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# Electron acoustic waves in a magnetized plasma with kappa distributed ions

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Electron acoustic solitary waves in a two component magnetized plasma consisting of fluid cold electrons and hot superthermal ions are considered. The linear dispersion relation for electron acoustic waves is derived. In the nonlinear regime, the energy integral is obtained by a Sagdeev pseudopotential analysis, which predicts negative solitary potential structures. The effects of superthermality, obliquity, temperature, and Mach number on solitary structures are studied in detail. The results show that the superthermal index $\kappa$ and electron to ion temperature ratio $\sigma$ alters the regime where solitary waves can exist. It is found that an increase in magnetic field value results in an enhancement of soliton electric field amplitude and a reduction in soliton width and pulse duration. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4743015]

## I. INTRODUCTION

Electron acoustic waves can exist in a magnetized plasma propagating nearly across the magnetic field with the ion temperature ($T_i$) much larger than the electron temperature ($T_e$), i.e., $T_i \gg T_e$. Although the condition $T_i \gg T_e$ is restrictive, it can be realized in laboratory and space plasmas. Satellite observations have shown existence of such plasmas in the Earth's magnetosphere, solarwind, in the distant magnetotail, and upstream of bowshock.[1–4] Several authors have attempted to study the properties of linear electron acoustic waves in a magnetized plasma.[5–9] Parametric excitation of electron acoustic waves in a pure-two-component electron-ion plasma has been studied extensively.[10–14]

Mohan and Buti[15] obtained a modified KdV equation for the electron acoustic waves in a current carrying magnetized plasma with ion temperature much greater than the electron temperature. Buti *et al.*[16] studied the nonlinear propagation of electron-acoustic solitons in a magnetized plasma by taking into account the exact electron and ion nonlinearities. Goswami and Bujarbarua[17] investigated the electron acoustic double layers in a multi component plasma with two ion species and cool background electrons. Sah and Goswami[18] found the modified electron acoustic solitons and double layers with relativistic drifting electrons and nondrifting thermal ions by using the reductive perturbation method.

Previous studies on the electron-acoustic solitons in pure two-component plasma with $T_i \gg T_e$ have considered electrons as mobile and ions with Maxwellian distribution. However, in space and astrophysical plasma environments, superthermal particles which deviate from the Maxwellian are also observed. These particles follow the kappa ($\kappa$)-like distributions that are characterized by the spectral index $\kappa$ and have Maxwellian-like core and high energy tail component in the power law form. It was first introduced by Vasyliunas[19] as an empirical fit to explain the observations of OGO 1 and OGO 2 solar wind data. A detailed analytical description of kappa distributions is given by Thorne and Summers.[20]

There have been a few studies on electron-acoustic waves with kappa-distributed electrons/ions. Mace *et al.*[21] studied the electron acoustic mode in a three component plasma using suprathermal distribution for hot electrons, stationary ions, and cool Maxwellian electrons. Electron acoustic solitary waves in a three component unmagnetized plasma consisting of fluid cold electrons, ions, and hot electrons having kappa distribution have been studied by Younsi and Tribeche.[22] They did not include the effect of cool electron temperature in their study. Thermal effects of cool electrons were included in the analysis of electron-acoustic solitons by Danehkar *et al.*[23] and Devanandhan *et al.*[24] It was found that inclusion of cool electron temperature shrinks the existence regime of the solitons, and electric field amplitude decreases with an increase in cool electron temperature. Sultana and Kourakis[25] examined the modulational instability of electron-acoustic waves in three-component unmagnetized plasma with cool electrons, hot kappa-distributed electrons, and ions. It was shown that superthermality affects the characteristics of solitary envelope structures. Further, electron acoustic solitons with superthermal electrons have been studied in an unmagnetized four-component beam plasma by Devanandhan *et al.*[26] The results showed that inclusion of an electron beam alters the minimum value of spectral index, $\kappa$, and Mach number for which electron-acoustic solitons can exist.

All the theoretical studies on electron-acoustic solitons with kappa distribution of electrons have considered multi-component, unmagnetized plasma with the electron temperature being larger than the ion temperature. So far, none of the study on EA solitons considered magnetized plasma. As a first step, we study the electron-acoustic solitons in a magnetized, pure two-component (electrons and ions with $T_i \gg T_e$) plasma with energetic hot ions having the superthermal distribution and cool fluid electrons. In Sec. II, the theoretical model is presented. The results are discussed and summarized in Secs. III and IV, respectively.

