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Low frequency solitons and double layers in a magnetized plasma with two temperature electrons

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Finite amplitude non-linear ion-acoustic solitary waves and double layers are studied in a magnetized plasma with cold ions fluid and two distinct groups of Boltzmann electrons, using the Sagdeev pseudo-potential technique. The conditions under which the solitary waves and double layers can exist are found both analytically and numerically. We have shown the existence of negative potential solitary waves and double layers for subsonic Mach numbers, whereas in the unmagnetized plasma they can only in the supersonic Mach number regime. For the plasma parameters in the auroral region, the electric field amplitude of the solitary structures comes out to be 49 mV/m which is in agreement of the Viking observations in this region. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4771574]

I. INTRODUCTION

Various theoretical models have been developed to study the ion-acoustic solitary waves. Lee and Kan[1] studied nonlinear low-frequency waves in a magnetized electron-ion plasma and showed that depending upon the speed of the nonlinear structure, three types of nonlinear waves, e.g., periodic ion-cyclotron, periodic ion-acoustic and ion-acoustic solitons can exist. Bharuthram and Shukla[2] discussed the dynamics and structure of multi-dimensional ion acoustic solitons and double layers in a magnetized plasma consisting of two electron species. Incorporating the departures from quasi-neutrality, they used a reductive perturbation technique to derive a general three-dimensional differential equation for the potential for small amplitude perturbations. In a later paper, Bharuthram and Shukla[3] presented the Sagdeev potential model for an unmagnetized plasma with cold ions and two distinct groups of Boltzmann-Maxwellian distributed hot electrons and obtained the conditions under which large amplitude stationary double layers can exist. Using a perturbation technique to derive a modified Korteweg-de Vries (mKdV) equation which governs the dynamics of a weak double layer, they also found that the parameter regimes for the existence of small and finite amplitude double layers are not complementary to each other. Baboolal *et al.*[4] have shown the cut-off conditions and existence domains for large-amplitude ion-acoustic solitons and double layers in plasmas consisting of the two Boltzmann electron species (hot and cool) and a single cold ion species. They found that below a temperature ratio threshold, both positive- and negative-potential solitons existed for an intermediate range of cool-electron density, with the negative potential solitons limited by double layers.

S3-3, Viking, Polar and FAST satellite observations have clearly indicated that the solitary wave structures are frequently observed along auroral field lines.[5–9] Motivated by the Viking satellite observations of the solitary structures with negative potentials, many theoretical models using multi-component plasmas have been developed to study the nonlinear ion-acoustic waves.[10–21] Berthomier *et al.*[7] studied ion acoustic solitary waves and weak double layers in two electron component unmagnetized plasma. They showed that the velocity, width, and amplitude of these structures are in agreement of the Viking observations in the auroral region. Lakhina *et al.*[22,23] studied large amplitude ion-acoustic and electron-acoustic waves in an unmagnetized multi-component plasma system consisting of cold background electrons and ions, a hot electron beam and a hot ion beam using the Sagdeev pseudo-potential technique. They found three types of solitary waves, slow ion-acoustic, ion-acoustic and electron-acoustic solitons. Lakhina *et al.*[24,25] have suggested that electron-acoustic solitons/double layers can explain the generation of magnetosheath electrostatic solitary structures and the broadband electrostatic ***noise*** in the plasma sheet boundary layer. Recently, Baluku *et al.*[26] have studied the ion acoustic solitary waves in two-electron temperatures unmagnetized plasmas. They reported finite-amplitude results and showed that positive double layers may occur for a restricted range of cool-electron densities, in addition to positive-potential solitons. They further showed that positive-potential double layers can form below a critical density ratio, associated with the third derivative of the Sagdeev potential evaluated at the origin for the phase velocity of the linear wave.

