of TECU/min^2 and ν_F given in hertz, and derived from ROTI ($= 2.5 S_4$) (dash-dotted line) with ROTI in units of TECU/min. Figures 5a and 5b are for the same satellite, PRN 5, for two different nights with different ν_F: (a) April 9, 1997, $\nu_F = 0.126$ Hz; and (b) April 17, 1997, $\nu_F = 0.053$ Hz.

from 30 s to 10 s indicates involvement of shorter-scale-length irregularities in the occurrence of intensity scintillations. Hence the scaling factor may also depend on the relationship between the characteristics of the spectra of TEC fluctuations and intensity scintillations, that cannot be recovered from (9), which holds only for scale lengths $d >> (\lambda z \phi / 2\pi)^{1/2}$ [*Bhattacharyya*, 1999].

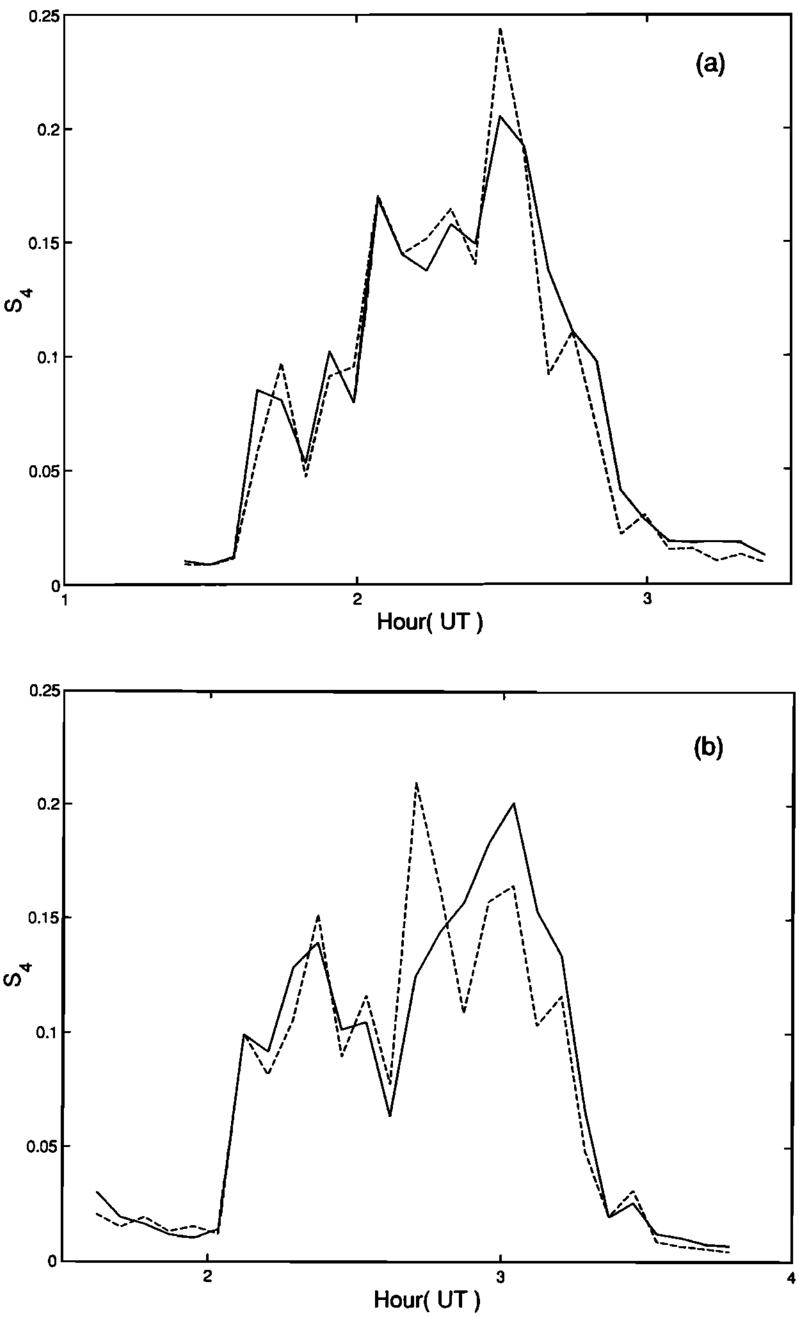


Figure 6. S_4 index computed from intensity scintillations on L1 (solid line) and S_4 derived from DROTI ($= \alpha \nu_F^2 S_4$) (dashed line) calculated using simulated 10 s data for TEC variations. Figures 6a-6c are for April 9, 1997, with $\alpha = 3.4 \times 10^3$ and data from all three satellites: (a) PRN 5, (b) PRN 9, and (c) PRN 26. Figure 6d is obtained with data from PRN 5 on April 17, 1997, using $\alpha = 9.4 \times 10^3$.