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## II. THEORETICAL MODEL

We consider a two-component, magnetized collisionless plasma consisting of fluid cold electrons and kappa distributed singly charged (Z = 1) hot ions (charge $q_i = Ze$). In our theoretical model, electrons are considered to be adiabatic and ambient magnetic field along z-direction, i.e., $\mathbf{B}_0 = B_0\hat{z}$ where $\hat{z}$ is the unit vector along the z axis. For simplicity, we consider the waves to be propagating in the *x–z* plane, so that there are no variations along y-axis (i.e., $\frac{\partial}{\partial y} = 0$). The normalized governing equations of electron acoustic mode in such a two-component magnetoplasma are given by,

$$\frac{\partial n_e}{\partial t} + \frac{\partial(n_e v_x)}{\partial x} + \frac{\partial(n_e v_z)}{\partial z} = 0, \tag{1}$$

$$\frac{\partial v_x}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_x = \frac{\partial \phi}{\partial x} - 3\sigma n_e\frac{\partial n_e}{\partial x} - v_y, \tag{2}$$

$$\frac{\partial v_y}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_y = v_x, \tag{3}$$

$$\frac{\partial v_z}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_z = \frac{\partial \phi}{\partial z} - 3\sigma n_e\frac{\partial n_e}{\partial z}. \tag{4}$$

Here $n_e$, v, and $\phi$ represents the density, velocity, and electrostatic potential of the cold electron fluid, respectively, and $\sigma = T_e/T_i$ is the electron to ion temperature ratio. We have used the normalization as follows: densities by equilibrium density $N_0 = N_{0e} = N_{0i}$; $N_{0e}, N_{0i}$ are the equilibrium electron and ion densities, respectively, time by inverse of electron cyclotron frequency $\Omega_e = \frac{eB_0}{m_e c}$, velocities by $c_s = \sqrt{\frac{T_i}{m_e}}$, lengths by effective electron Larmor radius $\frac{C_s}{\Omega_e}$, potential by $\frac{T_i}{e}$, and thermal pressure by $N_0 T_i$.

It must be pointed out here that for simplicity, we have taken adiabatic index $\gamma = 3$ in the energy equation, which is strictly valid for one degree of freedom. One can have $\gamma = 5/3$ by considering three degrees of freedom as has been done by Mahmood and Akhtar[27] to study ion-acoustic solitons in magnetized electron-positron-ion plasma with adiabatic ions. However, this complicates the analysis. Since equation of motion and continuity are treated exactly, we do not expect the results to change qualitatively by taking $\gamma = 5/3$ instead of $\gamma = 3$.

To model the effect of hot superthermal ions on solitary structures, we follow the kappa-distribution function given by Summers and Thorne,[20]

$$f_{oi}(v) = \frac{N_{0i}}{\pi^{\frac{3}{2}}\theta^3}\frac{\Gamma(\kappa)}{\sqrt{\kappa}\Gamma\left(\kappa - \frac{1}{2}\right)}\left(1 + \frac{v^2}{\kappa\theta^2}\right)^{-(\kappa+1)}, \tag{5}$$

where $\kappa$ is the superthermality index, $\Gamma(\kappa)$ is the gamma function, and the modified thermal speed is given by $\theta^2 = \left(2 - \frac{3}{\kappa}\right)\frac{T_i}{m_i}$. For thermal speed $\theta$ to be physical (i.e., real), we require $\kappa > \frac{3}{2}$. The Maxwell-Boltzmann equilibrium is obtained in the limit $\kappa \to \infty$. The distribution of ions in the presence of non-zero potential can be found by replacing $v^2$ by $v^2 + \frac{e\phi}{T_i}$ in Eq. (5). The number density of ions can be obtained by integrating distribution function $f_{oi}$ given by Eq. (5) over velocity space. Thus, the ion number density ($n_i$) in the normalized form can be written as,

$$n_i = \left[1 + \frac{\phi}{\kappa - \frac{3}{2}}\right]^{-\kappa+\frac{1}{2}}. \tag{6}$$

### A. Linear modes

Linearization of Eqs. (1)–(6) and making use of the plasma approximation ($n_e \approx n_i$), the linear dispersion relation in unnormalized form can be written as

$$\omega^4 - \omega^2\left\{\Omega_e^2 + k^2c_s^2\left(3\sigma + \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}}\right)\right\} + k^2c_s^2\Omega_e^2\cos^2\theta\left(3\sigma + \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}}\right) = 0. \tag{7}$$