In this paper, we investigate the finite amplitude ion-acoustic solitary wave and double layer structures in a magnetized plasma consisting of ions and two-temperature electrons. Cool and hot electrons are assumed to be Boltzmann distributed and ions follow the fluid dynamic equations. This is an extension of the work of Berthomier *et al.*[7] and Baluku *et al.*[26] to magnetized plasmas. In Sec. II, we present the governing equations and obtain the localized solution of nonlinear waves using the Sagdeev pseudo-potential approach. The Soliton characteristics and numerical results are also discussed in Sec. III. Our conclusions are presented in Sec. IV.

II. GOVERNING EQUATIONS

We consider low frequency arbitrary amplitude solitons in a collisionless, magnetized plasma consisting of cold ions and two distinct group of electrons, cool electrons (N_c, T_c) and hot electrons (N_h, T_h). We assume that the plasma is embedded in a uniform external magnetic field $\mathbf{B}_o = B_o\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector along the **z**-axis. Further, we assume that the waves are propagating in the (*x, z*) plane obliquely to the magnetic field. The Boltzmann distribution of densities of the cool (N_c) and hot (N_h) electron species are given as follows:

$$N_c = N_{c0}\exp\left(\frac{e\phi}{T_c}\right), \tag{1}$$

$$N_h = N_{h0}\exp\left(\frac{e\phi}{T_h}\right), \tag{2}$$

where ϕ is the electrostatic potential and N_{c0}, N_{h0} are the equilibrium densities of the cool and hot electrons, respectively. The dynamics of the cool ions is described by the fluid equations, namely, the continuity and the momentum equations,

$$\frac{\partial N_i}{\partial t} + \nabla.(N_i\mathbf{V}_i) = 0 \tag{3}$$

and

$$\frac{\partial \mathbf{V_i}}{\partial t} + (\mathbf{V_i}.\nabla)\mathbf{V_i} = -\frac{e\nabla\phi}{m_i} + e\frac{\mathbf{V}_i \times \mathbf{B}_o}{m_i c}, \tag{4}$$

where N_i, m_i, and V_i are the number density, mass, and the fluid velocity of the ions, respectively, e is the magnitude of the electron charge, c is the speed of light in vacuum.

We next present the governing equations in normalized form: the densities are normalized with respect to the total ion equilibrium density $N_{i0} = N_{c0} + N_{h0} = N_0$, velocities by the effective ion-acoustic speed $c_s = (T_{eff}/m_i)^{1/2}$, distance by effective ion Larmor radius, $\rho_i = c_s/\Omega$, time by inverse of ion gyro-frequency Ω^{-1}, where $\Omega = eB_0/m_i c$ and potential ϕ by T_{eff}/e. Here $\tau = T_c/T_h$ is the cool to hot electron temperature ratio, $f = N_{c0}/N_0$ is cool to hot electron density ratio, $T_{eff} = T_c/(f + (1-f)\tau)$ is an effective electron temperature $\alpha_c = T_{eff}/T_c$, $\alpha_h = T_{eff}/T_h$, and $\psi = e\phi/T_{eff}$.

Thus, Eqs. (1)–(4) in normalized form can be written as

$$n_c = f\exp(\alpha_c\psi), \tag{5}$$

$$n_h = (1-f)\exp(\alpha_h\psi), \tag{6}$$

$$\frac{\partial n_i}{\partial t} + \nabla.(n_i v_i) = 0, \tag{7}$$

$$\frac{\partial v_i}{\partial t} + v_i\nabla v_i = -\nabla\psi + \mathbf{v}_i \times \hat{\mathbf{z}}. \tag{8}$$

Further, Eqs. (7) and (8) in component form can be written as

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i v_x)}{\partial x} + \frac{\partial(n_i v_z)}{\partial z} = 0, \tag{9}$$

$$\frac{\partial v_x}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_x = -\frac{\partial\psi}{\partial x} + v_y, \tag{10}$$

$$\frac{\partial v_y}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_y = -\frac{\partial\psi}{\partial y} - v_x, \tag{11}$$

$$\frac{\partial v_z}{\partial t} + \left(v_x\frac{\partial}{\partial x} + v_z\frac{\partial}{\partial z}\right)v_z = -\frac{\partial\psi}{\partial z}. \tag{12}$$