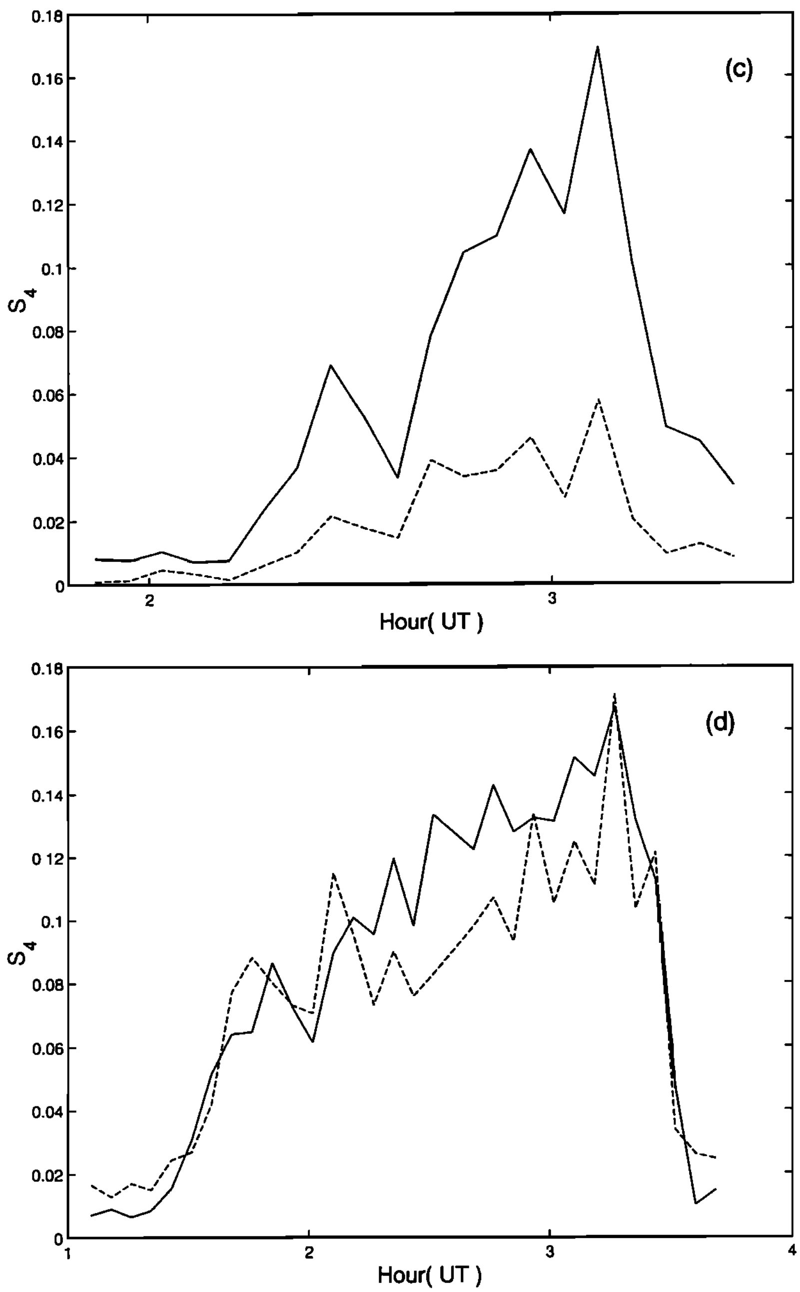


Figure 6. (continued)