For $\theta = 90°$, i.e., $\cos\theta = 0$, we get the cyclotron mode which is modified by the thermal effects and is given as

$$\omega^2 = \Omega_e^2 + k^2c_s^2\left(3\sigma + \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}}\right) \tag{8}$$

and for $\theta = 0°$, i.e., $\cos\theta = 1$, the cyclotron and the electron-acoustic modes decouple and are given by,
cyclotron mode

$$\omega^2 = \Omega_e^2 \tag{9}$$

and electron-acoustic mode

$$\omega^2 = k^2c_s^2\left(3\sigma + \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}}\right). \tag{10}$$

Equation (7) is quadratic in $\omega^2$, the solution of which can be written as

$$\omega_\pm^2 = \frac{1}{2}\left[\Omega_e^2 + k^2c_s^2\left(3\sigma + \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}}\right) \pm \sqrt{\left\{\Omega_e^2 + k^2c_s^2\left(3\sigma + \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}}\right)\right\}^2 - 4k^2c_s^2\Omega_e^2\cos^2\theta\left(3\sigma + \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}}\right)}\right]. \tag{11}$$

To identify the modes, we will analyze Eq. (11) in two regimes:

Case i: Large wavelength limit: When $\Omega_e^2 \gg k^2c_s^2\left(3\sigma + \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}}\right)$, the upper and lower frequency modes of Eq. (11) can be written as

$$\omega_+ = \Omega_e + \frac{k^2c_s^2}{2\Omega_e}\sin^2\theta\left(3\sigma + \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}}\right), \tag{12}$$

$$\omega_- = kc_s\cos\theta\left(3\sigma + \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}}\right)^{\frac{1}{2}}, \tag{13}$$

Case ii: Short wavelength limit: When $\Omega_e^2 \ll k^2 c_s^2 \left(3\sigma + \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}}\right)$, we get two modes as

$$\omega_+ = k c_s \left(3\sigma + \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}}\right)^{1/2}, \tag{14}$$

$$\omega_- = \Omega_e \cos\theta. \tag{15}$$

Equations (12)–(15) are identical to the Eqs. (12)–(15) of Mace and Hellberg.[28] In the large wavelength limit, $\omega_-$ mode given by Eq. (13) behaves like electron-acoustic mode in a magnetized plasma and in short wavelength limit, $\omega_+$ mode given by Eq. (14) behaves like electron-acoustic mode in unmagnetized plasma and is same as Eq. (10). In the limit $\kappa \to 0$, Eqs. (10) and (14) reduce to the usual electron-acoustic mode in a Maxwellian plasma. The modes described by Eqs. (12) and (15) are the cyclotron modes.

### B. Nonlinear analysis

We now focus our attention to study the nonlinear propagation of electron acoustic waves. We look for the solutions of Eqs. (1)–(6) that depend on x, z, and t through a single variable, $\xi = \alpha x + \beta z - Mt$, where M is the Mach number, $\alpha, \beta$ are the directions of cosines along x and z directions, respectively, i.e., $\alpha = \frac{k_x}{k} = \sin\theta; \beta = \frac{k_z}{k} = \cos\theta$. From Eqs. (1)–(4), we can express the transformed equations as

$$-M\frac{\partial n_e}{\partial \xi} + \alpha \frac{\partial (n_e v_x)}{\partial \xi} + \beta \frac{\partial (n_e v_z)}{\partial \xi} = 0, \tag{16}$$

$$(-M + \alpha v_x + \beta v_z)\frac{\partial v_x}{\partial \xi} - \alpha\frac{\partial \phi}{\partial \xi} + 3\alpha\sigma n_e \frac{\partial n_e}{\partial \xi} + v_y = 0, \tag{17}$$

$$(-M + \alpha v_x + \beta v_z)\frac{\partial v_y}{\partial \xi} - v_x = 0, \tag{18}$$

$$(-M + \alpha v_x + \beta v_z)\frac{\partial v_z}{\partial \xi} - \beta\frac{\partial \phi}{\partial \xi} + 3\beta\sigma n_e \frac{\partial n_e}{\partial \xi} = 0. \tag{19}$$