Using the transformation $\xi = (\alpha x + \gamma z - Mt)/M$, where $M = V/c_s$ is the Mach number, $\alpha = \sin\theta, \gamma = \cos\theta; \theta$ is the angle of propagation to a stationary frame, Eqs. (9)–(12) become

$$\frac{d}{d\xi}(L_v n_i) = 0, \tag{13}$$

$$L_v\frac{dv_x}{d\xi} = -\alpha\frac{d\psi}{d\xi} + Mv_y, \tag{14}$$

$$L_v\frac{dv_y}{d\xi} = -Mv_x, \tag{15}$$

$$L_v\frac{dv_z}{d\xi} = -\gamma\frac{d\psi}{d\xi}, \tag{16}$$

where $L_v = -M + \alpha v_x + \gamma v_z$. Our system of equation is closed with the quasi-neutrality condition

$$n_i = n_c + n_h = f\exp(\alpha_c\psi) + (1-f)\exp(\alpha_h\psi). \tag{17}$$

Solving coupled equations (13)–(17), and using appropriate boundary conditions for solitary wave structures (namely, $n_i \to 1$, $\psi \to 0$, and $d\psi/d\xi \to 0$ at $\xi \to \pm\infty$), and eliminating v_x, v_y, and v_z, we can reduce Eqs. (13)–(17) to an energy integral given by

$$\frac{1}{2}\left(\frac{d\psi}{d\xi}\right)^2 + V(\psi, M) = 0, \tag{18}$$

where $V(\psi, M)$ is the Sagdeev potential, given by

$$V(\psi,M) = -\frac{1}{\left[1 - M^2\left(\dfrac{\alpha_c f\exp(\alpha_c\psi) + \alpha_h(1-f)\exp(\alpha_h\psi)}{(f\exp(\alpha_c\psi) + (1-f)\exp(\alpha_h\psi))^3}\right)\right]^2}\left[-\frac{M^4}{2n_i^2}(1-n_i)^2 - M^2(1-\gamma^2)\psi\right.$$
$$+ M^2\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi) - 1) + \frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi) - 1)\right) - \frac{\gamma^2}{2}\left(\frac{f}{\alpha_c}(\exp(\alpha_c\psi) - 1) + \frac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi) - 1)\right)^2$$
$$\left. - M^2\gamma^2\left(\frac{\dfrac{f}{\alpha_c}(\exp(\alpha_c\psi) - 1) + \dfrac{(1-f)}{\alpha_h}(\exp(\alpha_h\psi) - 1)}{f\exp(\alpha_c\psi) + (1-f)\exp(\alpha_h\psi)}\right)\right]. \tag{19}$$

The ion density n_i in the above equation is given by Eq. (17). Equation (18) can be regarded as an "energy integral" of an oscillating particle of unit mass, with the velocity $d\psi/d\xi$ and the position ψ in a potential $V(\psi, M)$. We now look for the solitary wave solutions of Eq. (18).

III. SOLITON AND DOUBLE LAYER CHARACTERISTICS

In order to obtain soliton solution of Eq. (19), Sagdeev potential $V(\psi,M)$ must satisfy the following conditions: $V(\psi,M)=0, \mathrm{d}V(\psi,M)/\mathrm{d}\psi=0, \mathrm{d}^2V(\psi,M)/\mathrm{d}\psi^2<0$ at $\psi=0$; $V(\psi,M)=0$ at $\psi=\psi_m$, and $V(\psi,M)<0$ for $0<|\psi|<|\psi_m|$. For double layer solutions, an additional condition $\mathrm{d}V(\psi,M)/\mathrm{d}\psi=0$ at $\psi=\psi_m$ (ψ_m is maximum amplitude) should be satisfied.