The GPS data studied here are uniquely suited to a power spectral analysis of intensity scintillations carried out simultaneously with a similar analysis of TEC variations associated with ionospheric irregularities, for determining a relationship between the two. In earlier studies [*Rufenach*, 1972; *Yeh and Liu*, 1982; *Basu and Basu*, 1993], comparisons were made between power spectra of observed scintillations and those of in situ measurements of ionospheric density variations using probes on rockets and satellites. The latter yield one-dimensional spectra with a power law form. In order to relate the index which characterizes the power law behavior of scintillation spectra with the power law index associated with a one-dimensional spectrum of ionospheric electron density variations it is necessary to assume that the irregularities are isotropic, which is not the case with ESF irregularities [*Kelley et al.*, 1987]. In the present situation, however, the TEC variation data directly yield a one-dimensional spectrum of the phase variations imposed on the incident radio wave by a phase screen equivalent of the irregularities. For a one-dimensional phase screen with variations only along the x axis and also assumed to move relative to a fixed signal path with a speed v along the x axis, the spectrum of weak amplitude scintillations recorded by a fixed receiver is given by [*Yeh and Liu*, 1982]:

$$F(\Omega) = \frac{1}{v} S_{1\phi}\left(\frac{\Omega}{v}\right) \sin^2\left(\frac{\Omega^2 z}{2v^2 k}\right), \qquad (10)$$

where $S_{1\phi}(q)$ is the one-dimensional spatial spectrum of the phase variation $\phi(x)$ imposed by the screen, z is the distance of the phase screen from the receiver, and $k = 2\pi/\lambda$. With $\phi(x)$ related to fluctuations $\Delta N_T(x)$ in the TEC along the signal path, according to (1), the power spectrum of weak amplitude or intensity scintillations should fall off as Ω^{-m} for frequencies $\Omega/2\pi$ greater than the Fresnel frequency ν_F, if the one-dimensional spectrum of TEC fluctuations along the x direction is of a power law form: $S_{\Delta N_T}(q) \sim q^{-m}$.

Dual-frequency GPS data yield the power spectrum of fluctuations in TEC along the signal path, as the signal path sweeps across the irregularities. The TEC data are detrended with a low-pass six-pole Butterworth filter with a 3 dB cutoff at 0.0007 Hz to remove a basic trend due to changing path length through the background ionosphere with changing elevation of the satellite. A typical power spectrum of detrended TEC fluctuations along the signal path from PRN 5 to the receiver at Ancon, on April 9, 1997, is shown in Figure 7a. For frequencies lower than 0.1 Hz the spectrum has a power law form with an index of -2.57 ± 0.18. Measurements of intensity scintillations on the L1 signal from PRN 5 carried out simultaneously at Ancon, at a sampling frequency of 50 Hz, are averaged over five samples to obtain intensity data at intervals of 0.1 s, which is adequate for power spectral studies involving weak scintillations. These data are detrended, and their power spectrum is displayed in Figure 7b. For frequencies greater than the Fresnel frequency, which is 0.13 Hz in this case, the spectrum falls off according to a power law. The power spectral index of - 2.79 ± 0.15 is in agreement with the index derived from TEC fluctuation spectrum. Power law indices deduced from TEC fluctuation spectra pertain to irregularities of scale lengths $\geq d_F$, the Fresnel dimension, while those derived from intensity scintillation spectra are associated with irregularities of scale lengths $\leq d_F$. No inference can be drawn about shorter ($< d_F$) scale length irregularities from TEC fluctuation data because at frequencies $> \nu_F$, contributions from phase scintillations on both L1 and L2 are present in this data, in a combination given by (6), such that the resultant spectrum does not represent $S_{\Delta N_T}(q)$ for scale lengths $< d_F$. Thus the power law indices derived from weak intensity scintillation data and TEC fluctuation data are expected to be in agreement only if there is no break in the slope of a power law type of spectrum $S_{\Delta N_T}(q)$ for actual TEC fluctuations at some scale length in the neighborhood of d_F. This seems to be the situation for a majority of the cases, while in the remaining cases the power law index computed from intensity scintillation spectrum exceeds that of the longer-scale-length variations in TEC by nearly 1, which may be attributed to a break in the slope of the actual TEC fluctuation spectrum in the neighborhood of the Fresnel scale, as has been suggested by earlier studies [*Franke and Liu*, 1983; *Basu et al.*, 1983; *LaBelle and Kelley*, 1986; *Bhattacharyya and Rastogi*, 1986]. In either situation, no simple dependence of the scaling factor between DROTI and S_4 on spectral characteristics of TEC variations and intensity scintillations has emerged.