Integrating the Eqs. (16)–(19) by assuming quasi-neutrality, $n_e = n_i = n$ and using the appropriate boundary conditions ($n_{e,i} \to 1$, $v_{x,y,z} \to 0$, $\phi \to 0$ and $d\phi/d\xi \to 0$, at $\xi \to \pm\infty$), we obtain

$$n(\alpha v_x + \beta v_z) = M(n-1), \tag{20}$$

$$v_z = \frac{\beta}{M}\left[-1 + \left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-\kappa + \frac{3}{2}} + \sigma(n^3 - 1)\right], \tag{21}$$

$$v_x = \frac{1}{\alpha}\left[M\left(1 - \frac{1}{n}\right) - \frac{\beta^2}{M}\left\{-1 + \left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-\kappa + \frac{3}{2}} + \sigma(n^3 - 1)\right\}\right]. \tag{22}$$

Using Eqs. (6), (21), and (22) in Eq. (20) and following the ]similar procedure as adopted by Mahmood *et al.*,[29] we obtain

$$\frac{d^2 Q}{d\xi^2} = 1 - \left(1 + \frac{\beta^2}{M^2} - \frac{\sigma\beta^2}{M^2}\right)\left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-\kappa + \frac{1}{2}}, + \frac{\beta^2}{M^2}\left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-2\kappa + 2} + \frac{\sigma\beta^2}{M^2}\left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-4\kappa + 2} \tag{23}$$

where

$$Q = \left[\phi - \frac{M^2}{2}\left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{2\kappa - 1} - \frac{3}{2}\sigma\left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-2\kappa + 1}\right]. \tag{24}$$

Multiplying both sides of the Eq. (23) by $2\frac{dQ}{d\xi}$ and integrating by using the appropriate boundary conditions, we obtain the energy integral given below,

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + \psi(\phi, M) = 0, \tag{25}$$

where

$$\psi(\phi, M) = \frac{\psi_1(\phi, M)}{\psi_2(\phi, M)} \tag{26}$$

is the Sagdeev pseudo potential, whereas

$$\begin{aligned}
\psi_1(\phi, M) = {} & \left(1 + \frac{\beta^2}{M^2}\right)\left[1 - \left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-\kappa + 3/2}\right] + (M^2 + \beta^2)\left[1 - \left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{\kappa - 1/2}\right] \\
& - \left[1 - \beta^2\left(\frac{\kappa - \frac{1}{2}}{\kappa - \frac{3}{2}}\right)\right]\phi - \frac{\beta^2}{2M^2}\left[1 - \left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-2\kappa + 3}\right] - \frac{M^2}{2}\left[1 - \left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{2\kappa - 1}\right] \\
& + \sigma\left\{\frac{\beta^2}{M^2}\left[1 - \left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-\kappa + 3/2} + \left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-4\kappa + 3} - \left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-3\kappa + 3/2}\right] + \beta^2\left[1 - \left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{\kappa - 1/2}\right]\right. \\
& + \left(\frac{\beta^2 - 3}{2}\right)\left[1 - \left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-2\kappa + 1}\right] + \left(1 + \frac{\sigma\beta^2}{M^2}\right)\left[1 - \left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-3\kappa + 3/2}\right] \\
& \left. - \frac{\sigma\beta^2}{2M^2}\left[1 - \left(1 + \frac{\phi}{\kappa - \frac{3}{2}}\right)^{-6\kappa + 3}\right]\right\}
\end{aligned}$$

and

$$\psi_2(\phi,M) = \left[1 - M^2\left(\frac{\kappa-\frac{1}{2}}{\kappa-\frac{3}{2}}\right)\left(1+\frac{\phi}{\kappa-\frac{3}{2}}\right)^{2\kappa-2} + 3\sigma\left(\frac{\kappa-\frac{1}{2}}{\kappa-\frac{3}{2}}\right)\left(1+\frac{\phi}{\kappa-\frac{3}{2}}\right)^{-2\kappa}\right]^2.$$

Equation (25) yields solitary wave solutions when the Sagdeev potential satisfies the following conditions: $\psi(\phi,M) = 0, \mathrm{d}\psi(\phi,M)/\mathrm{d}\phi = 0, \mathrm{d}^2\psi(\phi,M)/\mathrm{d}\phi^2 < 0$ at $\phi = 0$; $\psi(\phi,M) = 0$ at $\phi = \phi_0$, and $\psi(\phi,M) < 0$ for $0 <|\phi|<|\phi_0|$. From Eq. (26), it is seen that $\psi(\phi,M)$ and its first derivative with respect to $\phi$ vanish at $\phi = 0$.