The soliton condition $\mathrm{d}^2V(\psi,M)/\mathrm{d}\psi^2<0$ at $\psi=0$ can be written as

$$\left.\frac{d^2V(\psi,M)}{d\psi^2}\right|_{\psi=0} = \frac{M^2-M_0^2}{M^2-M_1^2} < 0, \tag{20}$$

where

$$M_0^2 = \frac{\gamma^2}{f\alpha_c + \alpha_h(1-f)} = \gamma^2 \tag{21}$$

is the critical Mach number and

$$M_1^2 = \frac{1}{f\alpha_c + \alpha_h(1-f)} = 1, \tag{22}$$

since $f\alpha_c + \alpha_h(1-f) = 1$. Further analysis of Eq. (20) reveals that numerator $M^2 - M_0^2$ is always positive as for soliton solution to exist one must have $M > M_0$. Hence, in order to satisfy Eq. (20) denominator $M^2 - M_1^2 < 0$, i.e., $M < M_1$. Therefore, M_1 is the upper limit of the Mach number beyond which soliton solutions will not exist. Thus, the ion-acoustic soliton solution in the magnetized plasma case may exist in the interval

$$M_0 < |M| < M_1. \tag{23}$$

From Eq. (23), we obtain a condition

$$\gamma < |M| < 1 \tag{24}$$

which gives allowed values of Mach number M for an angle of propagation of solitary waves for fixed values of f, α_c, and α_h. It is interesting to point out that in the magnetized plasma case, the ion-acoustic solitons and double layer can exists only in the subsonic Mach number region as seen from Eq. (24). On the other hand, for the case of unmagnetized plasma consisting of cold ions and two-temperature Boltzmann electrons, ion-acoustic solitons and double layer can exist only in the supersonic Mach number regime,[7] i.e., $M > 1$.

Next, we numerically examine Eq. (18) and Sagdeev potential $V(\psi,M)$ given by Eq. (19) for different parameters such as M, f, θ, and τ.

The Figure 1 shows the Sagdeev potential $V(\psi,M)$ vs real potential ψ for different values of M for other fixed parameters namely, cool to hot electron temperature ratio, $\tau = T_c/T_h = 0.04$, cool electron number density, $f = N_{c0}/N_0 = 0.1$ and angle of propagation, $\theta = 15°$. The ion-acoustic soliton amplitude, ψ increases with increasing M. Further, soliton solutions are not found beyond $M > 0.9994$. This very well conforms to the upper Mach number limit $M < 1$ obtained analytically (cf. Eq. (24)). Figure 2 shows the variation of Sagdeev potential versus real potential ψ for different cool electron number density shown in the figure. The other fixed parameters are $\tau = 0.04$, $\theta = 15°$, and $M = 0.98$. It is observed that as cool electron density increases, the soliton amplitude increases and numerical computations show that the soliton solutions are not possible beyond $f > 0.35$.

The curves in Figure 3 show that as the angle of propagation θ increases (obliquity increases), the soliton amplitude increases. The chosen parameters are, cool electron density, f = 0.1 and other fixed parameters of Figure 2. It is

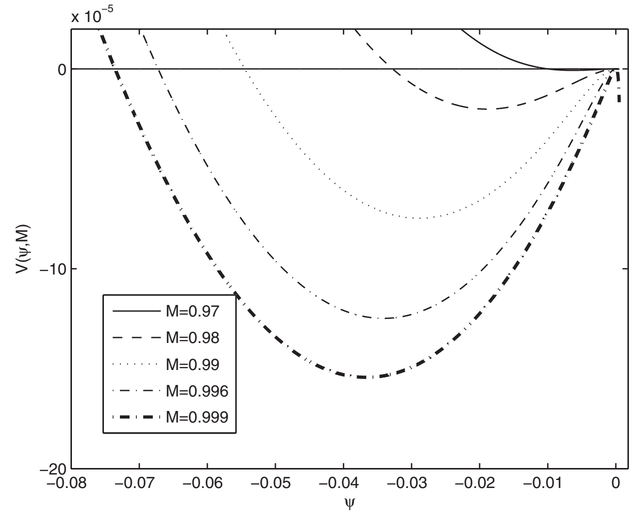

FIG. 1. The behavior of the Sagdeev potential $V(\psi,M)$ versus ψ for parameters, $\tau = 0.04, f = 0.1, \theta = 15°, M = 0.97, 0.98, 0.99, 0.996$, and 0.999.