6. Conclusions

With the availability of large amounts of data for relative TEC along GPS signal paths from the IGS network, in recent years, ionospheric scientists have started exploring the possibility of using this database for studies of ionospheric irregularities. The measure that was chosen was an index called ROTI, which was based on the $d(TEC)/dt$ values derived from the GPS 30 s TEC data [*Aarons et al.*, 1997; *Pi et al.*, 1997]. *Musman et al.* [1997] defined a "roughness" measure which was also based on the first derivative of TEC variations. Musman et al. found a good correlation between the "roughness" and plumes seen in backscattered power maps, obtained at the same site on two nights, with a portable radar which detected density irregularities of 3 m scale size. On the other hand, ROTI has been compared to the S_4 index computed from simultaneously recorded intensity

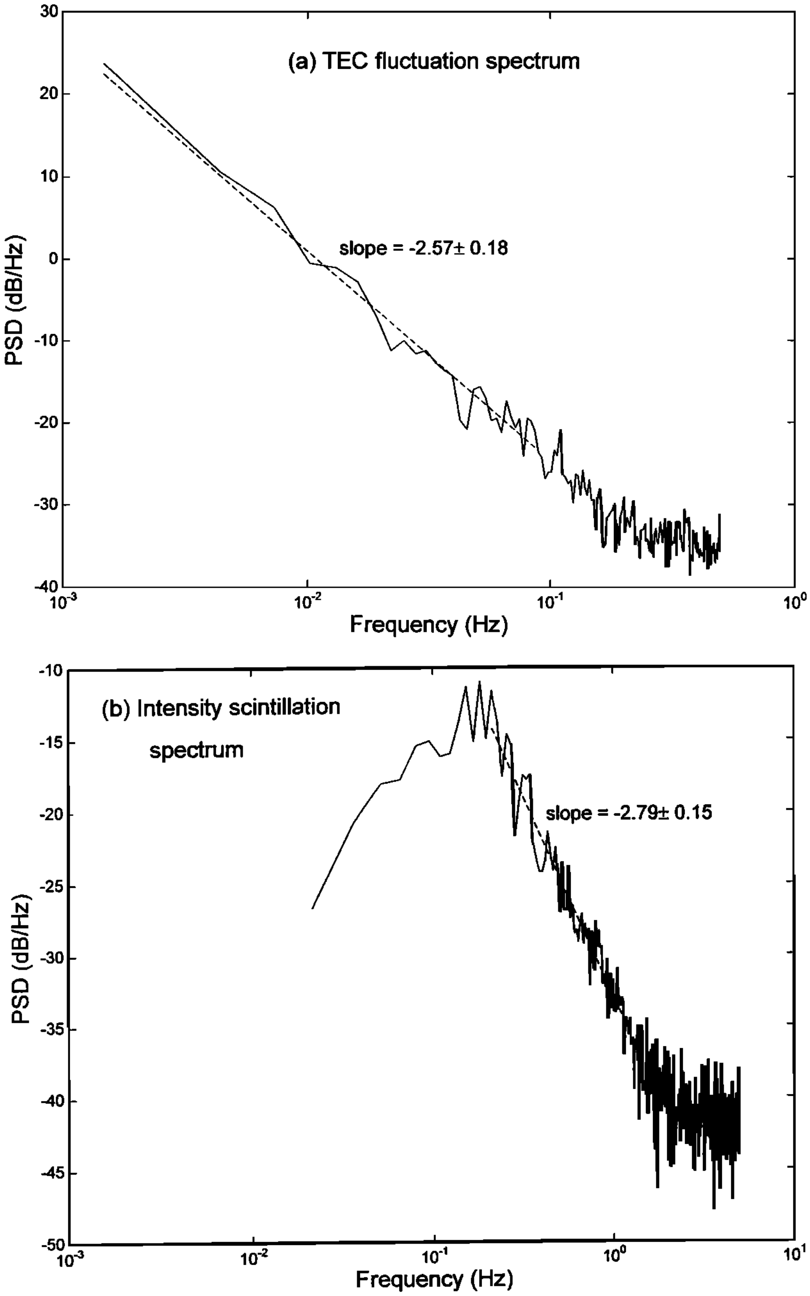


Figure 7. (a) Power spectrum of detrended TEC variations (detrend cutoff frequency is 0.0007 Hz) for the signal path from PRN 5 to Ancon, on April 9, 1997. (b) Power spectrum of detrended (detrend cutoff frequency is 0.01 Hz) 10 Hz intensity scintillation data for L1 signal from PRN 5 to Ancon during the same night.