It is worth mentioning here that Eq. (26) does not depend on the magnetic field, $B_0$, explicitly. This is due to the normalization used here. However, if one uses the normalization similar to that of Sultana *et al.*,[30] the Sagdeev potential term will have $\frac{\Omega_e^2}{\omega_{pe}^2}$ as a multiplying factor in Eq. (26), i.e., $\psi(\phi,M) \to (\frac{\Omega_e^2}{\omega_{pe}^2})\psi(\phi,M)$. Therefore, the critical mach number as well as maximum normalized potential amplitude of the solitons will not be affected as found by Sultana *et al.*[30] When we do the unnormalization to obtain actual electric field amplitude, width and pulse duration of the solitons, it is observed that electric field amplitude, pulse duration, and width changes due to the change in ratio of electron cyclotron frequency to the electron plasma frequency. Our results exhibit similar behaviour to the one obtained by Sultana *et al.*[30] for ion-acoustic solitons in magnetized plasmas with kappa distributed electrons.

Before proceeding further, we would like to analyze the second derivative of the Sagdeev potential. From Eq. (26), the soliton condition $\mathrm{d}^2\psi(\phi,M)/\mathrm{d}\phi^2 < 0$ at $\phi = 0$ can be written as,

$$\frac{d^2\psi(\phi)}{d\phi^2}|_{\phi=0} = \frac{M^2 - M_0^2}{M^2\{M^2 - M_1^2\}} < 0. \qquad (27)$$

The condition $\mathrm{d}^2\psi(\phi,M)/\mathrm{d}\phi^2 < 0$ at $\phi = 0$ is satisfied provided $\mathrm{M} > \mathrm{M}_0$, where $\mathrm{M}_0$ is the critical Mach number given by

$$M_0 = \beta\left(3\sigma + \frac{\kappa-\frac{3}{2}}{\kappa-\frac{1}{2}}\right)^{1/2} \qquad (28)$$

and

$$M_1 = \left(3\sigma + \frac{\kappa-\frac{3}{2}}{\kappa-\frac{1}{2}}\right)^{1/2}. \qquad (29)$$

It is clear that $M_1 \geq M_0$ as $\beta \leq 1$. Therefore, the soliton condition (i.e., inequality (27)) is satisfied when $M_0 < M < M_1$

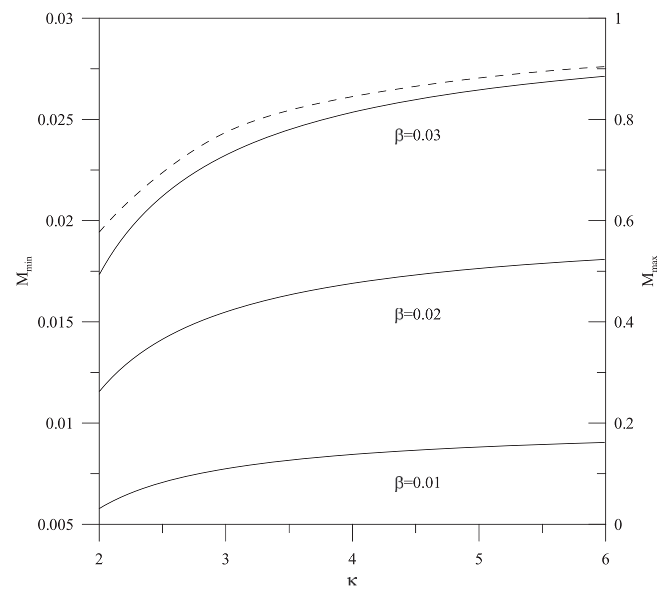


FIG. 1. Variation of minimum Mach number ($M_{min}$, solid line, L.H.S. y-axis) and maximum Mach number ($M_{max}$, dotted line, R.H.S. y-axis) with respect to $\kappa$ (from bottom to top) for $\beta = 0.01$, $\beta = 0.02$, and $\beta = 0.03$ at $\sigma = 0.0$.

for $\beta \neq 1$. Here, $M_0$ is the critical Mach number and $M_1$ is defined as the upper limit of the Mach number. It is interesting to note that for $\beta = 1$ inequality (27) cannot be satisfied, hence no soliton solution is possible.

Numerically, the upper limit of the Mach number is obtained in the following manner: For the fixed set of parameters, first critical Mach number is obtained from Eq. (28). From then onwards, Mach number is increased till the soliton solution cease to exist. The highest Mach number for which soliton solution exists is the upper limit of M. It is interesting to note that the upper limit of the Mach number obtained numerically matches with $M_1$ calculated from Eq. (29).