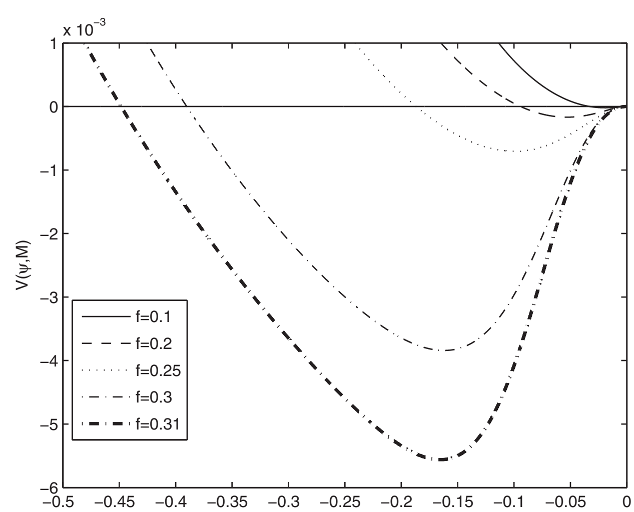

FIG. 2. Variation of Sagdeev potential $V(\psi,M)$ versus ψ for parameters, $\tau = 0.04, \theta = 15°, M = 0.98, f = 0.1, 0.2, 0.25, 0.3$, and 0.31.

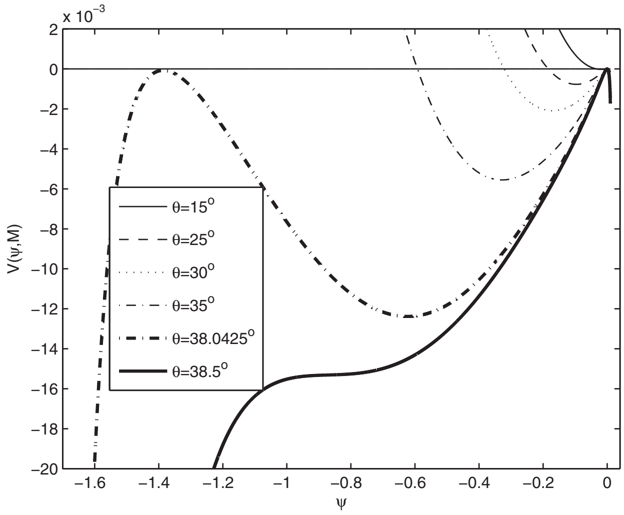

FIG. 3. Variation of Sagdeev potential $V(\psi, M)$ versus ψ for parameters, $\tau = 0.04$, $f = 0.1$, $M = 0.98$, $\theta = 15^\circ$, 30°, 35°, 37.5°, 38.0425° (double layer) and 38.5°.

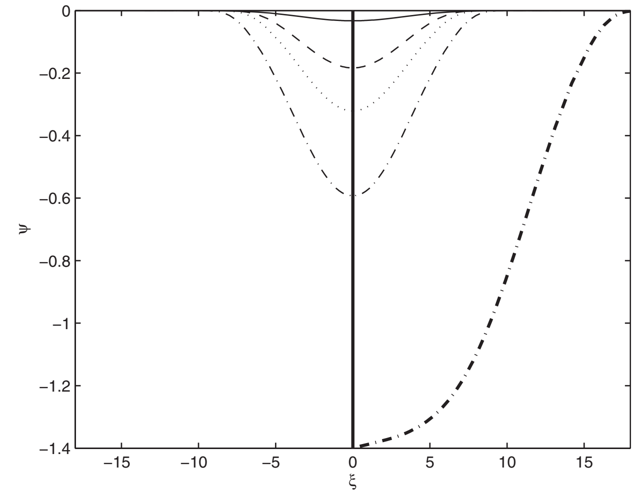

FIG. 5. Nonlinear wave structures: Variation of electrostatic potential ψ vs ξ for $\tau = 0.04$, $f = 0.1$, $M = 0.98$, $\theta = 15^\circ$ (—), $\theta = 25^\circ$ (- - -), $\theta = 30^\circ$ (…), $\theta = 35^\circ$ (.-), and $\theta = 38.0425^\circ$ (*-) (double layer)