scintillations on an L1 GPS signal [*Beach and Kintner*, 1999], which would largely be due to irregularities of scale sizes of a few hundred meters. Whereas phase scintillations, which arise because of the same irregularities and contribute to the measured TEC fluctuations, would follow the S_4 index closely, there is a distinction between the contribution from phase scintillations to the measured TEC fluctuations and actual TEC variations. This paper attempts to bring out this distinction using a model phase screen calculation and separating the two in actual data by the application of a suitable filter. Measures such as ROTI or the "roughness" are representative of gradients in the slant TEC. However, there is no quantitative relationship between these indices and the S_4 index which measures the strength of intensity scintillations, although it would appear that the drift speed v of the irregularities with respect to the signal path would play a role in quantifying such a relation, since this speed determines the spatial scale lengths which contribute to an index such as ROTI. Also, the slant distance z of the effective irregularity layer from the receiver would be an important factor since it determines the Fresnel scale size.

An equation which relates the second derivative, $d^2\Delta N_T/dt^2$, of TEC variations on timescales $\geq \nu_F^{-1}$ to the measured intensity is derived from an approximate version of the TIE. On the basis of this equation, a second-derivative-based index, DROTI, which can be easily extracted from GPS TEC data, appears to be a natural choice as a measure of scintillation activity due to ionospheric irregularities. The equation yields a scaling factor between DROTI and S_4 that depends on the Fresnel frequency ν_F which is determined by both z and v. This factor explains only a part of the variability of the relation between DROTI and S_4 from day to day and from one satellite to another. The spectral characteristics of TEC variations and intensity scintillations are also expected to have significant effects on DROTI and the S_4 index. However, no simple dependence of the scaling factor between DROTI and S_4 on these parameters emerged from power spectral analysis of the relevant data. A conclusion that can be drawn from the present study is that indices such as ROTI and DROTI are useful whenever the phase screen approximation is applicable to the layer of irregularities. As has been seen in the present paper, this is not always the case when the geometry of the signal path is such that considerable intensity scintillations are very likely to develop within the irregularity layer. In that situation, density gradients which cause scintillations may not be well represented in TEC variations. The contribution of phase scintillations to the measured TEC fluctuations, however, always follows the pattern of intensity scintillations, as they arise from the same irregularities whether they are present in a thick layer or a thin one. It is seen that in the presence of steep gradients in ionospheric electron density, phase scintillations may introduce as much error in the measured relative TEC as the variation in TEC because of the presence of the irregularities, and rapid changes in phase associated with scintillations may sometimes result in loss of receiver lock.

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References

Aarons, J., M. Mendillo, R. Yantosca, and E. Kudeki, GPS phase fluctuation in the equatorial region during the MISETA 1994 campaign, *J. Geophys. Res., 101*, 26,851-26,862, 1996.

Aarons, J., M. Mendillo, and R. Yantosca, GPS phase fluctuations in the equatorial region during sunspot minimum, *Radio Sci., 32*, 1535-1550, 1997.

Basu, S., and S. Basu, Ionospheric structures and scintillation spectra, in *Wave Propagation in Random Media (Scintillation)*, edited by V. I. Tatarskii, A. Ishimaru, and V. U. Zavorotny, pp. 139-155, SPIE - The Int. Soc. for Opt. Eng., Bellingham, Wash., and Inst. of Phys. Publ., Philadelphia, Pa., 1993.

Basu, S., J. P. McClure, S. Basu, W. B. Hanson, and J. Aarons, Coordinated study of equatorial scintillation and in situ and radar observations of nighttime F region irregularities, *J. Geophys. Res., 85*, 5119-5130, 1980.

Basu, S., S. Basu, J. P. McClure, W. B. Hanson, and H. E. Whitney, High resolution topside in situ data of electron densities and VHF/GHz scintillations in the equatorial region, *J. Geophys. Res., 88*, 403-415, 1983.

Beach, T.L., Global positioning system studies of equatorial scintillations, Ph.D. thesis, Cornell University, Ithaca, N.Y., 1998.