## III. RESULTS

In Figure 1, the minimum ($M_{min}$, solid curve) and maximum Mach number ($M_{max}$, dashed curve) vs kappa ($\kappa$) are plotted for various values of obliquity ($\beta$). Left hand side (L.H.S.) of y-axis corresponds to $M_{min}$ and right hand side (R.H.S.) of y-axis corresponds to $M_{max}$. It can be seen that $\beta$ modifies only the minimum Mach number, which corresponds to $M_0$ given by Eq. (27) and does not have any effect on the maximum Mach number $M_{max}$, which corresponds to $M_1$ and is given by Eqs. (29).

The necessary condition for electron acoustic modes to exist in a two-component electron-ion plasma is that the hot species (ions) should be subsonic (ion thermal velocity needs to be larger than phase speed of the wave) and cold species (electrons) should be supersonic (electron thermal velocity needs to be smaller than phase speed of the wave).[31] Using above criteria, from Eq. (13), we arrive at a condition that $\beta = \cos\theta < \sqrt{m_e/m_i}/\left(3\sigma + \frac{\kappa-\frac{3}{2}}{\kappa-\frac{1}{2}}\right)^{\frac{1}{2}}$. This shows that only very obliquely propagating electron-

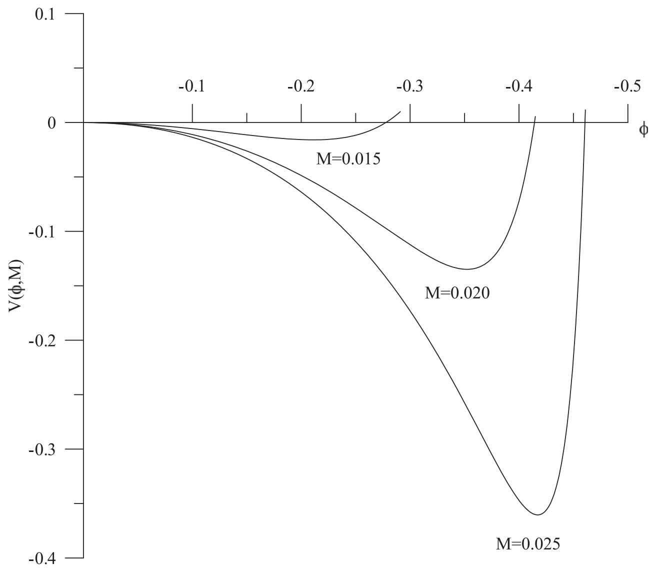


FIG. 2. Variation of Sagdeev potential with respect to $\phi$ for various values of Mach number (M) for $\kappa = 2.0$, $\sigma = 0.0$, and $\beta = 0.02$.

acoustic solitons can exist in a two-component magnetized plasma.

Figure 2 shows the variation of Sagdeev potential with normalized potential $\phi$ for various values of the Mach number. Other parameters are $\kappa = 2.0$, $\sigma = 0.0$, and $\beta = 0.02$. The solitary wave amplitude increases with increase of M and it is found that soliton solution does not exist beyond $M = 0.5773$.

Variation of normalized electric field vs $\xi$ for various values of $\beta$ is shown in Figure 3. Here, $M = 0.02$, $\sigma = 0.0$, and $\kappa = 2.0$. It is observed that the electric field amplitude increases with a decrease in $\beta$ value, i.e., it increases with