interesting to note that at $\theta = 38.0425^\circ$ double layer appears. For $\theta > 38.0425^\circ$ there is no soliton or double layer solutions. Figure 4 shows the variation of the Sagdeev potential with real potential for different values of the cool to hot electron temperature ratio $\tau = T_c/T_h$ for $\theta = 15^\circ$. Other fixed parameters are the same as in Figure 3. The soliton amplitude increases with the increase in cool to hot electron temperature ratio. It must be pointed out that in this case also double layer solution has been found to exist for cool to hot electron temperature ratio, $\tau = 0.0877117$. Figure 5 shows the potential ψ against ξ which has been obtained numerically by integrating Eq. (18) for the parameters $\tau = 0.04$, $f = 0.1$ and $M = 0.98$ for different values of θ. It is very obvious from Figure 5 that as we increase the propagation angle θ, the amplitude as well as width of the soliton increases.

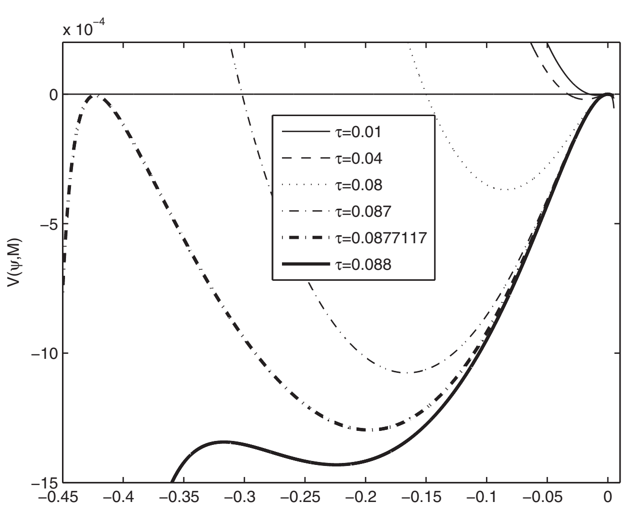

FIG. 4. Variation of Sagdeev potential $V(\psi, M)$ versus ψ for parameters, $\theta = 15^\circ$, $f = 0.1$, $M = 0.98$, $\tau = 0.01$, 0.04, 0.08, 0.087, 0.0877117 (double layer) and 0.088.

IV. CONCLUSION

We have investigated in detail nonlinear ion-acoustic solitary waves and double layers in magnetized plasma with two temperature electrons and ions. The model supports the negative potential ion-acoustic solitons and double layers, and they are found to ***exist*** only in the subsonic (i.e., $M < 1$) Mach numbers regime. In contrast, for the case of unmagnetized plasma these negative potential nonlinear structures can exist only in the supersonic (i.e., $M > 1$) Mach number regime. We have shown that amplitude of the ion-acoustic solitary waves increases with Mach number, increased obliquity and cool electron density. Here, we apply our results to the negative potential ion-acoustic solitary waves observed by the Viking satellite in the auroral region of the Earth's magnetosphere. Berthomier *et al.*[7] have reported ion-acoustic solitary structure in the auroral region with electric field amplitude of less than 100 mV/m, width of about 100 m, pulse duration of about 20 ms and soliton velocities in the range of $\approx 10 - 50$ km/s. For illustrative purpose, we have taken following parameters from the Viking observations,[7] namely, $n_c = 0.2\,\mathrm{cm}^{-3}$, $n_h = 1.8\,\mathrm{cm}^{-3}, T_c = 1\mathrm{eV}, T_h = 26\,\mathrm{eV}$ which gives $T_{eff} \approx 7eV$. The maximum electric field for $M = 0.98, \theta = 35^\circ$ comes out to be 49 mV/m and corresponding soliton width, pulse duration and speed comes out to be $\approx$270 m, 10 ms and 26 km/s, respectively. Thus, our results are in good agreement of the Viking observations.

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