Beach, T.L., and P. M. Kintner, Simultaneous Global Positioning System observations of equatorial scintillations and total electron content fluctuations, *J. Geophys. Res., 104*, 22,553-22,565, 1999.

Bhattacharyya, A., Deterministic retrieval of ionospheric phase screen from amplitude scintillations, *Radio Sci., 34*, 229-240, 1999.

Bhattacharyya, A., and R. G. Rastogi, Phase scintillations due to equatorial F region irregularities with two-component power law spectrum, *J. Geophys. Res., 91*, 11,359-11,364, 1986.

Bhattacharyya, A., K.C. Yeh, and S. J. Franke, Deducing turbulence parameters from transionospheric scintillation measurements, *Space Sci. Rev., 61*, 335-386, 1992.

Doherty, P., E. Raffi, J. Klobuchar, and M. B. El-Arini, Statistics of time rate of change of ionospheric electron content, in *Proceedings of the Institute of Navigation*, pp. 1589-1598, Alexandria, Virg., 1994.

Franke, S. J., and C. H. Liu, Observations and modeling of multi-frequency VHF and GHz scintillations in the equatorial region, *J. Geophys. Res., 88*, 7075-7085, 1983.

Fremouw, E. J., R. L. Leadabrand, R. C. Livingston, M. D. Cousins, C. L. Rino, B. C. Fair, and R. A. Long, Early results from the DNA Wideband satellite experiment: Complex-signal scintillation, *Radio Sci., 13*, 167-187, 1978.

Hofman-Wellenhof, B., H. Lichtenegger, and J. Collins, *Global Positioning System: Theory and Practice*, 3rd ed., Springer-Verlag, New York, 1994.

Kelley, M. C., *The Earth's Ionosphere*, chap. 4, pp. 113-120, Academic, San Diego, Calif., 1989.

Kelley, M. C., C. E. Seyler, and S. Zargham, Collisional interchange instability, 2, A comparison of the numerical simulations with the in situ experimental data, *J. Geophys. Res., 92*, 10,089-10,094, 1987.

Kelley, M. C., D. Kotsikopoulos, T. Beach, D. Hysell, and S. Musman, Simultaneous Global Positioning System and radar observations of equatorial spread *F* at Kwajalein, *J. Geophys. Res., 101*, 2333-2341, 1996.

Klobuchar, J. A., Ionospheric effects on GPS, in *Global Positioning System: Theory and Applications, vol. 1*, edited by B. W. Parkinson and J. J. Spilker Jr., pp. 485-515, Am. Inst. of Aeronaut. and Astronaut., Washington, D. C., 1996.

LaBelle, J., and M. C. Kelley, The generation of kilometer scale irregularities in equatorial spread *F*, *J. Geophys. Res., 91*, 5504-5512, 1986.

Musman, S., J.-M. Jahn, J. LaBelle, and W.E. Swartz, Imaging spread-*F* structures using GPS observations at Alcantara, Brazil, *Geophys. Res. Lett., 24*, 1703-1706, 1997.

Pi, X., A.J. Mannucci, U. J. Lindqwister, and C.M. Ho, Monitoring of global ionospheric irregularities using the worldwide GPS network, *Geophys. Res. Lett., 24*, 2283-2286, 1997.

Ratcliffe, J. A., Some aspects of diffraction theory and their application to the ionosphere, *Rep. Prog. Phys., 19*, 188-267, 1956.

Rino, C. L., A power law phase screen model for ionospheric scintillation, 1, Weak scatter, *Radio Sci., 14*, 1135-1145, 1979.

Rufenach, C. L., Power law wavenumber spectrum deduced from ionospheric scintillation observations, *J. Geophys. Res., 77*, 4761-4772, 1972.

Teague, M. R., Deterministic phase retrieval: A Green's function solution, *J. Opt. Soc. Am., 73*, 1434-1441, 1983.

Wanninger, L., The occurrence of ionospheric disturbances above Japan and their effects on precise GPS positioning, paper presented at the 8th International Symposium on Recent Crust Movements (CRCM '93), Kobe, Japan, December 1993.

Weber, E. J., et al., Equatorial plasma depletion precursor signatures and onset observed at 11° south of the magnetic equator, *J. Geophys. Res., 101*, 26,829-26,838, 1996.

Yeh, K.C., and C.H. Liu, Radio wave scintillations in the ionosphere, *Proc. IEEE, 70*, 324-360, 1982.

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