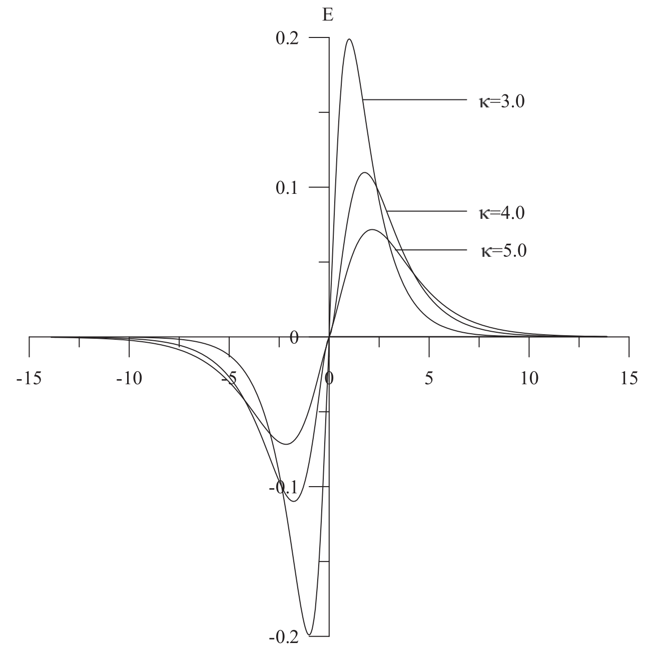


FIG. 4. Variation of normalized electric field profile with respect to $\xi$ for various values of $\kappa$ for $M = 0.02$, $\sigma = 0.0$, and $\beta = 0.02$.

increase in angle of propagation, $\theta$ and soliton width becomes narrower with the increased obliquity (less $\beta$). Figure 4 shows the variation of electric field vs $\xi$ for various values of $\kappa$ as shown on the curves. It is observed that soliton amplitude increases with an increase in superthermality (decrease in $\kappa$ values). We have also plotted the variation of normalized electric field vs $\xi$ for various values of $\sigma = T_e/T_i$ as shown on the curves in Figure 5 for $\kappa = 2.0$, $M = 0.04$, and $\beta = 0.04$. It shows that the inclusion of electron temperature reduces the electric field amplitude of the solitons.

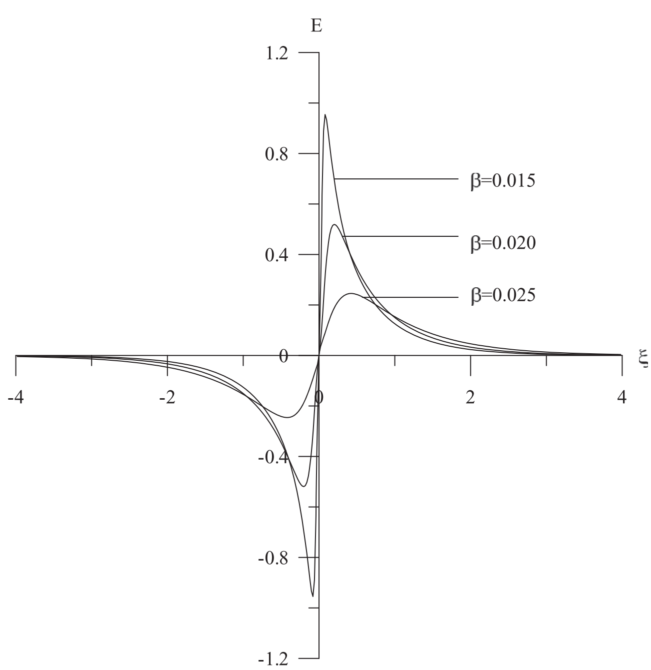


FIG. 3. Variation of normalized electric field profile with respect to $\xi$ for various values of $\beta$ for $\kappa = 2.0$, $\sigma = 0.0$, and $M = 0.02$.

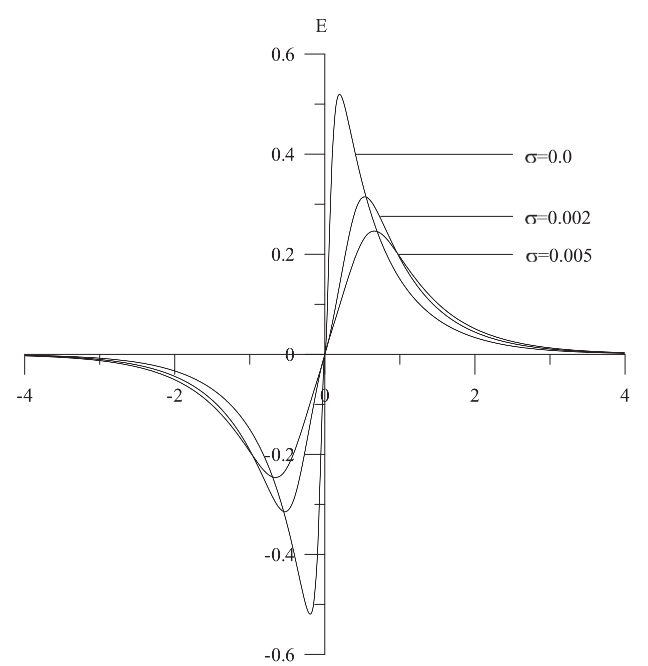


FIG. 5. Variation of normalized electric field profile with respect to $\xi$ for various values of $\sigma$ for $M = 0.04$, $\kappa = 2.0$, and $\beta = 0.04$.

TABLE I. Variation of the soliton velocity (V), electric field (E), soliton width (W), and pulse duration ($\tau$) with respect to $\sigma$ for ion temperature $T_i \sim 10\,\mathrm{keV}$, $\beta = 0.04$, $\kappa = 2.0$, and two different values of magnetic field $B_0$.

| | $B_0 = 10\,\mathrm{nT}$ | | | | $B_0 = 20\,\mathrm{nT}$ | | | |
|---|---|---|---|---|---|---|---|---|
| | V | E | W | $\tau = W/V$ | V | E | W | $\tau = W/V$ |
| $\sigma$ | ($\mathrm{km\,s^{-1}}$) | ($\mathrm{mV\,m^{-1}}$) | (km) | (ms) | ($\mathrm{km\,s^{-1}}$) | ($\mathrm{mV\,m^{-1}}$) | (km) | (ms) |
| 0.001 | 976-24315 | 0.17-234 | 386-72 | 394-3.0 | 976-24315 | 0.34-468 | 193-36 | 197-1.5 |
| 0.002 | 985-24424 | 0.44-199 | 324-86 | 328-3.4 | 985-24424 | 0.88-398 | 162-43 | 164-1.7 |
| 0.004 | 997-24638 | 0.75-166 | 284-100 | 286-4.0 | 997-24638 | 1.50-332 | 142-50 | 143-2.0 |
| 0.005 | 1006-24743 | 1.12-155 | 242-110 | 240-4.4 | 1006-24743 | 2.24-310 | 121-55 | 120-2.2 |

## IV. CONCLUSIONS

Arbitrary amplitude electron acoustic solitary waves in a two component electron-ion magneto-plasma have been studied using Sagdeev pseudo-potential analysis. Linear properties of the electron-acoustic waves in such plasmas have also been discussed. Our numerical results show that only negative solitary potential structures can be generated through this model. It is worth mentioning that only critical Mach number gets affected by the angle of propagation but not the upper limit of the Mach number for which soliton solution exist. The electric field amplitude is reduced by the inclusion of thermal effects. It is to be noted that, kappa distribution accounts for higher electric field values than the Maxwellian.

The Wind spacecraft data observations reported by Bale *et al.*[32] in the bow shock region have shown solitary waves with period of $\approx 3.5$ ms, average electric field of $\approx 150\,\mathrm{mV/m}$, electron temperature $\approx 20 - 40\,\mathrm{eV}$, and ambient magnetic field $B_0 \approx 10$ nT. Table I shows the soliton velocity (V), electric field (E), soliton width (W), and pulse duration ($\tau = W/V$) for $\kappa = 2.0$, $\beta = 0.04$, ion temperature $T_i = 10$ keV,[33] and $\sigma = T_e/T_i = 0.001 - 0.005$ for magnetic field $B_0 = 10\,\mathrm{nT}$ and 20 nT. The range of values given in columns 2 to 9, correspond to the Mach numbers just above critical value, $M_0$ (minimum) and at the upper limit, $M_1$ (maximum), respectively. It is obvious from the Table I that the range as well as the maximum electric field amplitude of the solitary wave decreases with the increase in the $\sigma$ values and the corresponding pulse width increases. It is clear from the Table I that for the Mach numbers lying close to the upper limit, the values of the maximum electric field amplitude ($\sim$155 – 234 mV/m) (cf. column 3) and minimum pulse duration ($\sim$3 - 4.4 ms) (cf. coulmn 5) are in good agreement with the observations for all values of $\sigma = 0.001$ to 0.005. Also, it can be seen from the Table I that electric field amplitude increases with the increase in magnetic field, whereas soliton width as well as pulse duration decreases.

Our theoretical model to study the electron-acoustic solitary waves in magnetized plasma is a two-component model consisting of energetic ions and electrons. $\kappa$-distribution may be helpful to explain the solitary waves in the space plasma environment where higher values of electric field are observed. Our model requires ion temperature to be much higher than electron temperature which can be satisfied in space plasma environments such as in bow shock, plasma sheet boundary layer, magnetotail regions of the Earth's magnetosphere. Usual three-component model in magnetized plasmas is being developed.

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